Roughness and Reflection in Machine Vision
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July 1994

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Abstract

We want to develop algorithms which allow a computer vision system to qualitatively estimate the roughness of reflective surfaces from a single image, under normal lighting conditions, and with incoherent light. In order to make such a system applicable to most imaging situations, we attempt to constrain the environment as little as possible, and thus, we will study the reflected images of step edges, since these are ubiquitous and algorithms for their detection are mature.

We first derive a six-parameter model of reflected step edges, and show that with this model it is possible to differentiate surfaces by roughness. In order for this differentiation to be correct, however, certain information about the imaging geometry must be known; we list what information is necessary, and describe the effect of changes in these parameters on the appearance of reflected edges.

We then use this knowledge of the appearance of the reflected edges to develop two methods of roughness estimation. The first method fits a curve described by the six-parameter model to the reflected edge profile data by a combination of gradient descent and singular value decomposition algorithms. The output of this algorithm is the best-fit values of the six parameters, one of which is the root-mean-square slope of the surface, which we use to quantify the roughness. We then discuss the imaging conditions for which this algorithm works well. The next method of roughness estimation treats the equation for the reflected radiance as a first kind Volterra integral equation. After solving the integral equation, it calculates a quantity which allows it to order surfaces by roughness. Although it does not calculate the roughness directly, as the first method does, it works for many imaging situations for which the first algorithm does not.
We test these algorithms on the images of step edges reflected in the surfaces of five steel disks of different roughness. We measure the root-mean-square slopes of these surfaces with a profilometer and compare the output of our algorithms with these values. We find that both algorithms order the surfaces correctly by roughness.
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Ad Majorem Dei Gloriam
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1. Introduction

Many things can influence the appearance of an image recorded by a camera, such as the lighting, the lens settings of the camera, and the properties of the objects in the scene. One important property which is presently a subject of much research in the field of physics-based vision is the effect of the roughness of reflective surfaces on their appearance. A complete description of the change in the reflective properties of surfaces with respect to roughness is of theoretical interest, but also has practical applications, from quality control in factories to general purpose vision systems, in which a robot attempts to gain information about its environment. The estimation of roughness could also aid a robotic vision system in material type classification and image segmentation, and thus could be one of the low-level steps in a complete vision system. Obviously, the understanding of the effects of surface roughness on object appearance and the development of roughness estimation algorithms is important to the advancement of computer vision research.

This thesis will address many of the issues of the effects of roughness on the appearance of reflective surfaces. It will study the reflected images of step edges, since these are present in many environments and are one of the most thoroughly studied and commonly used image features. It will present a model of the reflected images of step edges, and will show the effects of surface roughness on the appearance of the edges. Finally, it will discuss the minimum amount of information necessary for the estimation of surface roughness from edges, and will present algorithms that estimate the roughness of a reflective surface by studying a single image of the surface containing the reflection of one or more step edges.
1.1. Background

Although the analysis of the reflection of light from rough surfaces has a long history, the application of this knowledge in the computer vision field for the analysis of the appearance of rough surfaces occurred more recently. Theories of reflection found use in constrained laboratory settings from their inception, but the computational complexity inherent in most computer vision applications precluded their immediate use. As computational capabilities increased, however, computer vision researchers were able to use more complete and physically meaningful descriptions of reflection.

1.1.1. The Scattering of Light from Rough Surfaces

The study of the reflection of light from rough surfaces has its origin in the analysis of radar reflected from rough ground and the ocean. Rice performed some of the first work in this area shortly after the Second World War.[Rice 51] He assumed that the reflective surfaces had low slopes, and with this simplification, he was able to solve for the scattered light intensity. Additionally, he studied special cases of surface shape, such as sinusoids. His theory is presently important in the measurement of the roughness of very smooth surfaces, such as lenses and other optical elements.[Stover 90]

In 1963, Beckmann and Spizzichino proposed a model of the scattering of electromagnetic waves from rough surfaces which used a different set of assumptions.[Beckmann and Spizzichino 63] Their theory dealt with surfaces which have a Gaussian distribution of height values, and the calculations relied heavily on statistical methods. They derived a series solution for the scattered intensity pattern. This theory works well for surfaces that are rougher than those dealt with by Rice’s theory. With the advent of optical profilometers, this theory has gained favor with those
measuring the roughness of metal surfaces, and is now probably the most used description of rough surface scattering among those researchers.

Both Rice's and Beckmann and Spizzichino's theories treat light as a wave phenomenon. It is possible in some cases, however, to describe the reflection of the light by the laws of geometrical optics. Beckmann introduced such a description [Beckmann 57], and Du Castel and Spizzichino improved upon it.[Du Castel and Spizzichino 59] In 1967, Torrance and Sparrow [Torrance and Sparrow 67] used a geometrical description of reflection to explain the existence of "off-specular peaks" in the scattering patterns they found for rough metal surfaces, and showed how these peaks could result from the shadowing of surface "microfacets" by one another. The calculations involved in these geometrical optics theories are often simpler than those of wave theories such as Rice’s and Beckmann and Spizzichino’s. This accounts for much of their popularity. As shown by Beckmann and Spizzichino, however, limiting cases of their wave theory have an equivalent geometrical optics formulation, thus making their wave optics theory as easy to use as the geometrical optics theory in these cases.

Research into theories of the reflection of light from rough surfaces continues to this day. New surface models, such as fractal descriptions of surfaces [Jaggard and Sun 90], are popular. Most of the new theories have not yet influenced the field of computer vision.

1.1.2. Applications to Computer Vision

At different stages in the history of computer vision and computer graphics research, particular ones of these theories of reflection from rough surfaces have enjoyed popularity.

Starting in the 1970’s, computer graphics researchers began studying reflection from rough surfaces in order to create increasingly realistic images. They searched for theories which would
not only make convincing pictures, but which were also simple enough to use with the limited computational power of the times. One of the first of these models was proposed by Phong. \[\text{[Phong 75]}\] In this description, the distribution of the reflected light intensity, \( L \), is given by 
\[ L = (\cos \theta)^n, \]
where \( \theta \) represents the angle between the normal to the surface and the bisector of the directions of the incident and reflected light rays, and \( n \) is a number greater than 1. Increasing values of \( n \) represent decreasing roughness. There is little physical basis for this model, but the resulting images were quite good for the time.

As computational power increased, many in the computer graphics field saw the need for better descriptions of rough-surface reflection. The Torrance and Sparrow theory of reflection became popular at this time. Blinn was the first to use this theory to generate realistic images of rough metallic objects. \[\text{[Blinn 77]}\]

Researchers in the computer vision field also saw the potential of the Torrance and Sparrow geometrical optics theory for solving the inverse problem, that is, finding the roughness of a surface from its reflected radiance distribution. One of the first applications of this theory in computer vision was the work of Healey and Binford \[\text{[Healey and Binford 87]}\], in which rough cylindrical metal surfaces were imaged under point source illumination. The intensity pattern across the cylinder yielded the roughness as well as the radius of curvature of the surface.

The Torrance and Sparrow theory has found use more recently in the photometric sampling research of Ikeuchi and his collaborators. \[\text{[Kiuchi and Ikeuchi 93]}\][\text{Solomon and Ikeuchi91}] \[\text{[Nayar et al. 88]}\] Photometric sampling is an experimental technique in which an object is illuminated from several directions in turn, but viewed from a single direction. The illumination is usu-
ally a point source, although the method can work with extended sources. Analysis of the resulting images allows the determination of the object shape and roughness.

The need for multiple images, small samples, and specialized equipment make the photometric stereo technique unsuitable for general purpose vision systems. Ikeuchi and Sato have introduced a method of roughness estimation which uses the Torrance and Sparrow theory, and which requires only a single image under point source illumination of the object being studied.\[\text{Ikeuchi and Sato } 91\] It does, however, require knowledge of the object shape, and this is obtained with a laser rangefinder.

Novak also used the Torrance and Sparrow theory in her analysis of the color histograms of reflective dielectric surfaces.\[\text{Novak } 92\] The analysis yielded the roughness of the surfaces, but it required that the body reflection of the surfaces be Lambertian, so that the shape of the object could be estimated.

Further increases in the power of computers allowed researchers to move from the Torrance and Sparrow geometrical optics theory of reflection to the Beckmann and Spizzichino wave optics theory. Cook and Torrance were the first to use this theory for computer graphics.\[\text{Cook and Torrance } 82\] However, they used a simplified version of this theory which was also derived by Beckmann and Spizzichino.

Mundy and Porter were the first to use the theory of Beckmann and Spizzichino in a computer vision setting.\[\text{Mundy and Porter } 80\] They developed a method for the detection of defects on metal surfaces. Their method was similar to that of photometric stereo in that it calculated the roughness of surfaces from multiple measurements of the reflected intensity made with different
point light source directions. Like Cook and Torrance, they also used a simplified version of the theory.

Nayar used simulations to thoroughly study the wave optics theory of reflection. Thus he was the first to use the complete theory of Beckmann and Spizzichino in a computer vision setting. [Nayar et al. 91]

1.1.3. Our Research

There are shortcomings to all of the methods of roughness estimation listed above. First, many of the methods require multiple images of the reflective surface under study. The ability of humans to estimate the roughness of surfaces does not have this constraint; most people can guess the roughness of a surface from a single image. We believe that this ability is largely a matter of pattern recognition, and that with the analysis of the proper image features, we may design algorithms which determine roughness from a single image. Another shortcoming is that in all of the methods of roughness estimation described above, the researchers have used point light sources. Although point light sources are present in some imaging environments, they are not as common as many researchers would like to believe. In order to overcome these problems, our roughness estimation methods will analyze single images containing the reflections of step edges.

There are several reasons for our choice of studying step edges. We believe that edges are more common than point light sources. Additionally, step edges will be present in many environments, particularly those containing artificial objects. Finally, the methods of edge detection and analysis are mature, and are a common first step in many computer vision systems. Therefore, our methods will not require additional preprocessing steps.
In our research, we will use the Beckmann and Spizzichino model of reflection, since it is one of the most complete theories of reflection from rough surfaces, and because it has a geometrical optics formulation which is easy to use.

1.2. Thesis Outline

In the second chapter of the thesis, we discuss rough surfaces and the reflection of light from such surfaces. We begin by describing some of the different ways of parameterizing rough surfaces, and the statistics commonly used to quantify roughness. The chapter continues with a description of the different theories of the reflection of light from rough surfaces and how certain theories of reflection necessitate the use of certain surface parameterizations. We discuss in detail the model we will use in most of our work to describe rough surfaces and the theory we use to study the reflection of light from them.

In Chapter 3., we take up the subject of the reflected images of edges, the image feature around which we will build our work. We derive an equation which describes the appearance of the reflected edge image in a rough surface, and identify certain parameters, such as the shape of the reflective surface and the viewing angle, which greatly influence the reflected edge profile. Following this is a discussion of how variations in these parameters change the appearance of the reflected edge image. Simulations of reflected edge profiles accompany this discussion. We also show that there exist different parameter sets which produce reflected edges that appear almost indistinguishable, and conclude that the values of these parameters must be known for any method of roughness estimation to give accurate answers.

Chapter 4. introduces a six-parameter model of the profile of a reflected edge image. We then present our first method of roughness estimation; the algorithm fits a curve for our model to a
reflected edge profile. It iteratively finds the best fit solution, and from this we may read off the surface roughness. We test our method on several surfaces with different, known roughnesses. The results of the algorithm are then compared with roughness values found by stylus profilometry, a common roughness measurement method.

We begin Chapter 5. by giving examples of the reflected images of edges for which the iterative solution method of roughness estimation does not work. After explaining why the method failed in these cases, we introduce a second method of roughness estimation which is based on the theory of first-kind Volterra equations, and which can order surfaces by roughness. We show that this new method works on the images for which the first method did not.

In the Summary and Conclusions chapter, we reiterate the important findings of this thesis and discuss areas for future research.
2. Roughness and Reflection

2.1. Chapter Overview

Our study of the reflection of light from rough surfaces begins with a discussion of the definition of roughness. This is not as simple as it may seem at first, and our choice of definition for roughness often influences our choice of method for solving for the radiance of the reflected light. We then discuss the physical-optics and geometrical-optics approaches to quantifying the reflection of light from surfaces. We choose to model rough surfaces as having a Gaussian distribution of height values. We feel that this model adequately describes many real surfaces, and reflection from such surfaces may be treated by both the methods of physical optics and geometrical optics.

2.2. What is roughness?

In order to study the reflection of light from rough surfaces, we must first define what we mean by roughness. This is no easy task; myriad definitions exist, and each has its usefulness. Let us describe a few of the most common definitions of roughness.

2.2.1. Roughness Statistics

Perhaps the most common way to quantify the roughness of a surface is to calculate the values of some common statistical parameters for the surface. Let us assume, for ease of pedagogy, that the rough surface under study is a one-dimensional function, i.e. we know its height, \( h(x) \), for every position, \( x \). Our definitions will hold for two-dimensional surfaces with minor modifications. The arithmetic average roughness of the surface is defined to be [Stover 90]

\[
R_A = \frac{\int |h(x) - \mu| \, dx}{\int dx}
\]  

(1)
where the integral is over the whole surface, and $\mu$ is the mean height of the surface, or

$$\mu = \frac{\int h(x) \, dx}{\int dx}$$

In real-world situations, we will not know the height of the surface as a continuous function of $x$, but rather, we will know the height of the surface at discrete points, $x_i$. In this case, we estimate the arithmetic average roughness of the surface to be

$$R_A = \frac{1}{N} \sum_{i=1}^{N} |h(x_i) - \mu| \quad (2)$$

where $N$ is the number of sample points, and $\mu$ is now

$$\mu = \frac{1}{N} \sum_{i=1}^{N} h(x_i)$$

The arithmetic average roughness is the most commonly used roughness standard in machining applications.

Perhaps the most common roughness standard in optical circles is the \textit{root-mean-square}, or \textit{rms roughness}. This is defined to be

$$\sigma_{rms} = \left[ \frac{\int (h(x) - \mu)^2 \, dx}{\int dx} \right]^{\frac{1}{2}} \quad (3)$$

In the discrete case, this becomes
This statistic often appears in the equations describing the scattering of light from rough surfaces.

A less common roughness measure, but one which will figure prominently in this thesis, is the root-mean-square slope. It is given by

\[ m_{rms} = \left[ \frac{1}{N} \sum_{i=1}^{N} (h'(x_i) - \mu)^2 \right]^{\frac{1}{2}} \]  

(4)

where \( h'(x) = \frac{dh}{dx} \) is the slope of the surface, and

\[ \mu = \int h'(x) dx \int dx \]

is the mean slope. In the discrete case, this becomes

\[ m_{rms} = \left[ \frac{1}{N} \sum_{i=1}^{N} (h'(x_i) - \mu)^2 \right]^{\frac{1}{2}} \]  

(5)

where

\[ \mu = \frac{1}{N} \sum_{i=1}^{N} h'(x_i) \]

There are many other statistics which quantify surface roughness, but these three are some of the most common, and, for our work, most useful measures.
2.2.2. Roughness Models

Although we can derive statistics for rough surfaces, these values are often not very informative. Many different rough surfaces may have the same rms roughness, for example, but the pattern of the light scattered from each may be very different. Therefore, in order to calculate the scattering of light from a surface, we must assume a form for the distribution of its height or slope values.

As we stated in Section 2.2.1., we may parameterize a surface by means of its height values or its slopes, and the choice will often determine the method we use to solve for the reflected light radiance. Figure 1 presents a simple taxonomy of these models; this listing is by no means complete, but clarifies the relationships between some of the commonly used models.

The first thing to note is that the diagram shows that there is some overlap between the two formalisms, because some height-based models have a slope-based interpretation. Next, we may consider the directionality of the roughness. A surface which has constant height along one direction, although it may vary in the perpendicular direction, we call one-dimensionally rough. Such a surface will be made up of many parallel grooves. A surface which has similar roughness along any direction we call isotropically rough. There exist an infinite number of possible types of directional roughness between these two extremes, although most machine vision researchers consider only the limiting cases of isotropic and one-dimensional roughness. Two other common classifications of models assume that the heights or slopes follow a Gaussian distribution. Note that all Gaussian height models have a slope interpretation, and thus also fall into the category of slope-based models. Not all Gaussian slope models have a height-based interpretation, however. Other models, such as fractal height models, appear in the optics literature [Jaggard and Sun 90],
but are uncommon in machine vision research, probably due to their complex nature. These models do not appear in the diagram, since their relationship to other models is uncertain, although they would belong in the category of height-based models.

