Modeling Rate-dependent Hysteresis in Piezoelectric Actuator

Wei Tech Ang, Francisco Alija Garmón, Pradeep K. Khosla, and Cameron N. Riviere
The Robotics Institute, Carnegie Mellon University, Pittsburgh, PA 15213, USA
techang@cs.cmu.edu, alija@andrew.cmu.edu, pkk@ece.cmu.edu, and cam.riviere@cs.cmu.edu

Abstract – **Hysteresis of a piezoelectric actuator is rate-dependent. Most hysteresis models are based on elementary rate-independent operators and are not suitable for modeling actuator behavior across a wide frequency band. This work proposes a rate-dependent modified Prandtl-Ishlinskii (PI) operator to account for the hysteresis of a piezoelectric actuator at varying frequency. We have shown experimentally that the relationship between the slope of the hysteretic loading curve and the rate of control input can be modeled by a linear function. The proposed rate-dependent hysteresis model is implemented for open-loop control of a piezoelectric actuator. In experiments tracking multi-frequency nonstationary motion profiles, it consistently outperforms its rate-independent counterpart by a factor of two in maximum error and a factor of three in rms error.**

I. Introduction

A piezoelectric ceramic is an excellent choice as a micropositioning actuator because of its high output force, large bandwidth and fast response time. However, the existence of nonlinear multi-path hysteresis in piezoelectric material complicates the control of a piezoelectric actuator in high precision applications. The maximum hysteretic error is typically about 15% in static positioning applications. Still worse, the hysteresis is rate-dependent, increasing with the rate of control input, as shown in Fig. 1.

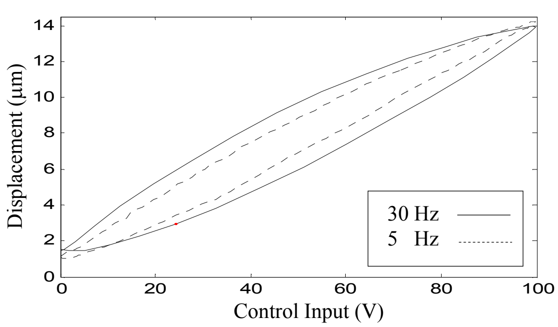


Fig.1. Hysteresis of piezoelectric material is rate-dependent. The plots show the response of a TS18-H5-104 (Piezo Systems, Inc.) piezoelectric actuator at two different driving frequencies.

While the formation theory of hysteresis [1] and its complex looping behavior [2] have been well documented, literature on piezoelectric hysteresis rate dependence is scarce. Lacking a more profound understanding of the underlying physics, any attempt to account for the hysteresis rate dependence has to be phenomenological. Current research in hysteresis modeling and compensation can be broadly classified into three categories: (I) electric charge control; (II) closed-loop displacement control; (III) linear control with feedforward inverse hysteresis model.

The first category exploits the fact that the relationship between the deformation of a piezoceramic and the induced charge has significantly less hysteresis than that between deformation and applied voltage [3, 4]. However, this approach requires specialized equipment to measure and amplify the induced charge and will inevitably reduce the responsiveness of the actuator. There has been little or no discussion of the effectiveness of this method in tracking more complex, nonstationary motion profiles, where the hysteretic rate dependence comes into play.

Most commercial systems (e.g. Polytec PI, Inc., Dynamic Structures and Materials, LLC., Melles Griot, Inc., Michigan Aerospace Corp. etc.) fall into the second category, normally using strain gauges as the feedback sensors. These systems can achieve sub-micron and even nano-level positioning precision but are generally only suitable in static positioning applications. Among the closed-loop schemes capable of tracking control, a few use different varieties of linear control schemes after linearizing the hysteretic nonlinearity [5, 6]; Tao *et al.* [7] uses adaptive control with an approximate model of the hysteresis; others propose using a neural network to learn the nonlinearity [8] or a combination of neural network with adaptive control [9].

