Influence of swing leg movement on running stability

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Abstract

The aim of this study was to investigate the role of the swing leg movement on running stability. A simple model was used describing a forward hopping motion. The model consisted of two sub-models, namely a spring-mass system for the stance phase and a functional control model for the swing phase (represented by a passive or actively driven pendulum). To verify the main simulation results, an experimental study on treadmill running was performed. The results of the model indicated that for certain running speeds and pendulum lengths, the behavior of the mechanical system was stable. The following characteristic dependencies between the model parameters were observed. (1) Pendulum length and hip muscle activity determined running height and therefore swing duration. (2) Horizontal velocity was inversely related to leg angle of attack. Increased speed corresponded to flatter leg angles at touch-down, which is in agreement with experimental studies and previous predictions of spring-mass running. It was shown that a biologically motivated control approach with oscillating leg movements is well capable of generating stable hopping movements. Due to its simplicity, however, the monopedal model failed to explain more detailed mechanisms like the swing-leg to stance-leg interaction or the functional role of the leg segmentation. This simple model is therefore considered as a functional mechanical template for legged locomotion, which could help to build more elaborate models in the future.

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1. Introduction

Running is the preferred way of humans for locomoting at higher speeds. Considering only a single leg, the gait cycle consists of a stance and a swing phase. Blickhan (1989) proposed a planar spring-mass model to describe the stance phase of running. The simulation and study of such dynamic models is useful to investigate functional aspects of human locomotion.

Since contact times are very short when running fast, it is important to know which mechanisms could provide the required movement stability. A number of studies investigating selected aspects of running, like the influence of kinematic conditions on the impact generation in heel-toe running (Gerritsen, Van den Bogert, & Nigg, 1995). Interestingly, only few models have been published that suggest strategies for achieving stable running or hopping movements (Herr, Huang, & McMahon, 2002; Herr & McMahon, 2001; McGeer, 1990; Seyfarth, Geyer, Gunther, & Blickhan, 2002; Seyfarth, Geyer, & Herr, 2003). In a simulation study of continuous running, Seyfarth et al. (2002) demonstrated that even a simple ‘fixed angle of attack’ strategy would be sufficient to obtain self-stabilizing running patterns, provided that an elastic leg operation is present. This corresponded nicely to the great success of the hopping robots of Raibert and coworkers (1986), which controlled the leg angle during swing depending on forward speed. In simulation studies by Herr (Herr et al., 2002; Herr & McMahon, 2001) and Seyfarth (Seyfarth & Geyer, 2002; Seyfarth et al., 2003), it was further demonstrated that the backward rotation of the swing leg prior to landing, called leg retraction, is a powerful strategy to further enhance running stability. McGeer (1990) presented results of a passive bipedal running machine, where an appropriate choice of design parameters enabled the machine to run without any active control. To move the legs backwards and forward, a torsional spring was added to the hip joint resulting in an elastically enforced pendulum motion during swing phase.

The control of the swing leg seems to be crucial for the appropriate placement of the swing leg at touch-down, which influences running stability. The question remains how the observed swing leg motion during running can be explained. As suggested in the literature, the ‘swing phase of human gait may be described as a ballistic motion’ (Mochon & McMahon, 1980). It was concluded that it is feasible to compare swing leg movement to a pendulum-like motion.

The aim of this work was to investigate the influence of a pendulum-like swing leg movement on the stability of spring-mass running.

2. Model

To keep the model simple, only one leg was used to describe forward hopping movements. This situation is similar to kangaroo hopping, where both legs are working in parallel. For systematic reasons, it was decided to exclude the mechanical interplay of the stance and swing leg dynamics at this stage of modelling. In the future, this might help to clarify the importance of two-legged systems in nature.

The monopedal running model of this investigation consisted of two sub-models. (1) The spring-mass model (Fig. 1a) described the movement of the center of mass (COM) during the stance phase, while (2) the pendulum model (Fig. 1b) simulated the leg orientation during the swing phase. In contrast to previous studies, the movement of the swing leg was not...
prescribed but represented a simple mechanical system with its own natural behavior. The initial conditions for the pendulum were derived from the leg configuration at the end of the stance phase. Since the body was reduced to a point mass during the stance and flight phase, the pendulum sub-model merely acted as a leg angle controller and, hence, had no physical representation. In a physical sense, the pivoting point of the pendulum was thought to be fixed, whereas the pendulum motion started at take-off. Therefore, the pendulum sub-model represented just an abstract control model that described the movement of the swing leg (backward and forward swing – retraction and protraction, respectively). Although this sub-model has no physical representation within the spring-mass model, it is thought to help to improve the understanding of the swing leg function.