The models which interest us the most are the isotropic and one-dimensional Gaussian models, which have characteristics of several of the previously listed classes. These are simple enough to use in computer vision research, and describe the surface reflection from common materials, such as metals and ceramics, quite accurately. The isotropic Gaussian height model and the one-dimensional Gaussian height model were proposed by Beckmann and Spizzichino. [Beckmann and Spizichino 63] The isotropic Gaussian slope model was proposed by Torrance and Sparrow
[Torrance and Sparrow 67], and is probably the most commonly used model in computer vision research. [Blinn 77][Cook and Torrance 82][Healey and Binford 88][Nayar et al. 90][Novak 92]

The one-dimensional Gaussian slope model is the one-dimensional analogue of this model. Note also that the isotropic Gaussian height model and the isotropic Gaussian slope model are equivalent in the limiting case, as are the one-dimensional Gaussian height model and the one-dimensional Gaussian slope model.[Beckmann and Spizzichino 63][Nayar et al. 91] We will discuss this in greater detail in Section 2.4.

We now have briefly described the most popular models of rough surfaces in computer vision research. Before we decide on one of these models for our work, let us describe the models of the reflection of light.

2.3. The Reflection of Light

As we stated previously, there are many ways to quantify the reflection of light from rough surfaces; these models may be classified by their treatment of the rough surface and of the light. There are two common ways to describe light. First, some models treat light as an electromagnetic wave phenomenon; we will call these wave models or physical optics models. Others, however, use the fact that light may be treated as rays if its wavelength is much smaller than the obstacles that it may encounter. These models use the laws of geometrical optics, such as the law of reflection and Snell's law, and thus we describe them as geometrical optics models.

2.3.1. The Theory of Reflection in the Physical Optics Regime

Let us now outline the analysis of the reflection of light when it is treated as a wave phenomenon. Figure 2 is a simple diagram of the quantities which will be important in our study. The diagram shows all quantities as lying in a plane, since most of the situations which we study will
have scattering in the plane of incidence, and also because this makes the diagram easy to understand. There are analogous results for scattering out of the plane which we will not show.

The direction of the incident light is denoted by the vector $k_i$, and that of the reflected light by $k_r$. These vectors have magnitude equal to $\frac{2\pi}{\lambda}$, where $\lambda$ is the wavelength of the light. We will assume for the moment that light is incident from only one direction, although it may reflect in many directions, and hence, there will be many values of $k_r$. The vector $\phi$ is the scattering vector, or $k_r - k_i$, the difference between the reflected and incident vectors. Our goal is now to find the intensity of the light for all directions of reflection, i.e. all values of $k_r$, given the intensity and direction of the incident light as well as the shape of the surface.

Because we are using the laws of physical optics, we treat the light as a wave. We solve for the reflected intensity by treating each point of the surface as a point light source, and sum the effects
of all sources, taking into account interference and diffraction. This is a difficult procedure, and the results will vary greatly depending on the surface model that we choose. The most common models used with the physical optics description of scattering are the Gaussian heights models, both the one-dimensional and isotropic forms. Beckmann and Spizzichino solved for the scattered light intensity distribution for these cases and described the procedure for solving for the distribution with the assumption of alternative surface models. Their derivation is too lengthy to include here, but we present their result. The intensity, $I$, for a one-dimensionally rough surface with a Gaussian distribution of height values is given by

$$I \propto e^{-\frac{1}{2} \left( \frac{\sin v_x X}{v_x X} \right)^2 + \frac{\sqrt{2} D^2 T}{2L} \sum_{m=1}^{\infty} \frac{m^4}{m!} \int_0^\infty \frac{e^{-\frac{v_z^2 t^2}{4m}}}{t} dt}$$

(7)

$x$ is the length of the part of the rough surface illuminated by the incident beam. $v_x = (-k)(\sin \theta_i + \sin \theta_r)$ and $v_z = k(\cos \theta_i - \cos \theta_r)$ are the $x$ and $z$ components of the scattering vector, respectively. $g$ is equal to $v_z \sigma$, where $\sigma$ is the rms roughness of the surface. $D$ is given by

$$D = \frac{1 + \frac{\cos(\theta_i + \theta_r)}{\cos \theta_i + \cos \theta_r}}{\cos \theta_i + \cos \theta_r}$$

and $T$ is the correlation length of the surface. A definition of the correlation length appears with the derivation. It is obvious that the expression is quite complicated; it is not surprising that it is difficult to use in computer vision applications. This is especially true for intermediate values of the roughness, i.e. $g = 1$, since the sum does not converge rapidly.

This formula is one of many that has been obtained for light scattering under the assumptions of physical optics; it is also one of the simplest. Let us now see how this result compares with
those of the geometrical optics approach.

2.3.2. The theory of reflection in the geometrical optics regime

We may not only treat light as waves, but also as rays and thus describe its reflection and refraction with the laws of geometrical optics. In this case, however, we ignore all diffractive effects, and this is equivalent to the assumption that all obstacles that the light encounters are much larger than its wavelength. For such obstacles, our calculated intensity distribution will be correct, but for surfaces which have roughness on a scale much smaller than the wavelength of the incident light, the calculated intensity distribution will be only an approximation to the true intensity distribution.

Let us now describe the reflection of light from a rough surface with the methods of geometrical optics. We first note that since we will be using the law of reflection, it is easiest to imagine that the surface is comprised of small facets which reflect like a mirror. Figure 3 is a diagram of such a surface. Once again, these facets are larger than the wavelength of the incident light. Since the angle at which these facets are tilted will affect the reflection of light, we will use slope-based surface roughness models. It is important to remember that, although we show the rays of light reflecting off single facets in Figure 3, slope-based models do not deal with the individual facets on the surface, but rather the statistical properties of the set of all facets on the surface.

We must now derive the radiance of light reflected by a rough faceted surface. This derivation parallels that given by Torrance and Sparrow [Torrance and Sparrow 67]. Consider a small planar patch of the surface under study, of size $dA$ (see Figure 4). Assume an infinitesimal source of radiance $L_t$ subtending a solid angle $d\omega$ with direction of incidence, in spherical coordinates, $(\theta, \phi)$. (The normal to the mean surface is in direction $(0,0)$. The reflected radiance is $dL_r$, with
Figure 3: A rough surface comprised of mirror-like facets. Light reflects off the facets according to the laws of geometrical optics.

Figure 4: The reflection geometry for the derivation of the scattered light intensity, under the assumption of geometrical optics models.
direction \((\theta_r, \phi_r)\). For a given incidence direction and reflectance direction only those facets whose normals bisect the angle between these directions will be able to reflect light from the source to the receiver. Let the direction of the normals to these facets be \((\theta_f, \phi_f)\). Let us denote the local angle of incidence, or the angle between the source direction and the direction of the normals to the privileged facets, as \(\theta_{\text{IL}}\).

Now, in order to calculate the radiance of the patch, we must know the total area of the privileged facets, which will reflect light from the source to the receiver. We therefore define a function, \(S(\theta_f, \phi_f)\), the facet area distribution function (FADF), which represents the total area of the privileged facets per unit solid angle subtended by the normals to these facets per unit area of the mean surface. Thus, the area of the facets present on an infinitesimal mean surface patch of area \(dA\) with normals in the infinitesimal solid angle \(d\omega_f\) is \(S(\theta_f, \phi_f) \, d\omega_f \, dA\). The power incident on the privileged facets is then

\[
P_i = L_i \, d\omega_i \cos \theta_{\text{IL}} \, S(\theta_f, \phi_f) \, d\omega_f \, dA
\]  

(8)

This light will then reflect off the facets and into the receiver, and may be attenuated upon reflection. From the point of view of the receiver, the power received from the facets may be written as

\[
P_r = FGP_i = dL_r \cos \theta_{\text{IL}} \, d\omega_f \, dA
\]  

(9)

where \(dL_r\) is the radiance of the patch as seen from the viewing direction, and \(F\) is the Fresnel coefficient for the surface. The Fresnel coefficient will be a function of the local angle of incidence, \(\theta_{\text{IL}}\). Some researchers [Blinn 77][Cook and Torrance 82][Nayar et al. 91][Torrance and Sparrow 67] also model the attenuation due to the shadowing of one facet by another, and quantify this with \(G\), the geometrical attenuation factor. It is a function of the directions of incidence
and reflectance. Torrance and Sparrow provide a derivation of this factor. Substituting the expression for the incident power into equation (9), we see that the radiance of the surface patch is

\[ dL_r = \frac{L_i d\omega_i \cos \theta_{il} FGS(\theta_r, \varphi_r) d\omega_f}{\cos \theta_i d\omega_r} \]  

(10)

Most researchers use the simplifying relation [Nayar et al. 91][Torrance and Sparrow 67]

\[ d\omega_f = \frac{d\omega_r}{4\cos \theta_{il}} \]  

(11)

We, however, will most often use the relation

\[ d\omega_i = d\omega_r \]  

(12)

This is merely a statement of the fact that if we hold \( \theta_f \) constant, all the facets which may reflect light from the source to receiver will have the same normal, and will reflect light in a specular fashion, so that the incident and reflected solid angles will be equal. We have found that numerical methods of calculating the reflected radiance often perform better if we use this relation. Additionally, the equation for the reflected radiance, which we will show presently, has the same form for both the one-dimensional and two-dimensional cases if we make this substitution.

For an extended source, the reflected radiance is then

\[ L_r = \int \int L_i F G \cos \theta_{il} S(\theta_r, \varphi_r) d\omega_f \]  

(13)

where the integral is over the hemisphere for which \( \theta_f \) is less than \( \pi/2 \).

There is also a normalization condition which any realistic FADF must satisfy. Consider one facet of the surface and let its area be \( dF \). The area of the projection of this element onto the mean
If we were to add up all the projected areas of all facets on the surface, we would obtain $dA$, the area of the patch, i.e.

$$dA = \int dF \cos \theta_f$$

We also know that

$$dF = S(\theta_f, \varphi_f) d\omega_f dA$$

so that

$$dA = \int S(\theta_f, \varphi_f) \cos \theta_f d\omega_f dA$$

and

$$\int S(\theta_f, \varphi_f) \cos \theta_f d\omega_f = 1$$

which is the normalization condition.

Just as we were able to apply physical optics methods of scattering analysis to many types of surface models, we may also apply the geometrical optics methods to many models. In fact, the above results pertain to any slope-based model, or FADF. Perhaps the most commonly used slope-based model, and perhaps also the most commonly used rough surface reflection model in computer vision work, is that proposed by Torrance and Sparrow [Torrance and Sparrow 67]. The model assumes a Gaussian distribution of facet normals, such that
The constant, \( c \), is related to the spread of the distribution of facet normals. The model is isotropic, and is hence independent of \( \phi \).

### 2.4. Our Choice of Model

In much of our subsequent research, we will need to decide whether we want to analyze scattering with the methods of physical optics or geometrical optics, and we will also require a model of the rough surfaces we study. Ideally, we would like to find a way to combine the power of the physical optics approach with the mathematical simplicity of the geometrical optics methodology.

There exist cases in which scattering may be described equivalently by the physical optics approach and a certain height model for the surface, and by the geometrical optics approach and a certain FADF. Beckmann and Spizzichino have shown [Beckmann and Spizzichino 63] that for surfaces with Gaussian height distributions where the rms height is much larger than the wavelength of the incident light, equation (7) reduces to

\[
dL_r = l \left( \left( \frac{\sin \nu X}{\nu X} \right)^2 + \frac{S(\theta, \phi) \cos \theta d\theta}{\cos \phi} \right)
\]  

(17)

where \( l \) is a constant which simply represents the radiance of the source. In other words, the scattered radiance consists of two terms, the first of which is focussed in a narrow range of angles, and which may only be derived by physical optics methods, and the second of which is equivalent to geometrical optics scattering. Nayar [Nayar et al. 91] calls the first component the “specular spike component of reflection”, and the second, the “specular lobe component”. It is interesting to note that the first term is the same as the diffracted radiance distribution resulting from a single-slit
source. The second term of equation (17) may be modified to include the effects of the Fresnel coefficient and the shadowing of one facet by another as was done in equation (13). This yields

\[ dL_r = \frac{1}{4} \left( \frac{\sin \nu X}{\nu X} \right)^2 \cos \theta_r \frac{FGS(\theta) \cos \theta_r}{\cos \theta_r} \] (18)

For a surface with a one-dimensional Gaussian height distribution, the FADF is

\[ S(\theta_r) = \frac{\exp\left(-\frac{\tan \theta_r^2}{\beta^2}\right)}{\beta \sqrt{\pi} \cos \theta_r} \] (19)

and for a surface with an isotropic Gaussian height distribution, the FADF is

\[ S(\theta_r, \phi_r) = \frac{\exp\left(-\frac{\tan \theta_r^2}{\beta^4}\right)}{\beta^2 \pi \cos \theta_r} \] (20)

In these functions, \(\beta\) is the roughness parameter, and the rms slope, i.e. the standard deviation of the facet slopes, is \(\beta/\sqrt{2}\) for the one dimensional case and \(\beta\) for the isotropic cases. As noted by Beckmann and Spizzichino and Nayar, these functions may be approximated by Gaussians in the limit of small \(\beta\), and thus the model of Torrance and Sparrow is a reasonable approximation to equation (20) in this limit. It is interesting to note that in one sense such a surface is smooth, since \(\beta\) is small and the surface has only low slopes, yet in another sense, the surface is rough, since the rms height is much larger than the wavelength of light. The above facet area distribution functions are more realistic than the Gaussian models, however, since they satisfy the normalization condition (equation (15)). We will use equations (17) and (19) in most of our work.
2.5. Conclusions

In this chapter, we have taken a brief look at a few of the many ways of modelling rough surfaces and the reflection of light from them. We described both the height-based and slope-based models for rough surfaces, and presented a normalization condition for all slope-based models, or Facet Area Distribution Functions. We noted that the two methods of describing light reflection, the physical optics approach and the geometrical optics approach, each have their strong points; the physical optics models yield very accurate results, while the geometrical optics models are easier to use. In order to combine the good points of both approaches, we will use the model of Beckmann and Spizzichino, which is derived using physical optics methods, but which has a geometrical optics formulation for the limiting case of rough surfaces. We must now use this model to calculate the effects of roughness on the reflected images of step edges.
3. Edges

3.1. Chapter Overview

We saw in Chapter 2. that the model of Beckmann and Spizzichino provides a powerful, yet easy-to-use, method of describing rough surfaces and the reflection of light from them. In this chapter, we calculate the appearance of the image of a reflected step edge, assuming this model. We then show how the shape of the edge varies with respect to changes in the roughness, the distance between the source and object, and the viewing angle. Following this is a discussion of how different sets of these parameters may produce reflected edge images which are almost indistinguishable. We therefore conclude that knowledge of the source-object distance and viewing angle is necessary for the functioning of any roughness estimation method which relies on the analysis of edges.

3.2. Edges as an Extended Source

As we stated in Chapter 2., we will study the images of edges reflected in rough surfaces. Let us begin this study by first defining a bit of our terminology. It should be clear from context when we use the word edge to refer to the physical edge of the source and when we use it to refer to the reflected image of the source. We will try to be clear in our usage, nonetheless, and will use terms like reflected edge image and image of the reflection of the edge whenever possible. The reflected image of the source will occupy a large part of the images we study. In order to make our results easier to understand, we will therefore often show plots of the intensity of the reflected edge image along the direction perpendicular to the image of the edge of the source. We will call these plots reflected edge profiles. Now, let us describe the physical step edge source as an extended source so that we may use equation (18) to calculate the appearance of the reflected image.
Consider a small patch of the reflective surface; perhaps it is the area viewed by one pixel. The *illumination environment* of this patch is the incident radiance for each incident direction, i.e., the brightness seen at the patch in each direction. Now let us assume that there is a step edge which can be seen reflected in the surface. The illumination environment will thus contain a bright region and a dark region. As we move from point to point on this surface, the two regions which define the step edge will change, and thus, so will the illumination environment. As an example, Figure 5 shows the hemisphere of possible directions of incident light for several points on a surface; the shaded regions show the directions from which light does impinge upon the surface. This type of diagram is similar to the illustrations in [Maxwell and Shafer 93].

Figure 5: The illumination environment for several points on a surface. The shaded regions of the hemispheres show the angles from which light is incident on the surface.
In most of this research, we will focus on one-dimensionally rough surfaces. The illumination environment containing a step edge is particularly simple for this case. We will perform our calculations in the coordinate system of the surface patch. Therefore, let the normal to the patch be along the z-axis, as shown in Figure 6, and let the grooves lie parallel to the y-axis (i.e. the height is constant for constant x). To begin, let us first consider the easiest case, in which the view vector lies in the plane of the surface normals, i.e., the x-z plane, and assume the edge of the source is parallel to the y-axis. As we see in Figure 7, light may be incident on the surface at many angles, \( \theta_i \), where \( \frac{\pi}{2} \leq \theta_i \leq \frac{\pi}{2} \). We assume that the illumination environment consists of a step edge of constant radiance, \( I \), and thus, \( L \) equals \( I \) for \( \theta_{io} \leq \theta_i \leq \theta_{i1} \), and zero otherwise. We also know the direction of the view vector, and hence \( \theta_r \), for the patch. Because the facets which reflect light from the source to the camera bisect the angle between these directions, we have

![Diagram of reflection geometry showing the bisector of illumination and reflection directions, macroscopic surface normal, and the reflection.](image-url)

Figure 6: The reflection geometry.
Figure 7: The one-dimensional reflection geometry

\[ \theta_f = \frac{\theta_i + \theta_r}{2} \]  

and we therefore know the range of values of \( \theta_f \) for which light will reflect from the source into the camera. Let us denote the starting and ending values of this range by \( \theta_{f0} \) and \( \theta_{f1} \), where 

\[ \theta_{f0} = \frac{\theta_{i0} + \theta_r}{2} \text{ and } \theta_{f1} = \frac{\theta_{i1} + \theta_r}{2} \].