The main idea of category III is to obtain a mathematical model that closely describes the complex hysteretic behavior, then to feed forward the inverse model to linearize the actuator response (see Fig.2).

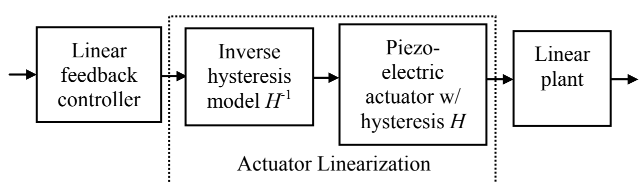


Fig.2. Piezoelectric actuator linearization with feedforward inverse hysteresis model.

Among the proposed hysteresis models, e.g. the generalized Maxwell's slip model [10], polynomial approximation [11], etc., the Preisach model [12] and its variations are by far the most well known and widely used in both closed-loop and open-loop systems. Ge and Jouaneh [13] combine the feedforward model with a PID feedback loop and their implementation attains a maximum error of about 1.5–3.5% in tracking 13 μm p-p stationary sinusoids from 0.1–20 Hz. However, this method does not work for nonstationary sinusoids because of the intrinsic properties of the classical Preisach model. Galinaitis [14] uses a Preisach type Kransnosel'skii-Pokrovskii (KP) operator to model and control piezoelectric actuators in open loop and reports a maximum tracking error of 3.9% with a 0.05 Hz stationary sinusoid. The tracking errors increase to about 6–9.4% when tracking sinusoids with reduced amplitudes.

Another important subclass of the Preisach model is the Prandtl-Ishlinskii (PI) model. The main advantages of this approach over the classical Preisach operator are that it is less complex and its inverse can be computed analytically, thus making it more attractive for real-time applications. These PI approaches reduce maximum hysteretic error to about 1–3% in open-loop control with quasi-static tracking [15, 16] and to about 1% in tracking of a nonstationary constant-rate saw-tooth profile with closed-loop adaptive control [17].

One basic assumption of the Preisach type model is that hysteresis is rate-independent. To date, little work has been done to explicitly model the rate dependence of hysteresis. Tan and Baras [18] extend the Preisach operator to model and control magnetostrictive actuators at > 5 Hz where hysteresis can no longer be assumed to be rate-independent. They report a maximum error of about 7.5% when tracking a nonstationary dynamic motion profile in closed-loop control. Smith *et al.* [19] report that hysteresis of piezoelectric materials is rate-dependent even at very low frequencies (< 1 Hz).

In this paper, we present an extension to the PI operator to also model the rate-dependent hysteresis characteristic of a piezoelectric actuator. We implement the rate-dependent PI hysteresis model with open-loop control and compare the experimental results with the rate-independent case. A discussion on the significance of the result and the limitations of our model is also presented.

II. Prandtl-Ishlinskii (PI) Hysteresis Model

This section describes the modeling of hysteresis using the modified PI operator proposed by Kuhnen *et al.* [16, 17], with a slightly different treatment to account for the one sided characteristic of many commercial piezoelectric actuators that are driven by non-negative control voltage.

A. Prandtl-Ishlinskii (PI) Operator

The elementary operator in the PI hysteresis model is a rate-independent backlash or linear-play operator. It is commonly used in the modeling of backlash between gears with one degree of freedom. A backlash operator is defined by

$$y(t) = H_r[x, y_0](t) = \max\{x(t) - r, \min\{x(t) + r, y(t-T)\}\} \quad (1)$$

where x is the control input, y is the actuator response, r is the control input threshold value or the magnitude of the backlash, and T is the sampling period. The initial consistency condition of (1) is given by

$$y(0) = \max\{x(0) - r, \min\{x(0) + r, y_0\}\} \quad (2)$$

with $y_0 \in \Re$, and is usually but not necessarily initialized to 0. Multiplying the backlash operator H_r by a weight value w_h, we have the generalized backlash operator,

$$y(t) = w_h \cdot H_r[x, y_0](t). \quad (3)$$

The weight w_h defines the gain of the backlash operator ($w_h = 1$ represents a 45° slope) and may be viewed as the gear ratio in gear mechanical play analogy (see Fig.3).