In the following, the equations of motion for the two sub-models as well as simulation details are presented.

2.1. Stance phase

A simple mechanical spring represented the action of the stance leg. During the first half of the stance phase, the leg spring (stiffness $k$, rest length $l_0$) was compressed and stored energy, which was released in the second half of the stance phase resulting in an acceleration of the COM counteracting the permanent influence of gravity (Fig. 1a). The acceleration of the mass in horizontal and vertical direction was given by Eqs. (1) and (2) with body mass $m$, gravity $g = 9.81 \text{ m/s}^2$ and $\alpha_{\text{leg}}$ depicting the leg angle measured with respect to the ground:

$$a_x = -\frac{k \cdot \Delta l}{m} \cdot \cos(\alpha_{\text{leg}}),$$

$$a_y = \frac{k \cdot \Delta l}{m} \cdot \sin(\alpha_{\text{leg}}) - g. \quad (1)$$

Here, $\Delta l$ denoted the amount of leg shortening $\Delta l(t) = l_0 - l(t)$. To minimize the notation, time dependent variables as $\Delta l(t)$ or $v(t)$ are abbreviated to $\Delta l$ and $v$.

The mechanical energy of the system was given by the sum of kinematic, potential and spring energy ($E = 0.5mv^2 + mgv + 0.5k\Delta l^2$) with $v$ describing the vertical position of the
COM relative to the ground. During the flight phase, no elastic energy was stored, resulting in a simple dependency between apex height $\Delta l$ and forward speed $v_x$ (Eq. (3)), with the apex height representing the highest point in flight ($v_y = 0$ m/s).

$$E = 0.5mv_x^2 + mg y_{\text{apex}}.$$  \hfill (3)

### 2.2. Swing phase

A mathematical pendulum of length $l_p$ and mass $m_{\text{leg}}$ described the change in leg angle $\varphi$ (which was defined with respect to the vertical axis in counterclockwise direction) during the swing phase (Fig. 1b). The equation of motion for the mathematical pendulum (Eq. (5)) was derived from Eq. (4), with $P$ representing the angular momentum and $\sum M_i$ the sum of acting torques.

$$\frac{d}{dt} P = \sum_i M_i,$$  \hfill (4)

$$\ddot{\varphi} = -\frac{g}{l_p} \sin \varphi.$$  \hfill (5)

The calculated rotation of the swing leg had no effect on the trajectory of the COM, which followed a ballistic curve. Furthermore, there was no influence of the COM movement on the pendulum, i.e., the pivoting point of the pendulum was fixed. As an extension of the simple mathematical pendulum, an actively driven pendulum was considered by introducing a rotational spring at the pivoting point of the pendulum, representing hip muscle activity. The rotational spring exerted a torque $M_S$ with a rotational stiffness $c$ and rest position $\varphi_0$ (Eq. (6)).

$$M_S = c(\varphi_0 - \varphi).$$  \hfill (6)

By applying Eq. (4), the swing dynamics for the actively driven pendulum were given by Eq. (7).

$$\ddot{\varphi} = -\frac{g}{l_p} \sin \varphi + \frac{c(\varphi_0 - \varphi)}{m_{\text{leg}} l_p^2}.$$  \hfill (7)

The parameters of the spring-mass model were set similar to those used by Seyfarth et al. (2002): $l_0 = 1$ m, $m = 80$ kg, $k = 20$ kN/m, $\varphi_0 = 0^\circ$ and $m_{\text{leg}} = 0.16 \cdot m = 12.8$ kg. The leg mass was derived from anthropometric data (Winter, 1979).