We are now able to calculate the reflected radiance for the surface patch.

Before we do this, however, we note that \( \theta_{i0} \) and \( \theta_{i1} \), and hence the values of \( \theta_{f0} \) and \( \theta_{f1} \), vary from point to point on the surface because the source subtends a different angle at each point. In order to find the form of the reflected image of the edge, we must know the values of \( \theta_{f0} \) and \( \theta_{f1} \) at every patch on the surface. These values will depend on the relative positions of the source and surface; for instance, for a source which is far from the reflective object, \( \theta_{f0} \) and \( \theta_{f1} \) will vary
slowly across the surface while they will change much more quickly across the surface for a source that is closer to the object. Similarly, \( \theta_r \) will be different for different points on the surface, and it depends on the relative positions of the reflective object and the camera. All of these variables depend on the shape of the object. To sum up, \( \theta_o \), \( \theta_f \), and \( \theta_r \) are all functions of the position on the object, \( x \). We denote this by \( \theta_o(x) \), \( \theta_f(x) \), and \( \theta_r(x) \). We also note that it is possible for the radiance pattern of the incident light to vary from point to point on the reflective surface. This will be true if the source radiance depends on the direction from which the light leaves the source. Most light bulbs exhibit this behavior, as do many materials which act as sources by reflecting light. However, if the distance from the surface to the source is much larger than the length of the surface under study, the radiance pattern of the incident light will change little across the reflective surface, since the angles subtended by the source change little over this length. Even in cases where the source is relatively close to the surface, we will assume that the source is Lambertian, that is, its radiance is independent of the direction of the emitted light. We do this in order to be able to calculate the reflected radiance pattern.

Now, let us return to equation (18), which gives the reflected radiance for a patch. Once again, it is

\[
dL_r = \int \left( \frac{(\sin x X)^2}{v X} + \frac{FGS(\theta) \cos \theta dL}{\cos \theta_r} \right) \, d\theta_f
\]  

or, for an extended source

\[
L_r = \int \left( \frac{(\sin x X)^2}{v X} + \frac{FGS(\theta) \cos \theta dL}{\cos \theta_r} \right) d\theta_f
\]

We will make the simplifying assumptions that \( F = 1 \) and \( G = 1 \). The first assumption holds
fairly well over a large range of angles for most metals. The second assumption holds when both $\theta_i$ and $\theta_f$ are less than approximately 45 degrees [Nayar et al. 91]. With these assumptions and the one-dimensional form of the FADF given by equation (19), the equation for the reflected radiance becomes

$$L_r = \int_{\theta_f(x)}^{\theta_f(x+\Delta)} \left( \frac{\sin \nu X}{\nu X} \right)^2 + \cos \theta_i \exp \left( -\frac{\tan \theta_f^2}{\beta} \right) \frac{\beta \pi (\cos \theta_f)^3}{\beta \pi (\cos \theta_f)^2} d\theta_f$$

Let us now focus on finding the integral of the second term in this equation. Because the angle at which light is incident on each facet must equal the angle at which light exits the facet,

$$\cos \theta_i = \cos (\theta_i - \theta_f) = \cos (\theta_f - \theta_f) = \cos \theta_f \cos \theta_r + \sin \theta_f \sin \theta_r$$

and thus

$$L_r(x) = \int_{\theta_f(x)}^{\theta_f(x+\Delta)} \left( \frac{\sin \nu X}{\nu X} \right)^2 + \exp \left( -\frac{\tan \theta_f^2}{\beta} \right) \frac{\beta \pi (\cos \theta_f)^3}{\beta \pi (\cos \theta_f)^2} d\theta_f$$

and

$$L_r(x) = \int_{\theta_f(x)}^{\theta_f(x+\Delta)} \left( \frac{\sin \nu X}{\nu X} \right)^2 + e^{\frac{\tan \theta_f^2}{\beta}} \frac{\beta \pi (\cos \theta_f)^3}{\beta \pi (\cos \theta_f)^2} d\theta_f$$

If we remove the assumptions that the edge of the source is parallel to the grooves of the rough reflecting surface and the view vector lies in the x-z plane, we only modify the equation
slightly. Note that the rough surface is still parameterized by a single variable, \( \theta_f \), so that we may still calculate the reflected radiance by means of equation (24). For this case, however, \( \cos \theta_{il} \) is given by

\[
\cos \theta_{il} = \cos \theta_f \cos \theta_r + \sin \theta_f \sin \theta_r \cos (\varphi_f - \varphi_r) \tag{27}
\]

Because the surface is "one-dimensionally" rough, however, \( \varphi_f = 0 \), and thus

\[
\cos \theta_{il} = \cos \theta_f \cos \theta_r + \sin \theta_f \sin \theta_r \cos \varphi_r
\]

so that

\[
L_r(x) = \int_{\theta_\mu(x)}^{\theta_n(x)} \left( \frac{\sin \nu_x X}{\nu_x X} \right)^2 \, d\theta_f + \frac{1}{2} \left[ \text{erf} \left( \frac{\tan \theta_f}{\beta} \right) - \text{erf} \left( \frac{\tan \theta_r}{\beta} \right) - \frac{\beta}{\sqrt{\pi}} \tan \theta_r \cos \varphi_r \left( e^{-\left( \frac{\tan \theta_f}{\beta} \right)^2} - e^{-\left( \frac{\tan \theta_r}{\beta} \right)^2} \right) \right] \tag{28}
\]

In order to complete this description of the reflection of light from rough surfaces, we must calculate the integral of the first term in equation (28), the specular spike component. As stated in Chapter 2., \( \nu_x \) is the x-component of the scattering vector, or

\[
\nu_x = (-k) (\sin \theta_r + \sin \theta_i)
\]

Therefore, the specular spike component of reflection is

\[
SINC^2 \left( (-k) X (\sin \theta_i + \sin \theta_r) \right) = \left( \frac{\sin (kX (\sin \theta_i + \sin \theta_r))}{kX (\sin \theta_i + \sin \theta_r)} \right)^2 \tag{29}
\]

where \( k \) is the wavenumber of the light, or \( 2\pi \) divided by the wavelength. Figure 8 shows the cen-
Figure 8: The central lobe of the square of the sinc function and a Gaussian of approximately the same width.

It is obvious that we may approximate the spike component with a Gaussian with minimal loss of accuracy. This fact was first noted by Nayar [Nayar et al. 91]. Therefore, we model the radiance of the spike component as:

$$dL_{r(spike)} = l\epsilon \left( \frac{k\zeta (\sin \theta_i + \sin \theta_r)}{\cos \theta_r} \right)^2$$

(30)

where we have set $x$ equal to $\frac{\zeta}{\cos \theta_r}$, and $\zeta$ is the cross-sectional length of the incident beam. If $\zeta$ remains constant, the length of the illuminated portion of the surface will change in this fashion.

By equation (21), we may write

$$\sin \theta_i + \sin \theta_r = 2 \sin \left( \frac{\theta_i + \theta_r}{2} \right) \cos \left( \frac{\theta_i - \theta_r}{2} \right) = 2 \sin \theta_i \cos \left( \frac{\theta_f - \theta_r}{2} \right) = 2 \theta_i \cos \theta_r$$
where we may make the final approximation because, as we stated in Section 2.4., the specular spike component is focused in a narrow range of angles and thus has non-zero radiance only for small values of $\theta_f$. Thus, for a point source, the reflected radiance of the spike component is

$$dL_{r(spike)} = le^{-\left(\frac{2k\xi}{\theta_f}\right)^2} = le^{-\left(\frac{\theta_f}{\alpha}\right)^2}$$

where we now let $\alpha$ quantify the angular spread of the spike component. We expect that $\alpha$ will remain constant with respect to roughness, since this has no effect on the wavelength of the incident light or the effective beamwidth. For a step edge, the reflected radiance of the specular spike component is

$$L_{r(spike)} = \int_{e^{(x,y)}}^{\theta_f(x,y)} e^{-\frac{\theta_f}{\alpha}} d\theta = 1 - \int_{e^{(x,y)}}^{\theta_f(x,y)} e^{-\frac{\theta_f}{\alpha}} d\theta$$

We now substitute this expression into equation (28) and find the equation for the radiance of light reflected from a rough surface to be

$$L_r(x) = l\int_{\frac{\theta_f}{\alpha}}^{\theta_f(x,y)} e^{-\frac{\theta_f}{\alpha}} d\theta +$$

$$\left(\frac{l}{2}\left(\epsilon r f\left(\frac{\tan \theta_f(x)}{\beta}\right) - e^{-\frac{\theta_f(x)}{\beta}} \cos \phi(x) \left(e^{-\frac{\left(\tan \theta_f(x)\right)^2}{\beta}} - e^{-\frac{\left(\tan \theta_f(x)\right)^2}{\beta}}\right)\right)\right)$$

In our experiments with rough surfaces and roughness estimation algorithms, we find that the ratio of the radiances of the two components of reflection does not always behave as predicted by equation (33). We therefore allow the magnitudes of the two components to vary independently in our algorithms. This is probably an over-generalization of the reflected radiance equation, since
there appears to be some relationship between the two radiances, but since we do not know the form of this relationship, we assume their independence. We thus model the radiance of the reflected light as

\[
L_r(x) = A \left( \text{erf} \left( \frac{\theta_f(x)}{\alpha} \right) - \text{erf} \left( \frac{\theta_o(x)}{\alpha} \right) \right) + B \left( \frac{1}{2} \left( \text{erf} \left( \frac{\tan \theta_f(x)}{\beta} \right) - \text{erf} \left( \frac{\tan \theta_o(x)}{\beta} \right) - \frac{\beta}{\sqrt{\pi}} \tan \theta_r(x) \cos \varphi_r(x) \left( e^{-\frac{(\tan \theta_f(x))^2}{\beta}} - e^{-\frac{(\tan \theta_o(x))^2}{\beta}} \right) \right) \right) \tag{34}
\]

where we have introduced \( A \) and \( B \) to describe the magnitudes of the two components. Any changes in the multiplicative constant \( l \) can be equivalently modelled by changes in \( A \) and \( B \), and therefore, \( l \) no longer appears in the equation. Further discussion of the behavior of the relative radiances of the specular spike and specular lobe components appears in Chapter 4.

Equation (34) allows us to calculate the radiance of the reflected edge image at any point on the rough reflective surface. The behavior of this equation with respect to its parameters, such as the roughness and the viewing angle, determines how much information we may obtain from the reflected image. For example, if the form of a reflected edge varies little with respect to roughness, then no method of roughness estimation based on edge analysis will work. Therefore, let us investigate the behavior of this equation, so that we may develop realistic methods of roughness estimation.

### 3.3. Limitations on the Differentiation of Edges

We may now use equation (34) to predict the appearance of edges under a variety of conditions: different roughnesses, different object shapes, and different source, object, and camera configurations. We want, however, to be able to solve the inverse problem, that is, given the
appearance of the edge, find the roughness of the object. The measured appearance of the edge, including the effects of noise and finite image size, will obviously affect our ability to determine the roughness. We will attempt to quantify such limitations. To do this, we will simulate the appearance of reflected edges for different roughnesses and different source, object, and camera configurations. If we find that for a particular configuration the reflected edge image does not change with respect to roughness, then we conclude that for this configuration, no information on the roughness of the surface may be obtained.

3.3.1. Source-Object-Camera Configuration for the Simulations

We will begin by studying some edge profiles for the simple case in which the reflective surface is planar, and the source is an infinite half plane parallel to the reflective surface. Figure 9 is a schematic of this configuration. We also assume that the camera is far from the surface, so that $\theta,$

![Figure 9: Schematic of the simulation configuration](image)
is approximately constant across the region of interest of the surface. In our simulations, the parameters of this configuration which we will change are the surface roughness, the perpendicular distance between the plane of the source and the plane of the reflective surface, and the viewing angle.

We choose this arrangement of source, object, and camera for several reasons. First, this is essentially the configuration that will be used in our later experiments. Therefore, any insight we gain from these simulations can be applied to our experiments. Secondly, the assumption of a planar surface allows us to ignore the effects of the object curvature on the appearance of the edge. This will make our initial set of simulations easier to understand. We will discuss the effects of surface curvature on reflected edge profiles briefly at the end of the chapter.

The choice of an infinite half-plane serves several purposes. Note that equation (34), the expression for the reflected radiance, is a difference between two similar terms, one dependent on the variable \( \theta_{0} \), which describes the angular position of one side of the bright region in the illumination environment, and another term dependent on the variable \( \theta_{1} \), which gives the position of the other side of the bright region. Thus, equation (34) actually represents the difference of the two edges which bound the bright region in the illumination environment. If we allow our source to be an infinite half plane, we essentially force \( \theta_{0} \) to go to \(-\frac{\pi}{2}\), and we will be able to study the appearance of a single edge. The closeness of the source serves another purpose. The values of \( \theta_{1} \) will vary from point to point on the reflective surface, while the values of \( \theta_{r} \) will not change much. Note that the error function terms in equation (34), which are the dominant terms, are symmetric with respect to \( \theta_{1} \) and \( \theta_{r} \) by equation (21). Thus, if the source were far from the object,
and the viewer near, it would be possible to obtain the same reflected edge image as with the opposite configuration. We would also see the same changes in the appearance of the edge with respect to changes in \( \theta_r \), as we did in the opposite case with respect to changes in \( \theta_{11} \). Thus, this initial set of simulations will be applicable to the case with the near camera and far source, as long as we are careful to reverse the roles of the variables.

Figure 10 is the first example of our sets of simulated edge profiles, and shows the intensity of the edge as a function of position along the surface. It shows five profiles graphed together to facilitate comparison. We choose the scale in these graphs to reflect the values in common imaging situations. Our camera records intensity values in the range from 0 to 4095, and in order to prevent clipping, we try to keep the maximum measured intensity in the range from 3000 to 4000. For this series of simulations, the values of the parameters in equation (34) are

![Figure 10: Edges for several values of beta. As the roughness increases, the top and bottom of the edge become more rounded.](image)
• $A$ - the magnitude of the specular spike component, 1000 intensity units

• $B$ - the magnitude of the specular lobe component, 1000 intensity units

• $\theta_r$ - the viewing angle, 0 degrees

• $D$ - the source to surface distance, 500 length units

• $\alpha$ - the width of the specular spike component, 0.0. For this value of $\alpha$, the specular spike component becomes a delta function.

$\beta$, the rms slope, takes on the values 0.00, 0.025, 0.050, 0.075, and 0.100. As expected, the higher the value of $\beta$, or the rougher the surface, the more the lobe component spreads out, and the more rounded the top and bottom of the edge profile become. Let us now change the values of the parameters, and see what effects this will have on the edge profiles.

3.3.2. The Effect of Changes in the Source-Object Distance

Consider equation (34) once again. The variable $\theta_{f1}$ gives the angle of the normal to those facets which will reflect light from the edge of the source into the camera. The functional form of $\theta_{f1}$ across the surface, $\theta_{f1}(x)$, largely determines the appearance of the reflected edge. For our simulations, with a planar surface, parallel planar source, and distant camera, equation (21) yields

$$\theta_{f1}(x) = \frac{\tan\left(\frac{x}{D}\right) + \theta_r}{2}$$

where $D$ is the perpendicular distance between the source and surface, and the origin of the coordinate system is at the point on the surface closest to the edge of the source. Figure 11 shows the relationship between the angles.
Now, let us find out the effect of changing $\theta_f(x)$ by varying the source to object distance. Figure 12 and Figure 13 show sets of simulated edges which all have $\theta_r$ equal to 0 degrees, $A$ equal to 1000 intensity units, $B$ equal to 1000 intensity units, $\alpha$ equal to 0.0, and $\beta$ equal to 0.00, 0.025, 0.050, 0.075, and 0.100. Figure 12 has the source to object distance equal to 750 length units, and Figure 13 has a value of 1000 length units. Therefore, these simulations differ from that of Figure 10 only in the distance from the source to the object.