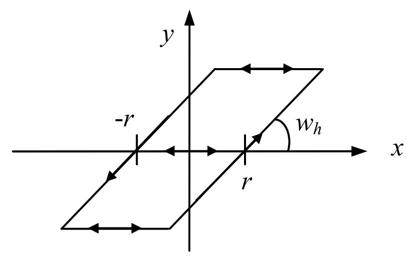


Fig.3. The rate-independent generalized backlash operator is characterized by the threshold or backlash magnitude, r, and the weight or backlash gain, w_h.

Complex hysteretic nonlinearity can be modeled by a linearly weighted superposition of many backlash operators with different threshold and weight values,

$$y(t) = \boldsymbol{w}_h^{\mathrm{T}} \cdot \boldsymbol{H}_r[x, \boldsymbol{y}_0](t). \quad (4)$$

with weight vector $\boldsymbol{w}_h^{\mathrm{T}} = [w_{h0} \ \dots \ w_{hn}]$ and $\boldsymbol{H}_r[x, \boldsymbol{y}_0](t) = [H_{r0}[x, y_{00}](t) \ \dots \ H_{rn}[x, y_{0n}](t)]^{\mathrm{T}}$ with the threshold vector $\boldsymbol{r} = [r_0 \ \dots \ r_n]^{\mathrm{T}}$ where $0 = r_0 < \dots < r_n$, and the initial state vector $\boldsymbol{y}_0 = [y_{00} \ \dots \ y_{0n}]^{\mathrm{T}}$. The control input threshold values $\boldsymbol{r}$ are usually chosen to be equal intervals.

Equation (4) is the PI hysteresis operator in its threshold discrete form. The hysteresis model formed by the PI operator is characterized by the initial loading curve (see Fig. 4). It is a special branch traversed by equation (4) when driven by a monotonically increasing control input with its state initialized to zero (i.e. $y(0) = 0$). The initial loading curve is defined by the weight values $\boldsymbol{w}_h$ and threshold values $\boldsymbol{r}$,

$$\varphi(r) = \sum_{j=0}^{i} w_{hj}(r - r_j), \ \ r_i \le r < r_{i+1}, \ i = 0, \dots, n. \quad (5)$$

The slope of the piecewise linear curve at interval i is defined by W_{hi}, the sum of the weights up to i,

$$W_{hi} = \frac{d}{dr}\varphi(r) = \sum_{j=0}^{i} w_{hj} \, . \quad (6)$$

The subsequent trajectory of the PI operator beyond the initial loading curve, with non negative control input is shown as the dotted loop in Fig. 4. The hysteresis loop formed by the PI operator does not return to zero with the control input and each of the piecewise linear segments now has a threshold width of $2r$ because of the backlash operators. This behavior of the PI operator closely resembles the hysteresis of a piezoelectric actuator.

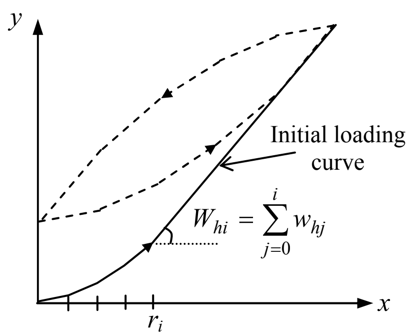


Fig.4. The PI hysteresis model with $n = 4$. The hysteresis model is characterized by the initial loading curve. The piecewise linear curve is defined by the equally spaced threshold values $\boldsymbol{r}$ and the sum of the weight values $\boldsymbol{w_h}$.