### 2.3. Sub-models interaction

The two sub-models spring-mass model and pendulum model interacted only at two instances within the gait cycle. (1) At take-off (TO), the pendulum was initiated by the angular configuration (angle, angular velocity) of the leg spring. TO took place when the actual leg length $l(t)$ exceeded the rest length of the leg spring $l_0$, i.e., when $l(t) > l_0$. (2) At touch-down, the swing leg angle determined the initial orientation of the leg spring (angle of attack $z_{\text{TD}}$). Touch-down (TD) occurred when $y \leq l_0 \sin(z_{\text{leg}})$. 
2.4. Initial conditions

The simulation of the model started at take-off. The initial leg angle at take-off ($\alpha_{TO,0}$) and the initial horizontal speed ($v_{x,0}$) had to be set very accurately, as they determined whether stability could be reached or not. These parameters completely determined the COM conditions and the initial conditions of the swing leg pendulum (angle, angular velocity). Stability was reached with $\alpha_{TO,0} = 96^\circ$ and $v_{x,0} = 1.5$ m/s.

2.5. Stability

For a given model configuration (system energy, pendulum length), running stability was defined by the number of successful steps predicted by the model. The simulations stopped if non-physiological conditions (e.g., $y < 0$ or $\alpha_{TD} > 90^\circ$) were detected or – for practical reasons – after executing 50 steps.

2.6. Implementation

The model was implemented in Simulink (The Mathwork, Inc.) and consisted of two sub-models: the spring-mass model and the pendulum model. In the present simulation, the ode45-integrator (Dormand-Prince) was used with a maximum step size of $0.001$ s and a relative and absolute tolerance of each $10^{-8}$. The integrator ode45 proved to be fast and accurate (energy fluctuations smaller than $0.01\%$) compared to other solvers provided by Simulink.

3. Simulation results

3.1. Predicted leg angle kinematics

The time course of the leg angle during the gait cycle is shown in Fig. 2. The angle swept during stance was nearly a linear function of time, while the touch-down and take-off angles were symmetric with respect to the vertical axis ($\alpha = 90^\circ$).

![Fig. 2. Predicted leg angle tracings $\alpha_{leg}(t)$ during stance and swing ($M_S = 0$). The transition between stance and swing (TO) is indicated by dots. (a) The angle swept during stance is nearly a linear function of time, whereas swing phase can be divided into three sections: take-off (TO) retraction (A), protraction (B) and landing retraction (C). (b) Leg angles characteristics is influenced by speed ($v_x = 0.6, 1.3, 1.7, 2.1$ and $2.4$ m/s) as indicated by the varying angular range of motion.](image-url)
The transition of the leg angle trajectory from stance to swing phase was continuous in value (i.e. leg angle) and in its first derivation (i.e. leg angular velocity). The tracing of the leg angle during swing could be subdivided into three parts: take-off retraction (A), protraction (B) and landing retraction (C). TO retraction was due to negative initial angular velocity of the pendulum at take-off. After reaching the maximal posterior position, the pendulum swung forward (protraction) in preparation of the next ground contact. Just prior to touch-down, the pendulum was starting to swing again backward (landing retraction).

Increasing horizontal velocity at constant pendulum length increased the leg angle swept during stance, whereas stance and swing durations were not affected much by speed (Fig. 2b). Higher velocities influenced pendulum dynamics by increasing the angular range of TO retraction, faster protraction and increased landing retraction. All the predicted leg angle trajectories had two common intersection points with \( \alpha = 90^\circ \): one at mid-stance and the other at mid-swing.

3.2. Model with pendulum-like swing leg (\( M_S = 0 \))

For a range of system energies \( E \) and pendulum lengths \( l_P \), stable movement patterns were predicted by the model (Fig. 3). Three types of stable solutions were present: hopping on place, fixed point stability and bi-stability.

For a given pendulum length, stable solutions were present within a range of energies. Let us consider the condition \( l_P = 0.2 \) m in Fig. 3. At low energies, bouncing in place (with \( \alpha_{TD} = \alpha_{TO} = 90^\circ \) and \( v_x = 0 \) m/s) was predicted. With higher system energies stable running patterns with nonzero horizontal speeds were observed. At highest energies, the solutions became bistable with the characteristic system state parameters (\( y_{\text{apex}}, v_x, \alpha_{TD} \)) alternating step-by-step. In the following, only fixed point stability is considered.