We see in the progression from Figure 10 to Figure 12 to Figure 13, that as the source-object distance increases, the curves spread out more. Thus, a fairly smooth surface at a large distance will have an intensity profile similar to that of a rougher surface with a smaller source-object distance. In fact, we can see that this effect of edge broadening at increasing source-object distances becomes more pronounced at greater roughness values. Therefore, changing the source-object distance for a rough surface will change the appearance of the reflected edge greatly, while it will
Figure 12: Edges for several values of beta. \( A = 1000, B = 1000, \theta_r = 0.0 \) degrees, \( D = 750, \alpha = 0.0 \).

Figure 13: Edges for several values of beta. \( A = 1000, B = 1000, \theta_r = 0.0 \) degrees, \( D = 1000, \alpha = 0.0 \).
cause less of a change for a smoother surface, and a perfectly smooth surface appears the same at all source-object distances. We conclude, therefore, that the distance between the source and object planes drastically affects the reflected image of the edge. This is especially true the rougher the surface.

3.3.3. The Effect of Changes in the Viewing Angle

Next, let us test the effect of changing $\theta_r$ on the appearance of the edges. We expect to see some change, since as we vary the viewing angle, different facets will reflect light from the source into the camera. Everyday experience, however, indicates that there might not be a large amount of change until $\theta_r$ approaches grazing angles, since the appearance of images reflected in metals tend to look the same for most viewing directions.

Figure 14 to Figure 16 show sets of simulated edge profiles which all have a source to object distance of 750 units, $A$ equal to 1000 intensity units, $B$ equal to 1000 intensity units, $\alpha$ equal to 0.0, and $\theta_r$ equal to 0, 15, 30, and 45 degrees. Figure 14 has $\beta$ equal to 0.0, Figure 15 has $\beta$ equal to 0.05, and Figure 16 has $\beta$ equal to 0.1. As expected, the curves with $\beta$ equal to 0.0, i.e. the profiles for a perfectly smooth surface, look the same for all angles $\theta_r$. For the other values of $\beta$, the edge profiles broaden as we increase $\theta_r$, although the effect is not as strong as in the simulations in which we varied the source to object distance. Also, this broadening is greater for greater roughness values. Another feature of these graphs is that for greater values of $\theta_r$, the last term of equation (34) becomes larger, and the lobe component changes slightly, but noticeably. This results in the apparent asymmetry of the graphs, which is actually because of the slight change in height of the lobe component.
Figure 14: Edges for several values of $\theta$. $A = 1000$, $B = 1000$, $D = 750$, $\alpha = 0.0$, $\beta = 0.0$.

Figure 15: Edges for several values of $\theta$. $A = 1000$, $B = 1000$, $D = 750$, $\alpha = 0.0$, $\beta = 0.05$. 
Our next conclusion is that the viewing angle does indeed affect the shape of the reflected edge profile, albeit not as drastically as did the source-object distance.

3.3.4. The Effect of Changes in the Specular Spike Width

For the sake of completeness, we must study $\alpha$, which determines the width of the specular spike component of reflection. The expression for the part of the edge profile due to the spike component is quite similar to that due to the lobe component, so we expect it to spread with increasing $\alpha$.

Figure 17 shows edge profiles where the distance from the source to surface is 750 length units, $\theta_r$ is 0 degrees, $A$ is 1000 intensity units, $B$ is 1000 intensity units, $\beta$ is 0.05, and $\alpha$ takes on the values 0.0, 0.015, and 0.030. The behavior is similar to that for a change of $\beta$, but, as expected, it is now the spike component which spreads. According to our model, $\alpha$ has no depen-
Figure 17: Edges for several values of alpha. $A = 1000$, $B = 1000$, $\theta_r = 0.0$ degrees, $D = 750$, $\beta = 0.05$.

dence on other parameters such as $\theta_r$, $\beta$, and the source to object distance. Our experiments will help us to determine if this is in fact true.

3.3.5. Ambiguous Edges

We now have an idea how the intensity profile across an edge varies as the roughness, viewing angle, and source to object distance change. With this knowledge, we may point out some limitations in the analysis of edges.

First, consider Figure 18. In it, we see two edge profiles. One of these has $A$ equal to 1000 intensity units, $B$ equal to 1000 units, $\alpha$ equal to 0.0, $\beta$ equal to 0.10, $\theta_r$ equal to 0 degrees, and a source to object distance of 750 length units. The other edge profile has $A$ equal to 1000 intensity units, $B$ equal to 1000 units, $\alpha$ equal to 0.0, $\beta$ equal to 0.15, $\theta_r$ equal to 30 degrees, and a source to object distance of 425 length units. Obviously, the surfaces and configurations are quite differ-
ent. Now, imagine that a configuration of source, object, and camera has the first set of parameters. The measured intensity curve of the edge will be similar to the first curve in Figure 18, albeit with noise added. If the noise level is greater than the difference between the curves, however, it will be impossible to determine which set of parameters generated the measured curve. For example, our camera has a signal-to-noise ratio of approximately 100 to 1. Therefore, at any position in the upper half of the profiles where the two curves differ by approximately 30 intensity units or less, the profiles will be indistinguishable. Since this represents a sizable portion of the edge profiles, the complete curves will probably be indistinguishable, and it will be difficult to calculate the values of the correct parameter set.

We may generalize this analysis and draw some conclusions. In the case of a perfectly smooth surface, the edge looks the same under all conditions, and there exist an infinite number of param-
eter sets which may generate the same edge profile. In this case, however, it is easy to determine the roughness, $\beta$, which is of interest to us. For other values of the roughness, however, there exist many parameter sets which may generate the same curve, to within noise levels, thus making any analysis of the roughness suspect. Therefore, our first conclusion about the limitations of edge analysis is that for any surface which is not perfectly smooth, the viewing angle and source to object distance must be known in order to determine the surface roughness.

Next, consider Figure 19. One of these edges has $A$ equal to 1000 intensity units, $B$ equal to 1000 units, $\alpha$ equal to 0.0, $\beta$ equal to 0.15, $\theta_r$ equal to 0 degrees, and a source to object distance of 1000 length units; the other has the same values for all parameters except $\beta$, which equals 0.20. In Figure 13, these curves look significantly different, and yet, in Figure 19, their differences are

![Figure 19: Part of the edge profiles for beta equal to 0.15 and 0.20. $A = 1000$, $B = 1000$, $\theta_r = 0.0$ degrees, $D = 1000$, $\alpha = 0.0$.](image)
small; in fact, the greatest difference between the curves is only about fifty intensity units. This is obviously due to the fact that we only show part of the complete edge profile. If we attempt to determine the roughness from one of these graphs, it will therefore be difficult to obtain an accurate value for $\beta$. For a fixed image width, this effect is more pronounced at greater roughness values and greater source-object distances, since these widen the reflected image of the edge. Therefore, our next conclusion about the limitations of edge analysis is that we should have the full edge profile in order to accurately measure the roughness. Although this may seem to be a minor point to make, the truncation of the reflected image of the edge may easily occur in real imaging environments.

3.4. The Effect of Surface Shape

We have seen how the image of a step edge reflected in a planar rough surface behaves with respect to source-object distance, viewing angle, and roughness. Another variable which we have not yet studied is the shape of the reflective surface. We will see that the effect of the object shape on the reflected edge image is mostly a combination of the effects we have already studied.

Consider a generic surface. The position of a patch of the surface and its orientation are well-defined, although they may take on any values. As we move across the surface, these two values will change, and thus, so will the viewing angle, $\theta_r$, and the angle of the facets which reflect light from the edge of the source, $\theta_{fl}$. With the proper placement and orientation of the patch, it is possible to obtain any value for the reflected radiance between zero and a maximum value which must be less than or equal to the radiance of the source. The proper placement and orientation of all patches on the surface can thus generate any given edge profile. Therefore, if we know the position and orientation of all patches on the surface, we may use equation (34) to calculate the
appearance of the reflected image of a step edge for a given roughness and configuration of source, object, and camera, or we may solve the inverse problem and calculate the roughness, given the same knowledge about the configuration. If we do not know the shape of the reflective object, however, such calculations are impossible, since a given edge profile can result from a multitude of configurations of source, camera, object position, and object shape. If we assume that the surface is continuous, this limits the number of configurations which can produce the reflected edge image, but not enough to produce a one-to-one mapping between edge profiles and surface shapes. We therefore conclude that it is necessary to know the shape of the reflective object in order to calculate the roughness of the surface.

The effect of the surface shape on the reflected edge image is even more complicated than this analysis seems to imply. Curved mirrors may act as lenses, and they suffer from many of the same distortions, such as spherical aberration and coma. Therefore, some blurring of the edge image may be due to the shape of the surface, rather than the roughness. As a result, roughness estimation algorithms will tend to calculate the value of the roughness as being greater than its true value. Again, in order to account for such effects, the shape of the surface must be known.

Despite all of these difficulties, there are a few general statements that we can make about the effect of surface shape on the behavior of reflected images of step edges. First, as long as the surface curvature is small relative to the extent of the edge, we may approximate the surface as a plane over the region of interest, and use the information we obtained in Section 3.3.. Secondly, if the surface has a single sign of curvature, the width of the edge varies monotonically with respect to the curvature. Consider Figure 20. Here we see a step edge and a convexly curved reflective surface. The values of $\theta_r$ and $\theta_i$ change more quickly with respect to $x$ than they do for a plane,
and the reflected image of the edge thus appears narrower than it does for a planar surface. Conversely, the reflected image appears wider for a concavely curved surface. Such considerations may prove useful in the analysis of edges by simplifying the amount of information necessary about the shape.

To reiterate, the shape of the reflective surface has a profound effect on the appearance of reflected step edges, and must be known for any roughness estimation algorithm.

3.5. Conclusions

In this chapter, we used the Beckmann and Spizzichino model of reflection to calculate a closed form expression for the image of a step edge reflected in a rough surface. We then showed how the reflected image changes as a function of the relative positions of the source and surface,
the viewing angle, and the surface shape. From our simulations, we were able to draw several conclusions. First, that in order to determine the roughness of a surface from the reflected images of step edges, the shape of the surface as well as its position relative to the source and the camera must be known. Additionally, the image should contain as much of the reflected edge profile as possible in order to prevent the calculation of incorrect parameter sets and roughness values. These limitations apply not only to the roughness estimation methods we present in this thesis, but to any method based on reflected edge images. Let us keep these results in mind and present our methods for roughness estimation.
4. An Iterative Roughness Estimation Method

4.1. Chapter Overview

In this chapter, we present an algorithm which calculates the roughness of a surface by iteratively fitting a curve to the reflected image of an edge with a gradient descent algorithm. We first describe the algorithm, and then discuss five rough surfaces on which we test the algorithm. The rms slopes of the surfaces are found by stylus profilometry, a conventional method of roughness measurement. We then use our algorithm on the images of step edges reflected in the five samples and compare the results of our method with those found with the profilometer. We find that our method orders all of the surfaces correctly by roughness. We also conclude that although this method ranks the surfaces well, it may not be applicable as a mensuration scheme.

4.2. The Algorithm

Our first algorithm is actually a quite obvious way of determining the roughness of a reflective surface. It fits a curve described by our model to the profile of a real reflected edge; the value of the rms slope which results in the best fit curve is then taken to be the rms slope of the surface. This procedure is not as simple as it may first seem, and we want to discuss some of the finer points in its application.

Let us begin by returning to equation (34), the equation for the appearance of a reflected step edge image under the assumption of the Beckmann and Spizzichino model of reflection. It is
\[ L_s(x) = A \left( \operatorname{erf} \left( \frac{\theta_i(x)}{\alpha} \right) - \operatorname{erf} \left( \frac{\theta_f(x)}{\alpha} \right) \right) + \]

\[ B \left( \frac{1}{2} \left[ \operatorname{erf} \left( \frac{\tan \theta_i(x)}{\beta} \right) - \operatorname{erf} \left( \frac{\tan \theta_f(x)}{\beta} \right) - \frac{1}{\beta \sqrt{\pi}} \tan \theta_s(x) \cos \varphi_r(x) \left( e^{-\left( \frac{\tan \theta_f(x)}{\beta} \right)^2} - e^{-\left( \frac{\tan \theta_i(x)}{\beta} \right)^2} \right) \right] \right) \] (35)

In order to use our roughness fitting algorithm, we must find the values of \( \theta_i \) and \( \theta_f \) for all points on the surface. These give the orientation of the normals to those facets which reflect light from one edge of the source into the camera. We already found these values for our simulations in Section 3.3.2. Let us briefly repeat the derivation. Figure 21 shows the source and reflective object as well as a ray of light that leaves the source and reflects off the surface. Note that since we assume that the camera is far from the surface, \( \varphi_r \) is a constant for all points on the reflective sur-

---

**Figure 21**: A schematic of the experimental configuration
face. For our experimental configuration, which we describe more completely in Section 4.4., \( \phi_r = 0 \), and the plane of reflection is perpendicular to both the reflective surface and the edge of the source, and we may derive all the necessary terms from Figure 22. Let \( x_{src} \) be a point on the source, and \( x \) be a point on the object. We see that

\[
\theta_i = \arctan \left( \frac{x_{src} - x}{D} \right)
\]

where \( D \) is the distance between the source plane and the object plane. Also, by equation (21),

\[
\theta_f = \frac{\theta_i + \theta_r}{2} = \frac{\arctan \left( \frac{x_{src} - x}{D} \right) + \theta_r}{2}
\]

Because we model the step edge as a half-infinite plane, \( \theta_0 \) is equal to \( -\frac{\pi}{2} \) for all points on the surface. If we let the position of the edge be the origin, i.e. \( x_{src} = 0 \) at the edge,

Figure 22: The reflection geometry for the experimental configuration
\[
\theta_{f1} = \frac{\arctan\left(\frac{-x}{D}\right) + \theta_r}{2}
\]

We therefore know the initial and final values of \(\theta_r\) for all points on the object, and we may use equation (35) to calculate the radiance of the reflected light.

Now, in order to fit the model to the data, we must account for several effects which change the appearance of the reflected image, but which are not included in equation (35). First, note that the dimmest parts of the image will not have a zero intensity, but rather, there will be a constant offset due to ambient light and perhaps camera effects such as the dark current, which is the current present in the CCD even when no light enters the camera. We model this with the constant, \(C\). Next, note that the value given for \(\theta_r\) is the result of a measurement and is therefore not exact; the same is true for \(\theta_i\). Apart from the term \(\tan \theta_r\), which is a small number for this experimental configuration and which is multiplied by other small numbers, \(\theta_r\) and \(\theta_i\) only appear as part of their sum. Therefore, we may model the errors in both \(\theta_r\) and \(\theta_i\) by replacing \(\theta_r\) with \(\theta_r + \Delta \theta\). The final model for the reflected edge is then

\[
L_r(x) = A \left( \text{erf}\left(\frac{\theta_{f1}(x)}{\alpha}\right) - \text{erf}\left(\frac{-\theta_i(x)}{\alpha}\right) \right) + B \left( \frac{1}{2} \left( \text{erf}\left(\frac{\tan \theta_{f1}(x)}{\beta}\right) - \text{erf}\left(\frac{-\tan \theta_i(x)}{\beta}\right) \right) - \frac{\beta}{\sqrt{\pi}} \tan (\theta_i(x) + \Delta \theta_r) \left( e^{-\left(\frac{\tan \theta_{f1}(x)}{\beta}\right)^2} - e^{-\left(\frac{\tan \theta_i(x)}{\beta}\right)^2} \right) \right) + C
\]

where

\[
\theta_{f1} = \frac{\arctan\left(\frac{-x}{D}\right) + \theta_r + \Delta \theta_r}{2}
\]
and $\theta_{\rho}$ is equal to $-\frac{\pi}{2}$. Our goal is to find the values of $A, B, C, \beta, \alpha$, and $\Delta\theta$ which best fit the data.

For any set of values of the parameters $A, B, C, \beta, \alpha$, and $\Delta\theta$, we can form a predicted edge profile, which we can then compare with the data by calculating the value of chi squared, which is a weighted sum of squared differences. The lower the value of chi squared, the better the fit. Let $R(x)$ be the radiance of the reflected edge at point $x$, and let $\rho(x)$ be the standard deviation in the radiance values measured at that point. If $L_r(x)$ is the radiance predicted by the model, then the value of chi squared is

$$\chi^2 = \sum_x \frac{(R(x) - L_r(x))^2}{\rho_r(x)}$$

(37)

where the summation is over all values of $x$, that is, over the whole edge profile. It is tempting to simply use a gradient descent algorithm on all six variables in order to minimize the value of chi squared. This method will not work, however, since the algorithm will likely get stuck in a local minimum of the search space. Therefore, the values of the parameters are found as follows.