B. Modified Prandtl-Ishlinskii (PI) Operator

The PI operator inherited the symmetry property of the backlash operator about the center point of the loop formed by the operator. The fact that most real actuator hysteretic loops are not symmetric weakens the model accuracy of the PI operator. To overcome this overly restrictive property, a saturation operator is combined in series with the hysteresis operator. A saturation operator is a weighted linear superposition of linear-stop or one-sided dead zone operators. A dead zone operator is a non-convex, non-symmetrical, and memory free nonlinear operator given by

$$S_d[x](t) = \begin{cases} \max\{x(t) - d, 0\}, & d > 0 \\ x(t), & d = 0 \end{cases} \tag{7}$$

$$z(t) = \boldsymbol{w_s}^{\mathrm{T}} \cdot \boldsymbol{S_d}[x](t). \tag{8}$$

where y is the output of the hysteresis operator, z is the actuator response, $\boldsymbol{w_s}^{\mathrm{T}} = [w_{s0} \ldots w_{sm}]$ is the weight vector, $\boldsymbol{S_d}[x](t) = [S_{d0}[x](t) \ldots S_{dm}[x](t)]^{\mathrm{T}}$ with the threshold vector $\boldsymbol{d} = (d_0 \ldots d_m)^{\mathrm{T}}$ where $0 = d_0 < r_n < d_1 < \ldots < d_m$. Equal intervals are chosen between d_1 and d_m. The last interval of the hysteresis operator, r_n, is selected to be at the midpoint of the control input range (see Fig.5).

The modified PI operator is thus

$$z(t) = \Gamma[x](t) = \boldsymbol{w_s}^{\mathrm{T}} \cdot \boldsymbol{S_d}[\boldsymbol{w_h}^{\mathrm{T}} \cdot \boldsymbol{H_r}[x, \boldsymbol{y_0}]](t). \tag{9}$$

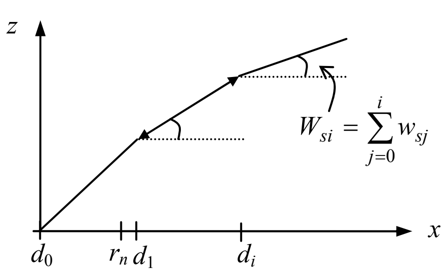


Fig.5. The saturation operator with $m = 2$. The threshold values $\boldsymbol{d}$ have equal intervals from d_1 to d_m. The slope of the piecewise linear curve at interval i is defined by the sum of the weights up to i. The value r_n is the midpoint of the full range of the control input.

C. Parameter Identification

To find the hysteresis model parameters, we first have to measure experimentally the responses of the actuator subjected to some control inputs. Then we set the threshold values $\boldsymbol{r}$ and $\boldsymbol{d}$ as described in the previous section. The weight parameters $\boldsymbol{w_h}$ and $\boldsymbol{w_s}$ are found by performing a least squares fit of equation (9) to the measured actuator response.

D. Inverse Prandtl-Ishlinskii (PI) Operator

The inverse of a PI operator is also of the PI type. The proof of existence of an inverse can be found in [15]. The inverse PI operator is given by

$$\Gamma^{-1}[z](t) = \boldsymbol{w'_h}^{\mathrm{T}} \cdot \boldsymbol{H_{r'}}[\boldsymbol{w'_s}^{\mathrm{T}} \cdot \boldsymbol{S_{d'}}[z], \boldsymbol{y_0'}](t) \tag{10}$$

The key idea of computing the inverse is to find the reflection of the resultant hysteresis looping curves about the 45° line as shown in Fig. 6.

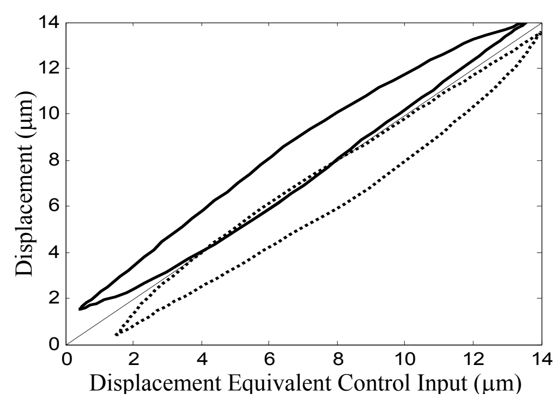


Fig.6. The solid curve is the modified PI model of a piezoelectric actuator. The inverse model in dotted line is the mirror image of hysteresis model about the 45° line.