The values of the system’s state parameters were dependent on system energy and pendulum length (Fig. 4). Apex height was merely influenced by pendulum length but not by energy (Fig. 4a). In contrast, horizontal speed and angle of attack both depended on system energy and pendulum length. High energies and short pendulum lengths corresponded to high velocities (\( v_x \), Fig. 4b) and flat angles of attack (\( \alpha_{TD} \), Fig. 4c).

![Diagram](image.png)

Fig. 3. The stability region (number of steps > 50, \( M_S = 0 \)) is divided into three areas of different movement behaviors (light-gray: fixed point stability, gray: bi-stability, black: hopping on place).
Fig. 5 further illustrates the dependencies between the model parameters. The apex height was directly proportional to pendulum height independent of system energy (Fig. 5a), i.e., the longer the pendulum the higher the flight trajectory. The relation between horizontal speed and angle of attack was inversely proportional and almost independent of pendulum length (Fig. 5b).

3.3. Model with elastically enforced pendulum

The model was further used to study the influence of hip muscle activity on running stability. This was simulated by introducing a linear rotational spring of stiffness $c$ at the pivoting point of the pendulum. It was found that apex height was reduced with increasing stiffness, resulting in shorter swing durations. Furthermore, longer pendulum lengths could be stabilized. The reciprocal relationship between angle of attack and forward speed (compare to Fig. 5b) was found to be independent of the rotational stiffness.

4. Experimental method

To verify the main simulation results, an experimental study on treadmill running (5 male and 4 female participants, age $28 \pm 9$ years, body mass $65 \pm 6$ kg, body height
1.73 ± 0.09 m) was performed. Leg kinematics (Qualisys), ground reaction forces (GRF) and electromyogram (EMG) of major leg muscles were measured. Sampling frequency was set to 150 Hz for kinematics and 2000 Hz for GRF and EMG. Treadmill speed ranged from 0.6 to 3.6 m/s with speed increments of 0.3 m/s. All participants were informed about the experimental protocol and signed a declaration approved by the local ethics committee.

The instances of touch-down and take-off were detected based on the low-pass filtered force data (cutoff frequency 30 Hz, first order digital Butterworth filter). The leg axis was defined based on hip, ankle and toe markers, resulting in the leg length (distance from hip to the middle of ankle and toe) and leg angle (orientation of the connecting line relative to the ground).

5. Experimental results

The results of the experimental study showed that leg kinematics was influenced by speed (Figs. 6 and 7). With increasing speed, flatter angles of attack (Fig. 6a) were

![Fig. 6. Experimental data of (a) leg angle $\alpha_{\text{leg}}$ at touch-down (solid line) and take-off (dotted line) at different running speeds. Angle of attack $\alpha_{\text{TD}}$ is decreased with increasing speed. (b) Stance $t_{\text{stance}}$ (solid line) and swing duration $t_{\text{swing}}$ (dotted line) at different speeds. Mean values (circles) and standard deviations (bars) of nine subjects.](image)

![Fig. 7. (a) Leg angle tracings $\alpha_{\text{leg}}$ during stance and swing of one subject at six different running speeds ($v_x = 0.6, 1.2, 1.8, 2.4, 3$ and $3.6$ m/s). Touch-down (TD) and the transition between stance and swing (TO) are indicated by bold and fine circles, respectively. Touch-down and take-off of the contralateral leg (TDc and TOc) are marked by bold and fine squares, respectively. (b) The angular velocity of the leg for a particular horizontal speed (2.4 m/s) is shown. See text for details.](image)
observed, whereas touch-down and take-off leg angles were not symmetrical with respect to the vertical axis (shifted forward by about 4°). Swing duration (≈0.5 s) was hardly affected by speed, whereas stance time clearly decreased with speed (Fig. 6b).

In Fig. 7a, leg angle tracings are shown for one participant. Higher velocities corresponded to smaller leg angles at touch-down. Increasing speed resulted in larger TO retraction and faster protraction. Furthermore, landing retraction was slightly increased with speed. The instance of touch-down and take-off of the contralateral leg (TDc and TOc) are indicated. In Fig. 7b, the angular velocity of the leg for one horizontal speed ($v_x = 2.4$ m/s) is shown. Between TO and TDc angular velocity was nearly kept constant. The onset of swing leg protraction occurred shortly after TDc.