First, the minimum value of the reflected edge is taken as the initial guess for $C$. Next, the reflected edge profile that would be obtained were the surface perfectly smooth is formed as a predicted edge. This edge profile has a minimum value of $C$ and maximum value equal to the maximum value of the true reflected edge. The value of chi squared for these two curves is then found for different values of $\Delta\theta$. The minimum with respect to $\Delta\theta$ is found by bracketing and bisection, and the value of $\Delta\theta$ which gives this minimum is taken as the best fit value. Given this value of $\Delta\theta$, the pixel at which the reflected edge is “located”, i.e., the pixel at which the discontinuity would occur if the surface were perfectly smooth, is calculated. The intensities of the data edge
profile a few (usually three) pixels on either side of this pixel are recorded, and their difference is taken as the initial value of $A$. The first guess for $B$ is then the maximum value of the reflected edge, minus $C$, minus $A$. Figure 23 shows a typical edges profile, and the initial values for these parameters.

At this point, the initial guesses for $A$, $B$, $C$, and $\Delta \theta$ have been determined, and the iterative search for the best fit parameters begins. A gradient descent algorithm finds the value of $\beta$ which minimizes chi squared for these initial values of $A$, $B$, $C$ and $\Delta \theta$, with $\alpha$ equal to 0.0. $\beta$ starts at 0.1; we choose this value because it seems to be in the middle of the range of $\beta$ for common objects, and the algorithm converges well when starting from this value. When this gradient descent subroutine reaches its minimum, $\beta$ is held fixed, and the minimum with respect to $\alpha$ is found by gradient descent. When this reaches a minimum, singular value decomposition is used to

![Figure 23: The initial guesses for the parameters $A$, $B$, and $C$ for an edge profile. The value of $\Delta \theta$ is related to the position of the edge.](image-url)
find the best fit values of $A$, $B$, and $C$ for these values of $\beta$ and $\alpha$. The algorithm cycles between fitting for $\beta$, for $\alpha$, and for $A$, $B$, and $C$ until convergence is reached.

In this section, we have presented an algorithm which utilizes both gradient descent and singular value decomposition to estimate the roughness of reflective surfaces. The algorithm uses a model with six parameters: $A$ and $B$, the magnitudes of the specular spike and specular lobe components of reflection, respectively, $C$, the offset of the edge profile, $\beta$, the rms slope of the surface, $\alpha$, the width of the specular spike component of reflection, and $\Delta \theta$, the sum of the error in the measurement of the viewing angle and the angle of incidence. The algorithm finds the values of the six parameters which best fit the measured reflected edge image in the least-squares sense. Let us now describe the results of using this algorithm on real images and surfaces.

4.3. The Surfaces

In order to test the accuracy of our method of roughness estimation, we had to determine if our calculated roughness values corresponded to those found by conventional means. This acted as a test of the appropriateness of the method as a mensuration scheme. Therefore, we measured our samples with a stylus profilometer, so that we could compare these measurements with those made with our algorithm. In this section, we describe these measurements, and we also discuss the validity of our assumption that the surfaces have a Gaussian distribution of height values, which allowed us to determine if the distribution of height values affected the reflected edge profile.

4.3.1. Height and Slope Statistics

In our experiments, we used samples of reflective surfaces with known roughness values. Our samples were circular steel disks, three inches in diameter, which had been milled to produce a
one-dimensionally rough surface on one face of each disk. The values of the arithmetic average roughness, \( R_A \), as stated by the manufacturer, were 1.5, 3.0, 4.5, 7.5, and 9.5 microinches.\[Alcoa 94\] As we mentioned in Chapter 2., the rms height and rms slope determine the distribution of the scattered light more than do other surface statistics such as arithmetic average roughness.\[Beckmann and Spizzichino 63\][Stover 90] Therefore, we could not use the roughness values stated by the manufacturer, but instead calculated the rms slopes of the surfaces from measurements made with a profilometer.

Before we describe our measurements, we mention a caveat. As noted by Bennett and Mattson \[Bennett and Mattson 89\], different values of surface roughness will be obtained for the same surface with different instruments. For example, stylus profilometers generally measure higher values for the roughness of a surface than do optical profilometers. Keeping this in mind, we performed the following measurements. A Tencor Alpha Step 200 stylus profilometer was used to measure the surface profiles of the samples. For each scan, the profilometer measured the surface height at 2000 points over a 400 micrometer distance, and ten scans were made for each sample.

We used these measurements to calculate the root-mean-square heights of the surfaces. The calculated rms height values appear in Table 1. The values in microinches are given for comparison with the stated roughness values. Remember that the roughness values stated by the manufacturer were the arithmetic average roughness, rather than the root mean square roughness, and therefore need not be the same as the values we calculate. Notice the unusually high value for the 7.5 microinches roughness sample; also note that the rms values place the samples in the same order as do the arithmetic average roughness values.

Our edge fitting algorithm produces estimates of the standard deviation of the facets normals,
### Measured rms roughness value

<table>
<thead>
<tr>
<th>Sample</th>
<th>Micrometers</th>
<th>Microinches</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 microinches</td>
<td>0.0366 ± 0.0002</td>
<td>1.44</td>
</tr>
<tr>
<td>3.0 microinches</td>
<td>0.0690 ± 0.0003</td>
<td>2.72</td>
</tr>
<tr>
<td>4.5 microinches</td>
<td>0.136 ± 0.001</td>
<td>5.34</td>
</tr>
<tr>
<td>7.5 microinches</td>
<td>0.241 ± 0.001</td>
<td>9.48</td>
</tr>
<tr>
<td>9.5 microinches</td>
<td>0.255 ± 0.001</td>
<td>10.05</td>
</tr>
</tbody>
</table>

Table 1

rather than rms height values. In most cases, it is difficult to relate these two parameters, but for a surface with a Gaussian distribution of heights, we know that the facet area distribution function for such a surface is [Beckmann and Spizzichino 63]

\[
S(\theta) = \frac{T}{2\pi \sqrt{\cos \theta}} \frac{T(\tan \theta)^2}{4\sigma^2} \tag{38}
\]

where \( \sigma \) is the rms height and \( T \) the correlation length of the surface. Comparison with equation (19) shows that \( \beta = \frac{2\sigma}{T} \). In order to find the correlation length of a surface with \( N \) height samples at positions \( x_i \), we form the autocorrelation function,

\[
C(\tau) = C(\Delta) = \frac{1}{N} \sum_{i=1}^{N} h(x_i)h(x_{i+\Delta})
\]
where $\Delta$ is the spacing between the height sample positions, and $\tau = j\Delta$. This function should have a maximum at $\tau_i = 0$. The correlation length, $T$, is defined as the position where the autocorrelation function reaches $e^{-1}$ times its peak value.

The correlation lengths of the surfaces were found by computing the autocorrelation function of each profile, and then fitting a function of the form

$$e^{-\left(\frac{\tau}{T}\right)^2}$$

in the least-squares sense to each. The correlation length was found from this fitted function. The resulting values appear in Table 2. Again, notice the large value for the 7.5 microinches roughness sample. Graphs of the scans from the profilometer confirm that this sample has a high amplitude, long wavelength variation imposed on the surface in addition to the roughness variations. This

**Calculated correlation length**

<table>
<thead>
<tr>
<th>sample</th>
<th>correlation length (micrometers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 microinches</td>
<td>5.8 ± 2.7</td>
</tr>
<tr>
<td>3.0 microinches</td>
<td>8.6 ± 3.8</td>
</tr>
<tr>
<td>4.5 microinches</td>
<td>5.1 ± 1.2</td>
</tr>
<tr>
<td>7.5 microinches</td>
<td>21.2 ± 2.4</td>
</tr>
<tr>
<td>9.5 microinches</td>
<td>4.2 ± 0.8</td>
</tr>
</tbody>
</table>

*Table 2*
long wavelength variation is probably due to chatter during the milling process; chatter is the motion of the milling head in the direction perpendicular to its travel, and is due to poor stabilization of the head. Visual inspection verifies the existence of chatter for this sample. Let us now calculate $\frac{2\sigma}{T}$ for the five samples. We denote this quantity $\beta_{\text{profi}}$ in order to differentiate it from estimates of the rms slope as calculated by other methods.

Table 3 shows the calculated values of $\beta_{\text{profi}}$. In order to make the roughness parameter easier to understand, we will express it both as a slope and as the angle of a normal to a facet, since this second representation is easier to visualize. Figure 24 shows how we may do this. First, however, we note that although we refer to $\beta$ as the root-mean-square slope of the surface, it is actually equal to the root-mean-square slope multiplied by the square root of two. With this knowledge,

<table>
<thead>
<tr>
<th>sample</th>
<th>$\beta_{\text{profi}}$</th>
<th>rms slope (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 microinches</td>
<td>0.0126 ± 0.0059</td>
<td>0.510 ± 0.239</td>
</tr>
<tr>
<td>3.0 microinches</td>
<td>0.0160 ± 0.0071</td>
<td>0.648 ± 0.288</td>
</tr>
<tr>
<td>4.5 microinches</td>
<td>0.0533 ± 0.0126</td>
<td>2.16 ± 0.510</td>
</tr>
<tr>
<td>7.5 microinches</td>
<td>0.0227 ± 0.0026</td>
<td>0.920 ± 0.105</td>
</tr>
<tr>
<td>9.5 microinches</td>
<td>0.121 ± 0.0231</td>
<td>4.89 ± 0.936</td>
</tr>
</tbody>
</table>

Table 3
we see that the root-mean-square angle of the normals to the facets is \( \tan(\beta/\sqrt{2}) \). There are several things that we note immediately from the table. First, the values of \( \beta_{\text{prof}} \) can be used to differentiate the surfaces by roughness, although the error in the measurements is large enough to cause some overlap between the values. More important, however, is the fact that the ordering of the surfaces by roughness is not the same as it was when the arithmetic average roughness was used as the roughness criterion. We see that the fourth sample, which has an arithmetic average roughness of 7.5 microinches, has a lower value of \( \beta_{\text{prof}} \) than does the third sample, which has an arithmetic average roughness of 4.5 microinches. Since the rms slope of the surface is the quantity in which we are interested, we must compare the values obtained by our roughness estimation algorithm with these calculated values of \( \beta_{\text{prof}} \), and we hope to find the same ordering. We will, however, continue to refer to the samples by their arithmetic average roughness values, since this will be a simple identification scheme.
In this section, we calculated the roughness parameter, \( \beta_{\text{prof}} \), for the five rough surfaces, and found that we may use these values to order the surfaces by roughness, although the ordering differs from that found when considering the arithmetic average roughness values. We will compare the results of our roughness estimation algorithm with these values of \( \beta_{\text{prof}} \), but before we do this, we must study the surfaces further to determine if our assumption that they have Gaussian height distributions is true.

4.3.2. Height Distributions

As we stated in the previous section, it is possible that some or all of the sample surfaces do not have a Gaussian distribution of height values. If this is true, then the facet area distribution function used in the roughness estimation algorithm is incorrect, and we cannot expect reasonable results. Therefore, we analyzed the raw height data from our profilometer scans to determine the actual height distributions for the samples.

The profilometer measured the height values in the range from -1.500 to 1.500 micrometers in units of 0.005 micrometers. For each sample, we counted the number of occurrences of each height value in all scans, and plotted this number versus the height value to form a height histogram. Figure 25 to Figure 29 show these histograms.

These distributions appear to be fairly close to Gaussian distributions, except for the 7.5 microinches sample. The distribution for the 7.5 microinches sample looks as if it might be the sum of two Gaussian-like distributions. The existence of a long surface wavelength due to chatter during the milling process might explain the appearance of this histogram. In any case, we must be careful in interpreting the results of our algorithm on this sample, since it obviously violates our assumption of a Gaussian distribution of height values.
1.5 microinches sample
Figure 25: Number of occurrences of each height value

3.0 microinches sample
Figure 26: Number of occurrences of each height value
4.5 microinches sample

Figure 27: Number of occurrences of each height value

7.5 microinches sample

Figure 28: Number of occurrences of each height value
Let us now consider the height distributions for the other four samples. Although they appear to be Gaussian, we must test this hypothesis. As we show in Appendix 1, if we have a scan of the surface with $S$ measured points, the probability, $W$, that there will be $n$ points with height values in the range $h$ to $h + \Delta h$ is given by the binomial distribution,

$$W = \binom{S}{n} P^n (1 - P)^{S-n}$$

where $P$ is the probability of a height measurement being in the range $h$ to $h + \Delta h$. $P$ is given by

$$P = \frac{e^{- \frac{(h-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma}} \Delta h$$

where, as before, $\mu$ is the mean height of the surface, and $\sigma$ is the rms height. We know that the standard deviation of the binomial distribution is $\sqrt{SP(1-P)}$ [Scheaffer and Mendenhall 75], and
we may thus model our histograms as binomial distributions, and find a value of chi squared for each set of parameters $\mu$ and $\sigma$. The values which give the best fit to the data appear in Table 4. Notice that the mean values are not zero, as they would be for a binomial distribution. The best fit values of the rms roughness are close to those shown in Table 1. Because this was a nonlinear fit, however, it was difficult to obtain bounds on the probable error of these values, and we cannot be certain if the rms roughnesses as found by profilometric methods fall in the range of acceptable values as found in this fit. We were able, however, to use the chi-square probability function to test the goodness of the fit. The results of this procedure show that the surfaces actually are not Gaussian. When we use our algorithm on the images of step edges reflected in these samples, we will see if this violation of the assumption of Gaussian height distributions affects the results.

The analysis of the height histograms has given us insight into the validity of the assumptions

<table>
<thead>
<tr>
<th>sample</th>
<th>mean height (micrometers)</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 microinches</td>
<td>-0.0025</td>
<td>0.0416</td>
</tr>
<tr>
<td>3.0 microinches</td>
<td>-0.0025</td>
<td>0.0733</td>
</tr>
<tr>
<td>4.5 microinches</td>
<td>-0.0075</td>
<td>0.153</td>
</tr>
<tr>
<td>7.5 microinches</td>
<td>-0.0025</td>
<td>0.248</td>
</tr>
<tr>
<td>9.5 microinches</td>
<td>-0.0175</td>
<td>0.313</td>
</tr>
</tbody>
</table>

Table 4
of the Beckmann and Spizzichino model of reflection. We see that the Gaussian distribution of heights is a reasonable, if not exact, model for properly milled surfaces. The analysis has also verified our guess that the 7.5 microinches sample does not have a Gaussian height distribution. Obviously, we must be careful in the interpretation of the scattering of light from rough surfaces unless we know that the surface has a Gaussian distribution of heights. Since few models of reflection exist for other types of surfaces, it is probably wise nonetheless to use the model of Beckmann and Spizzichino in the absence of other information, because it is thoroughly developed, and because the Gaussian distribution is the limiting distribution of the sum of values drawn from arbitrary distributions, as stated in the Central Limit Theorem. In the next section, we will study images of reflected step edges and will see that this model works fairly well even for surfaces which are mildly non-Gaussian.

4.4. Experimental Setup

Let us now show the results of using our roughness estimation algorithm on real images. In this section, we will discuss how we obtained the images and how we processed the image data prior to using the algorithm; most importantly, we will present the calculated roughness values. We will show the images that we used in our experiments so that we may visually order the surfaces by roughness and check if the algorithm agrees with our intuition.

Our experimental configuration, not coincidentally, was essentially the same as that used in our simulations, and thus, we could utilize the knowledge we gained from our numerical experiments. The camera was a 12-bit Photometrics Star 1 camera, and was located far enough away from the steel disks that we could assume that \( \theta_r \) was constant for all points on the reflective surface. A large piece of white paper was used as the step edge source. Because the source had a
much greater area than the steel plates, it was an appropriate choice to approximate a semi-infinite step edge. The piece of paper was lighted by a halogen lamp. Figure 30 shows the experimental configuration.

As we stated in the previous chapter, in order to obtain accurate estimates of the surface roughness, we must know the distance between the source and object as well as the viewing angle. Positions in the Calibrated Imaging Laboratory may be accurately measured with a pair of surveyor’s theodolites. The distance from the plane of the samples to the plane of the source was measured with these and was found to be 9.57cm. The distance from the disks to the camera was simply measured with a tape measure and was found to be approximately 120cm. The calculation of the viewing angle was a bit more intricate.

The camera in the Calibrated Imaging Laboratory rests on a six degree of freedom jig, which
positions the camera to within one thousandth of an inch in its translational degrees of freedom, and to within one hundredth of a degree in its rotational degrees of freedom. In order to calculate the viewing angle, the disk was first removed from its holding stand, and was replaced by a mirror. A pin was placed approximately three inches in front of the mirror. The camera was translated and rotated until the head of the pin and its image coincided. The optical axis of the camera was then perpendicular to the plane of the mirror. This jig position was set as the origin. Next, the camera was translated and rotated until the image of the edge could be seen in the mirror. The position and orientation of the camera were recorded, and from this, the viewing angle was calculated, and $\theta$, was found to be 7.29 degrees.

An improperly focused camera will cause the image of the step edge to blur and will make the surface appear rougher than it actually is. Therefore, while the mirror was in the stand, the camera was focused so that the image of the step edge appeared sharp. The camera focus remained the same for the imaging of all samples, since the experimental configuration remained the same. The mirror was removed, and the first sample was placed in the stand.