Cascading the inverse hysteresis model with the actual hysteresis model gives us the identity mapping between the control input $x(t)$ and actuator response $z(t)$,

$$z(t) = \Gamma[\Gamma^{-1}[x]](t) = I[x](t) = x(t) \tag{11}$$

The inverse model parameters can be found by

$$w'_{h0} = \frac{1}{w_{h0}}; \; w'_{hi} = \frac{-w_{hi}}{(\sum_{j=0}^{i} w_{hj})(\sum_{j=0}^{i-1} w_{hj})}, \; i = 1 \ldots n;$$

$$r'_i = \sum_{j=0}^{i} w'_{hj}(r_i - r_j), \;\; y'_{0i} = \sum_{j=0}^{i} w_{hj} y_{0i} + \sum_{j=i+1}^{n} w_{hj} y_{0j},$$

$$i = 0 \ldots n; \tag{12}$$

$$w'_{s0} = \frac{1}{w_{s0}}; \; w'_{si} = \frac{-w_{si}}{(\sum_{j=0}^{i} w_{sj})(\sum_{j=0}^{i-1} w_{sj})}, \; i = 1 \ldots m;$$

$$d'_i = \sum_{j=0}^{i} w'_{sj}(d_i - d_j), \;\; z'_{0i} = \sum_{j=0}^{i} w_{sj} z_{0i} + \sum_{j=i+1}^{m} w_{sj} z_{0j},$$

$$i = 0 \ldots m; \tag{13}$$

III. RATE-DEPENDENT PRANDTL-ISHLINSKII (PI) HYSTERESIS MODEL

A. Rate-dependent Hysteresis Slope

We propose in this section an extension to the modified PI operator to also model the rate-dependent characteristics of the piezoelectric hysteresis.

One of the advantages of the PI hysteresis model is that it is purely phenomenological; there are no direct relationships between the modeling parameters and the physics of the hysteresis. Therefore we would model the rate-dependent hysteresis with reference only to the experimental observations.

While the rate dependency of hysteresis is evident from Fig.1, the sensitivity of actuator saturation to the rate of control input is not apparent. Hence we assume that saturation is not rate-dependent and hold the saturation weights, $\boldsymbol{w_s}$, as well as the threshold values, $\boldsymbol{r}$ and $\boldsymbol{d}$, constant while attempting to construct a relationship between hysteresis and the rate of the control input $\dot{x}(t)$.

We model the slope of the hysteresis curve (i.e. sum of the hysteresis weights) at time t as the sum of the referenced hysteresis slope and a rate-dependent function,

$$W_{hi}(\dot{x}(t)) = \hat{W}_{hi} + f(\dot{x}(t)),\ i = 1 \ldots n. \quad (14)$$

Equation (14) will be reduced to the referenced hysteresis slope, $\hat{W}_{hi}$, or to the rate-independent case if the rate-dependent term is zero.

B. Rate-dependent Model Identification

The response of a piezoelectric actuator is measured subject to sawtooth control inputs at different rate values over the range of 0–1200 μm/s. With an amplitude of 12.5 μm p-p, this corresponds to the maximum rate of sinusoidal control input up to about 30 Hz with the same amplitude. We perform the modified PI parameters identification for the measured actuator response at different rate values. The sum of the hysteresis weights W_{hi}, $i = 0 \ldots n$, are plotted against the control input rate as shown in Fig. 7.