6. Discussion

The basic behavior of legs during locomotion could be described in terms of two properties: (1) the spring-like force development during leg compression in the stance phase and (2) the oscillation of the legs around the hip including retraction during stance and protraction during swing. So far, many studies looked at the mechanisms and consequences of spring-like leg operation during stance (Arampatzis, Bruggemann, & Metzler, 1999; Bullimore & Burn, in press; Dutto & Smith, 2002; Farley, Glasheen, & McMahon, 1993; Geyer, Seyfarth, & Blickhan, 2005; Seyfarth et al., 2002). Only little is known about strategies for achieving stable running movements and how the leg operation during the stance phase could be controlled using an appropriate leg control strategy during the swing phase. Therefore, it was asked whether a pendulum-like swing leg movement would suffice to achieve stable running movements. The aim was to identify and describe the function and influence of selected mechanical control parameters (e.g., swing leg length) on running stability. The model behavior was a result of the dynamics of two sub-models (spring-mass model, pendulum model), which interacted only at the transitions between stance and flight phases. First, a spring-mass model with ballistic flight phases was used during stance. Second, the movement of a mathematical pendulum prescribed the swing leg kinematics during flight resulting in the leg angle at touch-down. This template-based swing-leg control model (called pendulum model) was influenced by the dynamics of the stance leg and the effective length of the pendulum (representing leg flexion) and is, therefore, directly involved into the mechanics of running. The monopedal model described forward hopping, which is similar to kangaroo hopping.

6.1. Interpretation of simulation results

Stable running patterns were found provided that model parameters were carefully chosen. A distinct relation between different parameters was observed, in which pendulum length and horizontal speed played a crucial role.

Pendulum length determined apex height (Fig. 5a). The longer the pendulum, the larger the observed hopping amplitude independent of system energy (i.e., independent of speed). Hence, swing duration increased with pendulum length. The discovered correlation between pendulum length and swing duration is in agreement with the characteristics of a mathematical pendulum. In approximation, the time period of a mathematical pendulum for small amplitudes is given by $2\pi\sqrt{L/g}$. Furthermore, swing duration was influenced by hip muscle activity. High rotational hip stiffness decreased apex height and consequently
swing duration. For stable locomotion, the appropriate placement of the swing leg at touch-down is important; the cadence of the swing leg movement (here represented by the pendulum) takes on an essential role. A similar observation was made in a robotic study, where running stability was affected most by the ‘scissor frequency’ (McGeer, 1990).

There was also an inverse relationship between running speed and leg angle of attack (Fig. 5b). The faster the predicted running speed, the flatter the angle of attack. This result is in agreement with experimental studies (Fig. 6a), with the mechanical walking model of Garcia and with previous predictions of spring-mass running (Garcia, Chatterjee, Ruina, & Coleman, 1998; Seyfarth et al., 2002), where a similar relation between speed and angle of attack was found. Furthermore, the horizontal speed seemed to determine the angular range of the swing leg in humans (Fig. 7a) and of the pendulum of the model (Fig. 2b). The faster the model moved, the higher the angular range of the pendulum. A mathematical explanation for the discovered behavior is proposed: the movement of a mathematical pendulum can be described by a sine function with maximum amplitude \( \varphi_{\text{max}} \) and frequency \( \omega \), whereas \( \varphi_{\text{max}} \) is related to \( \dot{\varphi}_{\text{max}}/\omega \). Motivated by the results of the simulation, coupling of swing and stance phases resembles the motion of a mathematical pendulum (Fig. 2). The time course of the leg angle during stance merged into the cyclical pendulum motion, which may result from the interaction of stance and swing phases. As the model showed symmetrical behavior in respect of the vertical axis, the maximum angular velocity \( \dot{\varphi}_{\text{max}} \) can be derived from midstance and is defined as \( v_x/l_{\text{ms}} \), with \( l_{\text{ms}} \) denoting the leg length at midstance. The maximum amplitude is given by Eq. (8).

\[
\varphi_{\text{max}} = \frac{v_x}{l_{\text{ms}}} \sqrt{l_P/g}, \quad \text{with} \quad \omega = \sqrt{g/l_P}.
\]

For a given leg compression and pendulum length, the maximum amplitude of the pendulum increases proportionally to the horizontal velocity. This dependence of forward speed and range of motion of the swing leg in the inactive pendulum model as well as in human running supports the idea of the passive swing leg motion.