The sample was aligned so that its grooves were parallel to the edge of the source. This was done to ease the analysis of the resulting images. In order to apply the results of the theory, the intensity of the reflected step edge should be studied along the direction perpendicular to the reflected edge. For a generic configuration of source, reflective surface, and camera, it would therefore be necessary to find the reflected edge and determine its direction. This is not difficult in general, and would be the first step in a roughness estimation vision system. We felt that the extra calculation involved in edge detection would add nothing to the test of the theory, and thus the reflected edge was aligned vertically in the image. Similarly, the grooves of the rough surface lay
along this direction. This was done merely to simplify the calculations, and in no way represents a loss of generality in the test of the theory.

Sixteen images were taken of each sample and these were averaged to obtain one representative image for each sample. This averaging decreases the effects of camera noise. The use of sixteen images provides us with two extra bits of accuracy by decreasing the uncertainty of the measurement of the intensity at a pixel by a factor of four. The images used as data appear in Figure 31 to Figure 33 in order of increasing roughness. The change in the appearance of the reflected step edges with respect to roughness is as one would expect from intuition.

Although we averaged sixteen images of each sample to obtain each image used by our algorithm, this process only reduced the noise introduced by the camera. There are other sources of intensity variation in the images, however, which we will not model and therefore consider noise.

Figure 31: The reflected image of the step edge in the 1.5 microinches $R_A$ sample and the 3.0 microinches $R_A$ sample
Figure 32: The reflected image of the step edge in the 4.5 microinches $R_A$ sample and the 7.5 microinches $R_A$ sample.

Figure 33: The reflected image of the step edge in the 9.5 microinches $R_A$ sample.
For example, dust and fingerprints on the samples can cause regions of the reflected edge image to appear different from other corresponding regions of the image. Also, the radiance actually was not constant across the step edge source, and this caused intensity variations in the image. In order to mitigate the effects of this kind of "noise", the scanlines of each image were averaged to yield a representative reflected edge profile, which was then used for all calculations. Figure 34 to Figure 38 show these average scanlines. It is reassuring to note that the edge profile for the 7.5 micro-inches roughness sample looks similar to the other edge profiles. Obviously, the non-Gaussian nature of the height distribution for this sample does not drastically affect the image of the reflected edge.

1.5 microinches roughness

Figure 34: The average of the scanlines
3.0 microinches roughness
Figure 35: The average of the scanlines

4.5 microinches roughness
Figure 36: The average of the scanlines
7.5 microinches roughness
Figure 37: The average of the scanlines

9.5 microinches roughness
Figure 38: The average of the scanlines
The algorithm was then run on all five images; the best fit values of the parameters for each sample are shown in Table 5. Let us list the meanings of the six parameters again for the sake of clarity.

- $A$ - the magnitude of the specular spike component
- $B$ - the magnitude of the specular lobe component
- $C$ - the offset of the reflected edge profile
- $\Delta \theta$ - the error in the measurement of the viewing and incidence angles
- $\alpha$ - the width of the specular spike component
- $\beta_{opt}$ - the root-mean-square slope of the surface as determined by our optical roughness estimation method. It should be equal to the standard deviation of the FADF.

In order to make the roughness parameter easier to understand, we will once again express it in terms of the angles of the normals to facets, rather than as the slopes of facets. The last row of Table 5 shows these values in units of degrees. The best fit curves and the data for each sample are shown in Figure 39 through Figure 43. The solid lines represent the fitted curve, and the dashed lines show the average scanline plus and minus the standard deviation of the measured intensities in each column.

The fits to the data curves appear reasonable by eye, and the values of $\beta_{opt}$ listed in Table 5 seem to indicate that the algorithm can order the surfaces by roughness. Before we accept these conclusions, and before we further analyze the results of the roughness estimation algorithm, we must determine if the fits to the data are truly acceptable.
Best Fit Values of the Parameters

<table>
<thead>
<tr>
<th>sample (microinches roughness)</th>
<th>1.5</th>
<th>3.0</th>
<th>4.5</th>
<th>7.5</th>
<th>9.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1215</td>
<td>743</td>
<td>314</td>
<td>318</td>
<td>298</td>
</tr>
<tr>
<td>B</td>
<td>1125</td>
<td>2361</td>
<td>3070</td>
<td>2944</td>
<td>2633</td>
</tr>
<tr>
<td>C</td>
<td>169</td>
<td>250</td>
<td>378</td>
<td>307</td>
<td>534</td>
</tr>
<tr>
<td>$\Delta \theta_r$ (radians)</td>
<td>-0.0088</td>
<td>-0.0128</td>
<td>-0.0256</td>
<td>-0.0184</td>
<td>-0.0856</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.01</td>
<td>0.02</td>
<td>0.05</td>
<td>0.05</td>
<td>0.150</td>
</tr>
<tr>
<td>$\beta_{opt}$</td>
<td>0.080</td>
<td>0.087</td>
<td>0.103</td>
<td>0.099</td>
<td>0.190</td>
</tr>
<tr>
<td>rms angle (degrees)</td>
<td>3.24</td>
<td>3.52</td>
<td>4.17</td>
<td>4.00</td>
<td>7.65</td>
</tr>
</tbody>
</table>

Table 5

4.5. Analysis of the Fitted Parameters

In the last section, we saw that the fitting procedure appears to give good fits to the data, and finds an ordering of the surfaces by roughness consistent with our intuition. We must, however, check the quality of the fits to determine if they are justified. Our algorithm used chi-squared fitting, and there are common and powerful statistical tests for use with chi-squared fitting which measure the appropriateness of a fitted curve for describing data. If we determine that the fits are justified, we may then interpret the best fit values of the parameters and see if there are any patterns to the values.

We first assume that our model and fitted parameters are correct, and that the errors are distrib-
Figure 39: Best fit reflected edge profile and error bars for 1.5 microinches roughness sample
Figure 40: Best fit reflected edge profile and error bars for 3.0 microinches roughness sample

3.0 microinches roughness
Figure 41: Best fit reflected edge profile and error bars for 4.5 microinches roughness sample
7.5 microinches roughness

Figure 42: Best fit reflected edge profile and error bars for 7.5 microinches roughness sample
Figure 43: Best fit reflected edge profile and error bars for 9.5 microinches roughness sample
uted normally. $\chi^2$ is a weighted sum of the squared errors, and was defined in equation (37). Let $v$ represent the number of degrees of freedom in the problem, that is, the number of data points minus the number of parameters. If $\chi^2$ and $v$ are approximately equal, then the fitted curve lies within the error bars at most points along its length, and the fit is good. We can quantify this idea with the variable $Q$, which represents the value of the incomplete gamma function, and gives the probability that the value of chi squared would be higher purely by chance than the measured value. Thus, a value of 0.5 shows a good fit, and most researchers accept values greater than or equal to 0.05 as indicating a good fit. Values much lower than this indicate that our assumptions are violated, and that either the model or the values of the fitted parameters are probably incorrect.

The statistics quantifying the goodness of the fits are shown in Table 6. Although the predicted edge profiles are close to the data profiles, the fits are not acceptable.

<table>
<thead>
<tr>
<th>sample</th>
<th>$\chi^2$</th>
<th>$v$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 microinches</td>
<td>180974</td>
<td>54322</td>
<td>0.000000</td>
</tr>
<tr>
<td>3.0 microinches</td>
<td>180771</td>
<td>54322</td>
<td>0.000000</td>
</tr>
<tr>
<td>4.5 microinches</td>
<td>127972</td>
<td>54322</td>
<td>0.000000</td>
</tr>
<tr>
<td>7.5 microinches</td>
<td>161844</td>
<td>54322</td>
<td>0.000000</td>
</tr>
<tr>
<td>9.5 microinches</td>
<td>104902</td>
<td>54322</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

Table 6
It is obvious from the graphs of the fitted curves and the data, Figure 39 to Figure 43, that the fits, even though they are not technically acceptable, are nonetheless useful, and perhaps even reasonable. Why, then, do we obtain such low values of $Q$? There are several possible reasons. First, the model of the reflected light, equation (18), is technically incorrect. The specular lobe component of reflection may be described by geometrical optics and a Facet Area Distribution Function only for very rough surfaces (those for which the rms roughness is much greater than the wavelength of the light). [Beckmann and Spizzichino 63] In this case, there should be no specular spike component, and thus, for all our samples which showed an appreciable spike component, this equation is not appropriate. Notice the general downward trend in the values of $\chi^2$ with increasing roughness. This is presumably because the model is more applicable to the rougher surfaces. Another possible cause of error is the non-Gaussian distribution of the heights of the surfaces. Since the 7.5 microinches roughness sample had the height distribution which differed the most from a Gaussian, we would expect the error between its fitted and data profiles to be the highest of all the samples. However, the errors of the fits decrease monotonically as the values of $\beta_{prof}$ and $\beta_{opi}$ for the surfaces increase. We may thus conclude that although the non-Gaussian nature of the height distribution may contribute to the error, it is probably not the main cause. This indicates that our algorithm will probably function reasonably for surfaces which do not have a Gaussian height distribution. A third cause of the error is the fact that the source does not have a uniform radiance. This was the origin of the large error bars in the data curves; the noise in the camera is not this great. We could reduce this error by modelling the radiance pattern of the source; this could be a topic of future research, and would yield some improvement in the quality of the fits. Note also on the right hand side of all images that the intensity drops off, rather than levelling out or increasing. This is possibly due to the finite extent of the piece of paper which
acted as the step edge source, but more likely is due to the blurring by the camera of the dark region beyond the sample into the region of interest. This occurs because the camera is focused on the edge of the reflected image of the source, rather than on the sample surface itself. Therefore, some blurring of the edge of the steel disk occurs.

Perhaps the most likely cause of the low values of $Q$, however, is irregularities of the surfaces themselves because of contamination and inconsistencies in the milling process. Many of the short-range variations in the data curves are in fact reproducible, and hence indicate the presence of contaminants or regions of differing roughness which cause intensity variations much larger than the noise level. Consider Figure 44. This is a magnified version of the lower left part of Figure 39, the best fit and measured edge profiles for the 1.5 microinches roughness sample. As we can see, the variation in the measured curve is much greater than the error in the curve over much of this region. Other images of this sample, taken with different step edge sources and with different viewing angles, show the same pattern of intensity variations. Obviously, these intensity fluc-

![Graph](image)

**Figure 44:** A close-up view of part of the best fit curve and error bars for the 1.5 microinches roughness sample. The measured intensity variations are larger than the error and appear in all images of the sample.
tations are due to variations of the surface itself, which are, in turn, probably due to contamination. In order to improve our fits, then, we would have to model the location of surface defects.

The introduction of additional parameters to describe these sources of error would slow down the fitting algorithm and possibly affect its convergence. Since our fits capture the shape of the edge profiles well, we choose to accept them as they are and not modify our algorithm to model surface defects or any of the other causes of error listed above.

Since we have decided to accept the fits to the data as they are, we will forge ahead, undaunted, into our analysis of the fitted parameters. The first thing we note is that the values of the rms slope found by our method are not the same as those found with the profilometer and listed in Table 3. In fact, the values found by our curve-fitting method do not even fall into the range of possible values of $\beta_{\text{prof}}$ as calculated from the profilometric data, although the two sets of values are of the same order of magnitude. This discrepancy could be due to the poor quality of the estimates of the correlation length, or to the fact that two different methods of measurement were used for these calculations. It is also interesting to note that the optical values of the roughness parameter, $\beta_{\text{opt}}$, are higher than the values found with the profilometer, $\beta_{\text{prof}}$. We stated earlier that the opposite behavior is common in roughness measurement systems. Therefore, the existence of differences between $\beta_{\text{prof}}$ and $\beta_{\text{opt}}$ was really not unexpected, but the relative magnitudes of the values was surprising. This observation, plus the fact that the values of $\beta_{\text{prof}}$, and $\beta_{\text{opt}}$ are so different seems to indicate that the algorithm might not be able to accurately estimate the rms slopes of the surfaces. The second point of note about the fitted values is that the fitting process correctly orders the samples by roughness. The values found for $\beta_{\text{opt}}$ are reasonable, and
are similar to those found by other computer vision researchers.[Novak 92] These facts show that our algorithm might be useful for the accurate measurement of surface roughness. On the basis of these two conflicting pieces of information, we must therefore conclude that the algorithm can order the surfaces correctly by roughness, but that its use as part of a mensuration scheme is questionable.

One of the most interesting things to note about the results of the fitting process is that $\alpha$, the width of the specular spike component of reflection, increases with increasing roughness. This is not predicted by our model, and is a significant finding. It should be included in all future computer vision models of the reflection of light from rough surfaces. It might be possible to exploit this variation of $\alpha$ with respect to roughness in other roughness estimation systems. The fact that the specular spike component is so localized might cause problems, however, because the data will be present in only a small part of the image, but it will also avoid problems associated with the reflected image of the edge running off the bounds of the image. More research will be necessary to determine the functional dependence of the spike width on the surface roughness.

We next note that the value of $C$ tends to increase with increasing roughness. This is probably because the image does not contain the entire edge profile for the rougher samples, and the algorithm mistakenly assigns a higher value to $C$ to compensate for the brightness due to the missing part of the edge. This effect seems to be of minor consequence.

As we have seen, our method of fitting a six-parameter curve to the reflected edge profile does a good job of matching the appearance of the reflected edge, even though the curves are technically not acceptable fits to the data. The parameter $\beta$ allows us to order the samples by roughness, and, surprisingly, the parameter $\alpha$ also increases with increasing roughness, indicating that we
may use this parameter, too, in ordering the surfaces by roughness. Although the method might not be useful for the accurate measurement of the rms slope of a surface, it can differentiate surfaces on the basis of the rms slope.

4.6. Summary and Discussion

We have shown in this chapter that if we model the reflection of light from our samples with the theory of Beckmann and Spizzichino, we may obtain reasonable fits to the reflected images of step edges. We introduced an algorithm which fits a six-parameter function to the profiles, and orders the surfaces correctly by roughness; the non-Gaussian nature of the surfaces does not seem to affect this ability. Perhaps the most interesting discovery is that the width of the specular spike component is a function of the roughness. We have no explanation for this phenomenon, and we believe that more research will be necessary to determine its cause and its exact functional relationship. We noted that it might be possible to exploit this relationship in order to measure roughness.

Our roughness estimation algorithm seems to be an acceptable way of quantifying the roughness of surfaces. It is especially useful in situations in which the environment can be controlled, such as mensuration schemes. However, in a more general purpose vision system, we might not have such control of the environment, and perhaps we will not have all of the reflected image of the edge, which, as we noted in the previous chapter, could cause problems in the roughness estimation. Therefore, we intend to look for more robust methods to estimate the roughness; additionally, we will look for more general models of rough surfaces, for use with these new methods.
5. A Direct Roughness Estimation Method

5.1. Chapter Overview

The method of roughness estimation presented in Chapter 4. seems to work fairly well. It can order surfaces by roughness and provide good fits to the reflected edge profiles, but the veracity of the calculated rms slope may be questioned. There are some other shortcomings of the method which we will discuss presently. We would like to find a method of roughness estimation which does not suffer from these difficulties. Therefore, we present a modified form of the reflected radiance equation and discuss its significant features. We will show that it is an example of a type of thoroughly studied, easily solvable equation. We then present a method of roughness estimation which uses this equation and calculates a roughness measure directly, thereby avoiding the problems of local minima in the search space. We first test this method on the images of our samples from Chapter 4., then use this algorithm on images of our samples for which the iterative method failed. We show that the new method of roughness estimation obtains results for the situations in which the previous method did not work. We finally discuss the shortcomings of this method and directions for future research.

5.2. Shortcomings of the Iterative Method

We saw in the last chapter that we may find the roughness of a reflective surface by fitting a six parameter function to the image of a reflected edge. In order to do this, we need to know the position and orientation of the surface, as well as the position of the source and camera. The method works fairly well but we were unable to determine if it would be useful as a mensuration tool, since we could not measure the rms slope of the surface by other means. There are some other problems with this method, also, largely due to its iterative nature. Let us show the situa-
tions in which this method fails, and then find the cause of these failures.