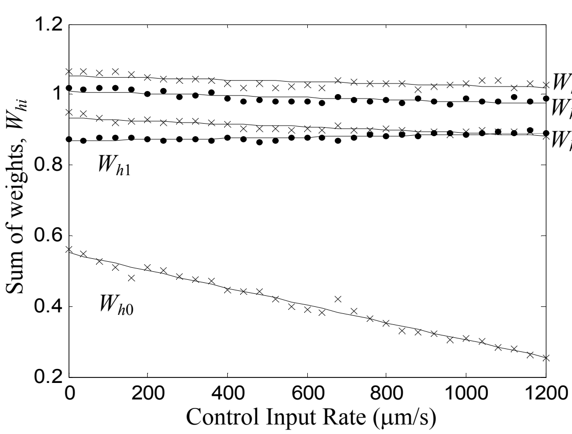


Fig.7. Plot of the hysteresis slopes W_{hi}, $n = 4$, vs. input rate.

We observed that the hysteresis slope of the piezoelectric actuator varies linearly with the rate of input. Thus the rate-dependent hysteresis slope model would be:

$$W_{ri}(\dot{x}(t)) = \hat{W}_{ri} + c_i \dot{x}(t),\ i = 0 \ldots n \quad (15)$$

where c_i is the slope of the best fit line through the W_{hi}'s and the referenced slope $\hat{W}_{hi}$ is the intercept of the best fit line with the vertical W_h axis or the slope at zero input rate. The individual rate-dependent hysteresis weight values can be calculated by

$$w_{hi}(\dot{x}(t)) = W_{hi}(\dot{x}(t)) - W_{h(i-1)}(\dot{x}(t)),\ i = 1 \ldots n;$$
$$w_{h0}(\dot{x}(t)) = W_{h0}(\dot{x}(t)) \quad (16)$$

C. Rate-dependent Modified Prandtl-Ishlinskii Operator

The rate-dependent modified PI operator is defined by

$$z(t) = \Gamma[x,\ \dot{x}](t) = \boldsymbol{w_s}^{\mathrm{T}} \cdot \boldsymbol{S_d}[\boldsymbol{w_h}(\dot{x})^{\mathrm{T}} \cdot \boldsymbol{H_r}[x, \boldsymbol{y_0}]](t). \quad (17)$$

The inverse rate-dependent modified PI operator is also of the PI type:

$$\Gamma^{-1}[z](t) = \boldsymbol{w'}_h(\dot{x})^{\mathrm{T}} \cdot \boldsymbol{H_{r'}}[\boldsymbol{w'}_s{}^{\mathrm{T}} \cdot \boldsymbol{S_{d'}}[z], \boldsymbol{y_0'}](t). \quad (18)$$

The inverse rate-dependent parameters can be found by (12), replacing $\boldsymbol{w_h}$ with the rate-dependent $\boldsymbol{w_h}(\dot{x}(t))$.

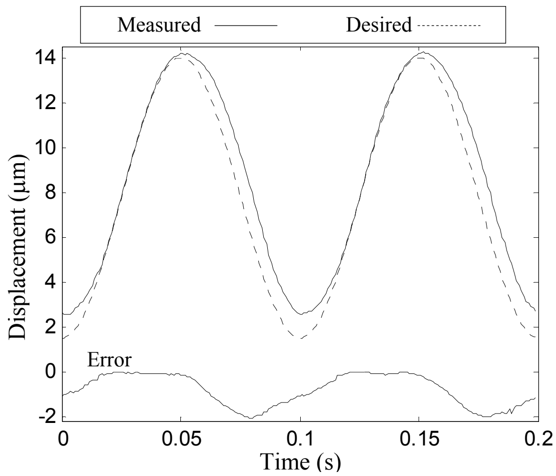
IV. EXPERIMENTAL RESULTS

To model the hysteretic nonlinearity of a TS18-H5-104 (Piezo Systems, Inc.) piezoelectric actuator, we use a modified PI model of $n = 9$, and $m = 2$, for both the rate-independent and rate-dependent case.