6.2. Proposition for economic running strategies

In considering the results of the proposed model, one may ask whether economic running strategies can be identified. During flight, system energy was distributed in terms of apex height and horizontal speed (Eq. (3)). The higher the hopping amplitude, the slower the forward movement: in the model, short pendulum lengths corresponded to low apex heights. Translated into human running, short swing leg lengths may be advantageous for increasing performance as more energy is converted into forward than vertical motion. However, shortening of the swing leg in humans is generally associated with muscle work. The question is to what extent passive mechanisms contribute to the observed swing leg kinematics in humans. Mochon and McMahon (1980) found that the swing phase in walking may function without any muscle action – except that at take-off the initial conditions (positions and velocities of the lower limb) had to be set. One may further speculate that the segmentation of the leg (thigh, shank and foot) and the elasticity of hip muscles contribute to passive leg shortening. Moreover, the active plantar flexion at the ankle joint during the push-off phase may support leg flexion. Therefore, it is proposed that performance increases by shortening the swing leg.
6.3. Application in rehabilitation and robotics

Our objective was to better understand the interaction of the mechanical parameters between stance and swing phase. Such a functional understanding is of importance for many applications as in therapy, diagnostics and robotics. In a recently developed robotic gait orthosis, paraplegic patients are walking on a treadmill with induced leg movements (Colombo, Wirz, & Dietz, 2001). Using such a locomotor training many incomplete paraplegic patients are able to regain locomotor function. Replacing stereotypical kinematic leg control during swing with a more functional leg program could have an impact on the therapeutical effect of the locomotor therapy.

The pendulum-like swing-leg control of an elastically operating leg is a technically and biologically reasonable approach to relax the control effort for locomotion. Sinusoidal patterns at the hip joint can be found in human running and walking as well as in simple hopping robots (Seyfarth, Lipfert, & Rummel, 2005). Interestingly, this harmonic oscillatory movement of the hip joint does not imply a sinusoidal movement of the foot with respect to the body. The gait specific knee flexion has an important influence on the foot trajectory (Seyfarth, 2005). However, this cannot be discussed in the framework of the spring-mass model. To this aim, a more precise representation of the leg geometry would be required.

Many simple hopping robots used fixed landing limb angles (Raibert, 1986). In the hexapod robot RHEx a continuous leg rotation with no leg protraction was implemented (Saranli, Buehler, & Koditschek, 2001). In the simulation presented here it was demonstrated that a biologically motivated control approach with oscillating leg movements was capable of generating stable hopping movements. This represents in a simplified way the action of neural pattern generators (Grillner & Wallen, 1985). In contrast to a neural pattern generator, however, the pendulum dynamics was used here as a control template generating the required oscillating leg kinematics.

6.4. Limitations and prospect

Two fundamental restrictions of the model were observed. Firstly, running was stable at speeds only up to 2.5 m/s. This limitation could not be resolved by adding a rotational stiffness or by implementing a variable pendulum length (unpublished results). Secondly, the model predicted constant stance times independent of speed. This is due to the inverse relationship between speed and angle of attack. However, this is not in agreement with spring-mass running (Seyfarth et al., 2002) and experimental studies (Fig. 6b), where stance time decreases with speed. This discrepancy might be due to the characteristics of the proposed pendulum model, which restricted – as a result of the additional pendulum condition – the solution space of stable spring-mass running (Seyfarth et al., 2002).

It is clear that the present model is far too simple to describe the real swing leg movement, which is largely influenced by intra-body dynamics and muscular activity influencing the leg movement. However, the analysis of simple models will help us to better understand more complicated models. In the future, a more detailed representation of human running will be pursued by including a second leg. The initiation of the pendulum during swing may then be triggered by the touch-down of the opposite leg (Fig. 7). Such bipedal models will provide a framework for investigating the interactions between stance and swing leg in both running and walking.
References