Consider Figure 45. Here we see the average profile of a reflected edge for the 1.5 micro-inches arithmetic roughness sample. The perpendicular distance from the plane of the reflective surface to the source is 0.322 meters and the viewing angle is 8 degrees. If we run our iterative algorithm on this image, we find the best fit values of the parameters to be $A$ equal to 1321, $B$ equal to 712, $C$ equal to 438, $\beta$ equal to 0.042, and $\alpha$ equal to 0.01. Figure 46 shows the best fit curve and the error bars for each pixel. If we compare this fit to that found for the same sample in Chapter 4 and shown in Figure 39, it is obvious that the new fit to the reflected edge profile is not as good as the earlier one, even though the viewing angle is almost the same and the source-object distance and object-camera distance differ little from the values in the previous experiment. The value of $\beta$ is also quite different from that calculated in the last chapter. We took eight other images with this sample and with different configurations of source, sample, and camera; the results of the fitting algorithm appear in Table 7. Again, we see that the values are quite different than those found in the last chapter, and that they vary greatly amongst themselves. They seem to decrease with increasing viewing angle, $\theta_r$, and with increasing source-object distance. Obviously, there appears to be a problem with the repeatability of the method, and with its sensitivity to the viewing angle and the position of the source relative to the reflective object.
Figure 45: Average reflected edge profile for the 1.5 microinches roughness sample

Figure 46: Best fit curve and error bars for 1.5 microinches roughness sample
<table>
<thead>
<tr>
<th>image</th>
<th>source distance (m)</th>
<th>camera distance (m)</th>
<th>$\theta_r$ (deg)</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.322</td>
<td>1.540</td>
<td>8</td>
<td>0.042</td>
</tr>
<tr>
<td>2</td>
<td>0.368</td>
<td>1.540</td>
<td>20</td>
<td>0.054</td>
</tr>
<tr>
<td>3</td>
<td>1.423</td>
<td>1.540</td>
<td>20</td>
<td>0.017</td>
</tr>
<tr>
<td>4</td>
<td>0.320</td>
<td>1.540</td>
<td>45</td>
<td>0.013</td>
</tr>
<tr>
<td>5</td>
<td>0.598</td>
<td>1.545</td>
<td>30</td>
<td>0.013</td>
</tr>
<tr>
<td>6</td>
<td>0.223</td>
<td>1.545</td>
<td>30</td>
<td>0.036</td>
</tr>
<tr>
<td>7</td>
<td>0.071</td>
<td>1.545</td>
<td>30</td>
<td>0.073</td>
</tr>
<tr>
<td>8</td>
<td>0.714</td>
<td>1.545</td>
<td>30</td>
<td>0.023</td>
</tr>
<tr>
<td>9</td>
<td>1.016</td>
<td>1.545</td>
<td>30</td>
<td>0.024</td>
</tr>
</tbody>
</table>

Table 7: The calculated rms slope for the 1.5 microinches roughness sample for several source-object-camera configurations.

There are other problems with the iterative method of roughness estimation. Consider Figure 47, which is the best fit curve and error bars for image number eight. The method used by the algorithm to find starting values for the fit was described in Chapter 4. If we instead start our algorithm from the best fit values for image number five, i.e. $A$ equal 1277, $B$ equal 723, $C$ equal 525, $\beta$ equal 0.013, and $\alpha$ equal 0.00, it will converge to the solution $A$ equal 1165, $B$ equal 837, $C$ equal 554, $\beta$ equal 0.007, and $\alpha$ equal 0.00. Figure 48 shows the best fit edge profile and error bars for this solution. The first solution has $\chi^2 = 235922$ and the second has $\chi^2 = 128496$, showing that the second is a better fit. Thus, we see that there exist local minima in our search space, and that we may get different estimates of the roughness, depending on the starting guesses in our
Figure 47: Best fit edge profile and error bars for image number eight. This fit used the default starting values in the roughness estimation algorithm.

Figure 48: Best fit edge profile and error bars for image number eight. This fit used the best fit parameters for image number five as its starting values.
The cause of these problems is not hard to discover. As we discussed in Section 3.3., unless we have the full reflected edge profiles, it is hard to differentiate some profiles which belong to surfaces with different roughnesses. In fact, almost all edge profiles, regardless of roughness, will appear the same if we consider a small enough portion of the edge profile, so that any differences between the curves is less than the noise in the images. Figure 19 shows this clearly. In Figure 49, we plot a shifted version of the average edge profile for image number 5 and the error bars for image number 8. We see that the average edge profile lies between the error bars for most of its extent, and therefore, images number 5 and 8 are essentially indistinguishable. Since the geometrical configurations which formed these images were quite different, the fitted parameters differ, even though the images are quite similar. This explains the widely varying values found in the most recent series of experiments. The indistinguishability of the profiles also contributes to the

![Graph](image_url)
problems with local minima. The existence of local minima is determined by the form of the fitted equations, but the indistinguishability of profiles makes it more difficult to find the correct starting values, and easier to fall into a minimum, since there is less curve to differentiate solutions. Conversely, if we have the full reflected edge profile, we should be more likely to find the proper roughness value for a surface. Image number 7 bears out this hypothesis. The source to object distance for the experiments of Chapter 4. was 0.096 meters and the viewing angle was 7.29 degrees. The distance for image number 7 is 0.071 meters, meaning that we probably have most of the reflected edge profile in this image, even though the viewing angle is 30 degrees. The fitted value of $\beta$ is 0.073, which is the largest of any values found in this series of experiments, and which is fairly close to the value found in Chapter 4., which was 0.080. Therefore, we believe that the iterative method worked correctly on this image.

We may rightly ask if our results from the last chapter may be trusted. There were five samples in the experiments of Chapter 4., with arithmetic average roughnesses of 1.5, 3.0, 4.5, 7.5, and 9.5 microinches. As we stated above, unless we have the full edge profiles, our results are questionable. The images for the first four samples contained large portions of the profiles, and this indicates that we may trust the calculated values. The image for the 9.5 microinches sample did not show the full edge profile, but the surface roughness was ordered properly. Therefore, we feel that the value for this sample may not be exactly correct, but is probably close to the correct value. Additionally, we used our iterative algorithm on these images, but used different starting values, and found that it converged to the same results. Thus, we believe that we can trust our results from the last chapter.

Our most recent series of experiments has shown that there are some shortcomings to the iter-
ative method of roughness estimation. We can only conclude that in order for the method to work, we must have most of the reflected edge profile, but that if we do have this, the method works well. Let us now look for a method which does not require the full edge profile, and which will not become trapped in local minima of the search space.

5.3. Reflected Radiance Equation

5.3.1. Modified Reflected Radiance Equation

We now wish to devise a method of roughness estimation which does not have the problems of our iterative method. In order to do this, we will modify the reflected radiance equation slightly, so that we may use alternative methods of solution.

Let us return to the equation for the reflected radiance due to an extended source, equation (23). Once again, this is

\[ L_r = \int_{\theta_f}^{\theta_f} \left( A \cos^2 \theta_f + B \frac{\cos \theta_f - \theta_f}{\cos \theta_f} \right) d\theta_f \]

(39)

where \( \theta_f \) is the facet normal angle, \( \theta_v \) is the viewing angle, \( S(\theta) \) is the FADF, \( \alpha \) quantifies the spread of the specular spike, \( A \) and \( B \) are constants relating the strength of the spike and lobe components, and \( I \) gives the radiance of the source. We have taken the liberty of replacing the first term in the integrand by its approximation, which we derived in Chapter 3. As we noted in that chapter, \( A \) and \( B \) can take on any values and can thus represent any source intensity. Therefore, we will absorb \( I \) into the definitions of \( A \) and \( B \) and \( I \) will no longer appear in the equation. Now, let us consider the limits of integration. As we showed in Chapter 3., for every point on the surface, there will be a range of values of \( \theta_f \) for which light will be incident on the surface from the
source and will then reflect into the camera. This range will change from point to point on the surface. We may phrase this another way if we change our focus of attention from the source-object-camera configuration to the reflective object itself.

Consider a generic FADF. The value of the FADF for an infinitesimal range of values of $\theta_f$ is, by definition, the ratio of the area of facets with their normals in this range to the area of the mean surface. For essentially all surfaces, the FADF will be bounded, that is, there will be two values of $\theta_f$, beyond which the FADF equals zero. Obviously, the values of $\theta_f$ will always lie between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, since a higher value would imply that the surface has “folded over” on itself, and we study only single-valued surfaces. For most surfaces, however, the range of facet angles will be much smaller than this. Let us call the endpoints of this range $\theta_{f_{\text{min}}}$ and $\theta_{f_{\text{max}}}$. If the range of integration does not overlap the range of non-zero values of the FADF, there will be no light reflected. If the range of integration overlaps the range of non-zero values of the FADF, there will be some facets on the surface which will reflect light from the source to the camera. Figure 50 shows several schematics of a source, object, and camera. The schematics show the range of all facet orientations, the directions of some incident light rays, and the direction of the reflected ray for different points on the surface. In all the diagrams, the angles of the facet normals which actually do reflect light from the source to camera are filled in with black. In this series of diagrams, the range of facets which reflect light varies, but, so long as light is reflected, this range always contains the endpoint closest to the source. Therefore, in equation (39), the final value of the range of integration varies as a function of the position on the surface, but the initial value remains the same. For the region of the surface for which one of the endpoints of the range of integration stays fixed, we may write equation (39) as
Figure 50: Different groups of facets will reflect light from the source into the camera for different points on the surface. The directions shown in black are the normals to those facets which reflect light from some point on the source into the camera.
As before, $\theta_r$ is a function of $x_{obj}$. Naturally, we will obtain the same form for the equation if $\theta_{f_{max}}$ remains constant rather than $\theta_{f_{min}}$. If we know the relative positions of the source, reflective surface, and camera, it is not difficult to calculate which points are in the region for which the starting value of the range of integration is fixed.

Let us now modify the equation slightly in another way. Consider the ratio

$$\frac{\cos(\theta_f - \theta_s)}{\cos \theta_r}$$

The reason for studying this function will become apparent shortly. Let us multiply the first term in the integral by this ratio; the FADF in the second term is already multiplied by it. As long as the ratio remains close to one, the approximation will be justified. It is easy to see that the ratio approximates unity for small values of $\theta_f$. Now, the values of $\alpha$ remain fairly small; in fact, for our roughest sample, the 9.5 microinches sample, the value of $\alpha$ was found to be only 0.150. This implies that the value of the exponential in the first term of the integral will be appreciable only for small values of $\theta_f$, and thus, the ratio will contribute to the integral only when its value is approximately equal to one. For example, for $\theta_f = 0.150$, $\alpha = 0.150$, and $\theta_r = 45$ degrees, the exponent will equal $e^{-1}$, and the ratio will be equal to approximately 1.138. Its value will be even smaller for most of our imaging situations. Therefore, for all values of $\theta_f$, for which the exponent is appreciably larger than zero, our introduction of the ratio will change the value of the integral only slightly. With this change to equation (40), we may factor the integrand, and the equation for
the reflected radiance becomes

\[ L_r = \int_{\theta_{j,\text{min}}}^{\theta_{f,\text{obj}}} \left( \frac{\theta_f^2}{2} + B S(\theta_f) \frac{\cos(\theta_f - \theta_r)}{\cos \theta_r} \right) d\theta_f \]  

(41)

Let us multiply both sides of the equation by \( \cos \theta_r \) to make the important things about this equation more clear. We then obtain

\[ L_r(x_{obj}) \cos \theta_r(x_{obj}) = \int_{\theta_{j,\text{min}}}^{\theta_{f,\text{obj}}} \left( \frac{\theta_f^2}{2} + B S(\theta_f) \right) \frac{\cos(\theta_f - \theta_r(x_{obj}))}{\cos \theta_r(x_{obj})} d\theta_f \]  

(42)

There are several interesting things about the form of this equation. First of all, note that it is very similar to a convolution. The only difference between this equation and a convolution is that the limits of integration of a convolution would be \(-\pi\) and \(\pi\), rather than finite values which vary from point to point on the surface. Therefore, it might be possible to use Fourier methods to solve for the quantity

\[ \frac{\theta_f^2}{2} + B S(\theta_f) \]

Additionally, it is possible to make a different approximation and cast the reflected radiance equation into true convolution form; details appear in Appendix 2.

Next, note that this is also an integral equation. A general integral equation is of the form

\[ \frac{b}{a} u(x) = f(x) + \int_{a}^{b} K(x, t) u(t) dt \]

where \( u(x) \) is the unknown function, and \( f(x) \) and \( K(x, t) \) are known. If \( \lambda \) equals zero, this is called
a first-kind integral equation. If the upper limit of integration, b, is equal to x or a function of x, it is called a Volterra integral equation. Therefore, equation (42) is a first kind Volterra integral equation. Unlike many other first kind integral equations, first kind Volterra equations are usually stable with respect to inversion. Just as with the convolution, we may use the methods of solution for Volterra equations to find

\[ Ae^{-\frac{\theta^2}{\sigma^2}} + BS(\theta) \]

This is the weighted sum of the FADF and a Gaussian. We will use this function to quantify the roughness of surfaces.

Therefore, by modifying the reflected radiance equation slightly, we put it into a form which can be solved in two different ways. The function for which we will solve is a weighted sum of the FADF and a Gaussian, and we must discover how we may use this to estimate roughness.

5.3.2. Solution of the Modified Reflected Radiance Equation

We showed in the last section that if we make some small changes to our description of the reflection of light from rough surfaces and the reflected radiance equation, the form of the reflected radiance distribution is governed by an equation which is similar to a convolution and is also a first kind Volterra equation. We must decide whether we want to solve the equation with the methods applicable to convolutions or those for use with Volterra equations.

We have decided to attack the problem of solving equation (42) with the methods suitable for Volterra equations. We feel that these methods are simple, and yield good results. In this section, we develop numerical, rather than analytical, methods of solution.
In order to solve the equation, first consider a point on the surface. Because $L_r$ and $\theta_r$ are known at every point, we may calculate $L_r \cos \theta_r$, which is given by

$$L_r \cos \theta_r = \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \cos (\theta_f - \theta_r) U(\theta_f) \, d\theta_f$$

where $U(\theta_f) = Ae^{-\left(\frac{\theta_f}{\alpha}\right)^2} + BS(\theta_f)$. We may then discretize the variable of integration, and approximate the integral by the sum

$$L_r \cos \theta_r = \sum_{j=1}^{N} \cos (\theta_{f_j} - \theta_r) U(\theta_{f_j}) \Delta \theta_{f_j}$$

We must decide on the number of function evaluations, $N$, which will be used in the sum, and the size of the steps, $\Delta \theta_{f_j}$. Figure 50 will help us with this endeavor. Let us start at a point on the surface where the reflected edge profile is dim, and move perpendicular to the reflected edge, in the direction of increasing brightness. For most singly-curved surfaces, a larger portion of the facets will reflect light into the camera as we move from point to point on the surface in this direction. The change in the endpoint of the range of facets will be the obvious choice for $\Delta \theta_{f_j}$. For every point in the edge profile, then, the number of function evaluations will be the same as the position of the point in the profile. For example, for the first point in the profile, there will be one term in the sum, for the second, two terms, for the third, three terms, and so on. Therefore, if we let $i$ denote the position of the measured radiance along the edge profile, we will obtain a matrix equation of the form
The matrix will be lower triangular, and, schematically, the equation looks like

\[
(L, \cos \theta_r)_i = \sum_{j=1}^{i} (\cos (\theta_f - \theta_r) \Delta \theta_f) \left ( U(\theta_f) \right )
\]

(44)

The equation can generally be solved by simple backsubstitution, and the solution vector, \( U(\theta_f) \), is simply the values of the function \( U(\theta_f) \) at the discrete points, \( \theta_f \). If the first point in the edge profile has a radiance of zero, that is, no light is reflected, we may obtain the complete solution vector from the matrix equation, since we have information for all angles for which \( U(\theta_f) \) is non-zero. If the first point of our recorded profile has a positive radiance, the process of solving the equation will assign a large value to \( U(\theta_f) \), the first value in the solution, in order to account for this radiance. Therefore, in most cases, the calculated solution will have a large value at one endpoint which will be incorrect. Therefore, we drop the first component of all calculated solution vectors. In fact, we have found empirically that it is wise to drop the first two components of the solution vector.

There are several advantages that this method of solution has relative to the iterative method presented in the last chapter. First, there is no possibility of the solution becoming trapped in a local minimum. Secondly, because the function \( U(\theta_f) \) is a sum of a Gaussian and the FADF for the surface, it might be possible to solve for the FADF for surfaces which do not have a Gaussian distribution of heights, and therefore do not follow the model of Beckmann and Spizzichino. Lastly, even if we have only part of the reflected edge profile, we can still find part of the function

\[
\begin{bmatrix}
(L, \cos \theta_r)_1 \\
(L, \cos \theta_r)_2 \\
(L, \cos \theta_r)_3
\end{bmatrix} =
\begin{bmatrix}
\cos (\theta_f - \theta_r) \Delta \theta_f & 0 & 0 \\
\cos (\theta_f - \theta_r) \Delta \theta_f & \cos (\theta_f - \theta_r) \Delta \theta_f & 0 \\
\cos (\theta_f - \theta_r) \Delta \theta_f & \cos (\theta_f - \theta_r) \Delta \theta_f & \cos (\theta_f - \theta_r) \Delta \theta_f
\end{bmatrix}
\begin{bmatrix}
U(\theta_f) \\
U(\theta_f) \\
U(\theta_f)
\end{bmatrix}
\]
\(U(\theta_f)\). Of course, we must determine if the corruption of the data by noise will prevent the achievement of these goals.