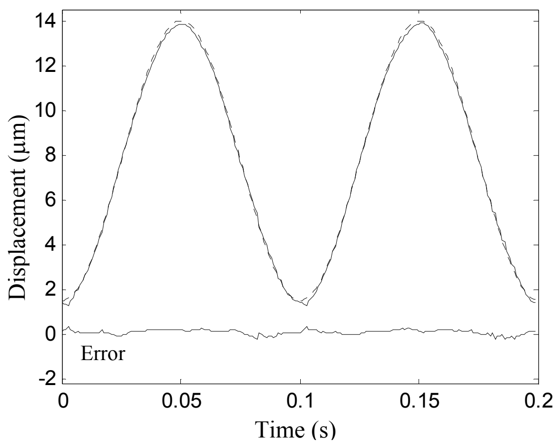
Fig. 8 and 9 compare the experimental results of the rate-independent and rate-dependent PI operators tracking a 10 Hz, 12.5μm p-p stationary sinusoid and a multi-frequency, nonstationary dynamic motion profile (modulated 5, 20, and 35 Hz sinusoids with time-varying amplitudes). The rate-independent model parameters are identified from the measured response of the piezoelectric actuator subjected to a 10 Hz, 12.5 μm p-p stationary sinusoidal control input. Table 1 summarizes the performance of the rate-independent and rate-dependent hysteresis models in tracking experiments.

Table 1. Measured experimental errors of the rate-independent and rate-dependent hysteresis models in tracking a 10 Hz, 12.5μm p-p stationary sinusoid (Stat) and a multi-frequency, nonstationary dynamic motion profile (Dyn).

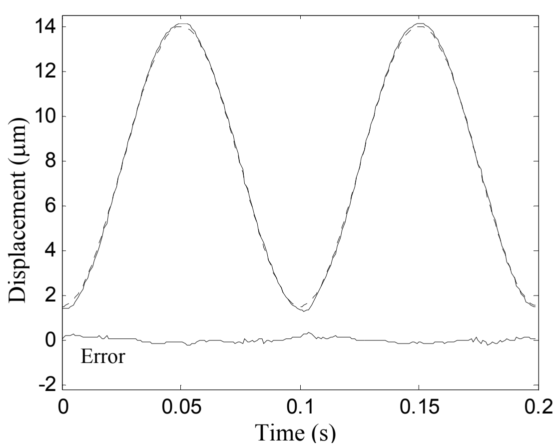
		Without model	Rate-independent	Rate-dependent
Rmse (μm)	Stat	1.07	0.13	0.11
	Dyn	0.84	0.40	0.11
Rmse / p-p ampl.(%)	Stat	8.6	1.0	0.9
	Dyn	6.7	3.2	0.9
max error (μm)	Stat	2.07	0.33	0.34
	Dyn	2.09	1.35	0.56
max error / p-p ampl.(%)	Stat	16.6	2.6	2.7
	Dyn	16.6	10.7	4.4



(a) Without model.

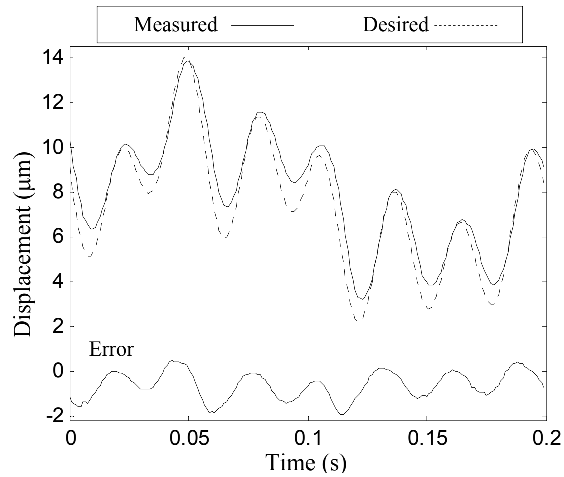


(b) Rate-independent.

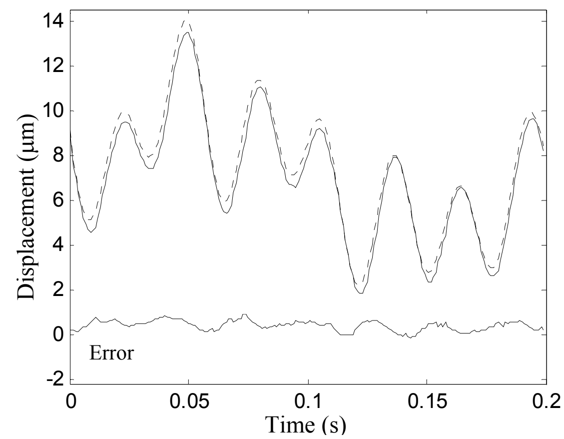


(c) Rate-dependent

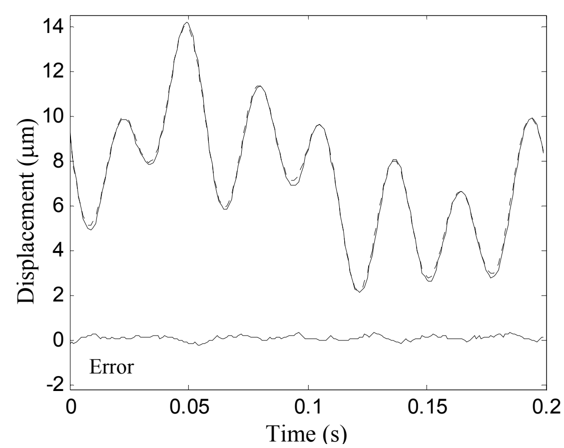
Fig.8. Experimental open-loop tracking results of a stationary 12.5 μm p-p sinusoid at 10 Hz. The rate-independent model parameters are identified from measured response a TS18-H5-104 (Piezo Systems, Inc.) piezoelectric actuator subjecting to the same control input.



(a) Without model.



(b) Rate-independent.



(c) Rate-dependent.

Fig.9. Experimental open-loop tracking results of a multi-frequency, nonstationary dynamic motion profile. The rate-independent model parameters are identified from measured response a TS18-H5-104 (Piezo Systems, Inc.) piezoelectric actuator subjecting to a 10 Hz, 12.5 μm p-p stationary sinusoidal control input.

V. Discussions

The proposed rate-dependent PI hysteresis model reports very similar performance as the rate-independent model in tracking a stationary sinusoid. In tracking more dynamic motion profiles involving frequencies other than the one on which the rate-independent model is based, the rate-dependent model consistently outperforms the other by at least a factor of two in maximum error and a factor of three in rms error. Although the proposed model registers a higher maximum error in the dynamic tracking case, the rms error, a more important tracking performance yardstick, remains almost the same.

Creep is not modeled in the proposed model because its effect is negligible for sinusoids of > 1 Hz. If quasi-static tracking is desired, since the rate-dependent model and its inverse are also of the PI type, the creep model proposed by Krejci *et al.* [15] can be easily incorporated.

One limitation of all PI-type hysteresis models is that it has singularity when the slope of the hysteresis loading curve becomes zero. The inverse model near the singularity is highly sensitive to noise and extra care has to be taken in real-time implementation to avoid modeling error. The singularity of the proposed rate-dependent model occurs at around 1450 μm/s or the maximum rate of a 36 Hz, 12.5 μm p-p sinusoid.

VI. Conclusion

We presented a rate-dependent Prandtl-Ishlinskii (PI) hysteresis model to account for the behavior of a piezoelectric actuator in multi-frequency tracking. The proposed method uses a linear function to model the relationship between the slopes of the hysteretic loading curve and the rate of control input. Compared with a rate-independent model, our model yields significantly better experimental results.

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