The first-kind Volterra integral equation form of the reflected radiance equation provides a new way of describing the reflection of light from rough surfaces, and new methods of solution for the FADF. We must now see how we can use the results of this equation to quantify surface roughness.

5.4. Width Measures

Although we may use the new method of solution to find the FADF for any surface for which we have geometrical reflection, it is probably unwise to take the resulting function too literally, since noise may corrupt the shape of the calculated function. Perhaps, however, there exist other quantities which we may derive from \(U(\theta_f)\) in order to estimate the surface roughness.

5.4.1. The Variance

In earlier chapters of this thesis, we repeatedly referred to the “width” of the Facet Area Distribution Function, denoted \(\beta\), and stated that it represented the roughness of the surface. This quantity is related to the standard deviation of the distribution function. Perhaps we can find a similar quantity for the function \(U(\theta_f)\).

Although we have not yet assumed a specific model of the FADF in the calculations of this chapter, in order to continue with our calculations, we must now do so. As in other sections of this thesis, we assume the model of Beckmann and Spizzichino. Therefore, we substitute this FADF into the function \(U(\theta_f)\), to obtain
Let us now calculate the "width" of $U(\theta_f)$.

Perhaps the most common way of quantifying the width of a probability density function is to find its variance or its standard deviation, which is the square root of the variance. If $F(x)$ is a probability density function, normalized so that

$$\int F(x) \, dx = 1$$

the mean value of the distribution is

$$\mu = \int F(x) \, x \, dx$$

and the variance of the distribution is defined to be

$$\text{var} = \int F(x) \, (x - \mu)^2 \, dx$$

Let us perform these calculations for the function $U(\theta_f)$. We will not perform a strict normalization or calculation of the variance, but rather related calculations.

We must first normalize the function; therefore, we must find the value of its integral so that we may divide by this number to normalize $U(\theta_f)$. The calculations are actually easier if, rather than finding the value of the integral, we find a related quantity which we denote $N$. We thus form

$$N = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left( Ae^{-\left(\frac{\theta_f}{\alpha}\right)^2} + B \frac{e^{-\left(\frac{\tan \theta_f}{\beta}\right)^2}}{\sqrt{\pi} \beta (\cos \theta_f)^{\frac{3}{2}}} \right) \cos \theta_f \, d\theta_f$$

(47)
As we commented in Section 5.3.1., \( \alpha \) is a small number, and therefore, the first term of the integral will contribute to \( N \) only for small values of \( \theta_j \). Therefore, we will approximate \( \cos \theta_j \) by 1 in the first term of equation (47). For \( \theta_j \) equal to 0.150, which we found for \( \alpha \) for our roughest sample, \( \cos \theta_j \) is approximately 0.989, and it differs from 1 by only 1.1 per cent; our approximation is thus reasonable. With this change, equation (47) becomes

\[
N = \int \frac{\pi}{2} A e^{-\left(\frac{\theta_j}{\alpha}\right)^2} d\theta_j + \int \frac{\pi}{2} B e^{-\frac{(\tan \theta_j)^2}{B^2}} \cos \theta_j d\theta_j
\]

We solve equation (48) to find

\[
N = A \sqrt{\pi} \alpha + B
\]

Because \( U(\theta_j) \) is symmetric about zero, its mean value is zero. We may now find a quantity akin to the variance by forming

\[
\text{var} = \frac{1}{(A \sqrt{\pi} \alpha + B)} \int \frac{\pi}{2} \left[ A e^{-\left(\frac{\theta_j}{\alpha}\right)^2} + B e^{-\frac{(\tan \theta_j)^2}{B^2}} \right] \cos \theta_j (\tan \theta_j)^2 d\theta_j
\]

This time, we approximate \( \cos \theta_j (\tan \theta_j)^2 \) by \( \theta_j^2 \) in the first term of the integral. For \( \theta_j \) equal to 0.150, the difference is only about 0.38 percent of the correct value. We then find

\[
\text{var} = \frac{A \sqrt{\pi} \alpha^3 + B \beta^2}{2 (A \sqrt{\pi} \alpha + B)}
\]

We would now like to determine if this is a useful quantity for characterizing roughness. We
mean "roughness" in the intuitive sense, rather than actually choosing rms heights or rms slopes as our roughness parameter. Although we do not know the exact dependence of the four parameters, A, B, α, and β on the roughness, our experiments have given us some rules of thumb. As the surface roughness increases, α and β increase. In fact, we could choose these parameters to be our definition of roughness if we so desired. B tends to increase or remain constant with increasing roughness, while A decreases with increasing roughness. How, then, does this variance-like quantity change with increasing roughness? We let ρ represent this intuitive roughness and calculate

\[
\frac{d(\text{var})}{d\rho} = \frac{\partial(\text{var})}{\partial A} \frac{dA}{d\rho} + \frac{\partial(\text{var})}{\partial B} \frac{dB}{d\rho} + \frac{\partial(\text{var})}{\partial \alpha} \frac{d\alpha}{d\rho} + \frac{\partial(\text{var})}{\partial \beta} \frac{d\beta}{d\rho}
\]

(52)

We know that

\[
\frac{\partial(\text{var})}{\partial A} = \frac{B \sqrt{\pi \alpha} \left( \alpha^2 - \beta^2 \right)}{2 (A \sqrt{\pi \alpha + B})^2}
\]

\[
\frac{\partial(\text{var})}{\partial B} = \frac{A \sqrt{\pi \alpha} \left( \beta^2 - \alpha^2 \right)}{2 (A \sqrt{\pi \alpha + B})^2}
\]

\[
\frac{\partial(\text{var})}{\partial \alpha} = \frac{A \left( 2A \pi \alpha^3 + B \sqrt{\pi} \left( 3 \alpha^2 - \beta^2 \right) \right)}{2 (A \sqrt{\pi \alpha + B})^2}
\]

and

\[
\frac{\partial(\text{var})}{\partial \beta} = \frac{\beta B}{A \sqrt{\pi \alpha + B}}
\]

The parameters must be larger than zero, and β is always larger than α. Therefore,

\[
\frac{\partial(\text{var})}{\partial A} < 0
\]
\[ \frac{\partial (\text{var})}{\partial B} > 0 \]

\[ \frac{\partial (\text{var})}{\partial \beta} > 0 \]

and \( \frac{\partial (\text{var})}{\partial \alpha} \) can take on both positive and negative values. Our “rules of thumb” state that

\[ \frac{dA}{dp} < 0 \]

\[ \frac{dB}{dp} > 0 \]

\[ \frac{d\alpha}{dp} > 0 \]

and

\[ \frac{d\beta}{dp} > 0 \]

Thus, the derivative of this quantity with respect to roughness can take on both positive and negative values. As a result, there exist local minima in the space of its possible values. Numerical experiments confirm this. Since this variance-like quantity does not vary monotonically with respect to roughness, it is not a suitable parameter to quantify roughness.

We must now look for a function which, like the variance, measures the width of the distribution, but which varies monotonically with respect to the roughness.

5.4.2. Our Width Measure

In the last section, we studied a width measure that was related to the traditional measure of
the width of a distribution, the variance. We showed, however, that it is unsuitable for estimating the roughness. We must now look for a related, but more useful, function. In this section, we will introduce a new function which we have developed that serves this purpose.

Consider once again the function,

\[ U(\theta_f) = Ae^{\left(\frac{\theta_f}{a}\right)^2} + B e^{\frac{\left(\tan \theta_f\right)^2}{\beta^2}} \]

At \(\theta_f = 0\), the function takes on its maximum value, \(A + \frac{B}{\sqrt{\pi \beta}}\). Let us use this as our normalization constant, rather than \(N\). This means that we make the maximum value of the distribution equal to one. Now, the function we want to study is

\[ W = \frac{1}{\left(A + \frac{B}{\sqrt{\pi \beta}}\right)} \int_0^{\pi/2} \left( Ae^{\left(\frac{\theta_f}{a}\right)^2} + B e^{\frac{\left(\tan \theta_f\right)^2}{\beta^2}} \right) \cos \theta_f (\tan \theta_f)^2 d\theta_f \quad (53) \]

For lack of a good name, let us simply refer to this new function as our width measure, \(W\). It is equal to

\[ W = \frac{A \sqrt{\pi a}^3 + B \beta^2}{2 \left(A + \frac{B}{\sqrt{\pi \beta}}\right)} \quad (54) \]

We again let \(\rho\) represent the intuitive roughness and calculate

\[ \frac{dW}{d\rho} = \frac{\partial W}{\partial A} \frac{dA}{d\rho} + \frac{\partial W}{\partial B} \frac{dB}{d\rho} + \frac{\partial W}{\partial \alpha} \frac{d\alpha}{d\rho} + \frac{\partial W}{\partial \beta} \frac{d\beta}{d\rho} \quad (55) \]
We know that

\[ \frac{\partial W}{\partial A} = \frac{B \pi \beta \left( \alpha^2 - \beta^2 \right)}{2 \left( A \sqrt{\pi \beta} + B \right)^2} < 0 \]

\[ \frac{\partial W}{\partial B} = \frac{A \pi \beta \left( \beta^2 - \alpha^2 \right)}{2 \left( A \sqrt{\pi \beta} + B \right)^2} > 0 \]

\[ \frac{\partial W}{\partial \alpha} = \frac{3A \alpha^2 \beta}{2 \left( A \sqrt{\pi \beta} + B \right)} > 0 \]

and

\[ \frac{\partial W}{\partial \beta} = \frac{A B \pi \left( 2 \beta^2 + \alpha^2 \right) + 3B \beta^2 \sqrt{\pi \beta}^2}{2 \left( A \sqrt{\pi \beta} + B \right)^2} > 0 \]

The "rules of thumb" are the same as they were in Section 5.4.1., and therefore, \( W \) increases with increasing roughness for all physically realizable values of the parameters. Thus, our width measure, and the Volterra equation description of the reflected step edge which allows us to calculate it, provide a new method of roughness estimation. Let us perform experiments to determine the usefulness of this method.

5.4.3. The Effects of Noise

As in any experiment, our results will be corrupted by noise. Therefore, we must study how noise will affect our calculations, and then devise methods to correct for this. We obtain many reflected edge profiles in our analysis of the image, and we may average these to reduce the effects of the noise. The question is, should we average the profiles before we perform our calculations, or should we average the results of our calculations? In the solution of the Volterra equa-
tion, noise will corrupt the calculation of $U(\theta_j)$. The matrix form of the equation is linear, so that we will obtain the same answer regardless of whether we average the data before solving the equation, or if we solve the equation for each profile and then average the results. The calculation of $W$, however, is nonlinear, and tends to magnify the effects of noise since we square the term $\tan \theta_j$. Therefore, we should average the scanlines before we perform our calculations in order to minimize the noise going into the calculation of $W$.

5.5. Experiment

In the previous section, we stated that we wished to quantify the width of the function, $U(\theta_j)$, and showed the problems with using the variance, a common width measure. We then derived a function, which we simply call our width measure and denote by $W$, which we may use as a criterion for roughness estimation, since it varies monotonically with respect to roughness for a surface with a Gaussian distribution of heights. Let us now calculate $W$ for the images of step edges reflected in some real surfaces.

5.5.1. Experimental Configuration

In our experiments, we will use the same rough surfaces as we did in Chapter 4. In fact, we will use the same experimental configuration and images as we did in that chapter, although we will also perform additional experiments. In Section 5.2., we mentioned these additional images briefly, when we showed that the iterative method of roughness estimation would not work on them. We will discuss them again in greater detail now. Let us describe the experimental configuration one more time. Figure 51 and Figure 52 show the experimental configuration and label the important physical quantities. The source is a large sheet of white paper, illuminated by a halogen lamp, and is parallel to the face of the reflective surface. The distance from the source to the
Figure 51: Schematic of the experimental configuration

Figure 52: The reflection geometry for the experimental configuration
surface is known, as are the distance from the camera to the surface, and the viewing angle, \( \theta_r \).

As before, given a point on the surface and a point on the object, we may calculate the facet normal angle, \( \theta_f \), from

\[
\theta_f = \frac{\theta_i + \theta_r}{2} = \frac{\arctan \left( \frac{x_{src} - x_{obj}}{D} \right) + \theta_r}{2}
\]

The only difference between the new experiments and our old ones is that in some cases, the source-object distance will be of the same order of magnitude as the object-camera distance. Therefore, we cannot approximate \( \theta_r \) as being constant across the reflective surface. We know the distance from the camera to the object, however, which we denote \( C \). As we see in Figure 53, we may find \( \theta_r(x_{obj}) \), the viewing angle at the point \( x_{obj} \), from

\[
\theta_r(x_{obj}) = \arctan \left( \frac{C \sin \theta_r + x_{obj} - x_{edge}}{C \cos \theta_r} \right) = \arctan \left( \tan \theta_r + \frac{x_{obj} - x_{edge}}{C \cos \theta_r} \right)
\]

where \( \theta_r \) is the viewing angle at the position of the reflected edge, and \( x_{edge} \) is the position of the reflected edge.

This description of the experimental configuration is complete enough for our calculations; let us now describe how our new algorithm estimates the roughness from the reflected images of step edges.

5.5.2. The Direct Roughness Estimation Algorithm

We obtain the data and form the matrices of equation (44) as follows. First, a region of the image containing an edge is supplied by hand, as in the earlier experiments. The orientation of the
Figure 53: Geometry for the calculation of the viewing angle as a function of position.

e is also given. Next, the image is scanned perpendicular to the edge to obtain reflected edge profiles, and the profiles are averaged to yield a representative profile.

The algorithm then starts at the side of the average edge profile with the lower intensity, and, as per equation (43), records \( L_r(x_1) \cos \theta_r(x_1) \), where \( x_1 \) is the position of the point under study. It next calculates the angle, \( \theta_{edge1} \), at which a facet would need to be oriented in order for light to reflect from the edge of the source, off this point of the surface, and into the camera. All light seen at this point is thus reflected from facets which lie between \(-\frac{\pi}{2}\) and \(\theta_{edge1}\). It then finds the midpoint of this range, denoted \( \theta_{mid1} \). The \([1,1]\) element in the matrix is then \( \cos (\theta_{mid1} - \theta_r) \Delta \theta_1 \), where \( \Delta \theta_1 = \theta_{edge1} - (-\frac{\pi}{2}) \).
The algorithm then moves to the next point, denoted \( x_2 \), and records \( L(x_2) \cos \theta(x_2) \). It calculates \( \theta_{edge2} \), the angle at which a facet would need to be oriented in order for light to reflect from the source, off the second point on the surface, and into the camera. \( \theta_{mid2} \) is then the midpoint of the range \( \theta_{edge1} \) to \( \theta_{edge2} \), and \( \Delta \theta_2 = \theta_{edge2} - \theta_{edge1} \). The \([2,1]\) element of the matrix is then \( \cos (\theta_{mid1} - \theta_{r2}) \Delta \theta_1 \) and the \([2,2]\) element is \( \cos (\theta_{mid2} - \theta_{r2}) \Delta \theta_2 \). The algorithm continues in this fashion and fills the matrix. The matrix is lower triangular, and may be solved immediately by backsubstitution, which is done. As we stated before, we ignore the first two components in the solution vector of \( U(\theta_j) \), since they will be large to account for any offset in the data.

The next step in the algorithm is to calculate the width measure, \( W \), for the derived distribution. Our analysis of the width measure in Section 5.4.2. was performed using continuous forms of the functions in question, but the numerical solution of the Volterra form of the radiance equation will yield a solution vector where each component is the value of the function \( U(\theta_j) \) at a point. We therefore calculate \( W \) as follows. First, we find the maximum value of \( U(\theta_j) \) and divide the function by this value. Next, we estimate the value of the mean of the function with the expectation value of \( \theta_f \). This is denoted \( \langle \theta \rangle \) and is given by

\[
\langle \theta \rangle = \frac{\sum_{i=1}^{N} U(\theta_{fi}) \theta_{fi} (\Delta \theta_{fi})}{\sum_{i=1}^{N} U(\theta_{fi}) (\Delta \theta_{fi})}
\]

where there are \( N \) samples of \( U(\theta_j) \). We expect this value to be close to zero. We then estimate the value of \( W \) by calculating
For the smoother of our samples, the results of this calculation are often at the level of machine precision for our computer. This indicates that most of the terms in the sum of equation (57) are smaller than the machine precision, and that the calculation suffers severely from the results of roundoff error. In order to solve this problem, we multiply each term in the sum by a constant, which we choose to be 10000. There is no special significance to this number, other than that the calculated values of the pseudovariance for our samples are generally in the range from 0 to 100.

5.5.3. Old Data

Let us now use our algorithm on the images from Chapter 4, and calculate the width measure for each. Pictures of these images and the average scanlines for the regions of interest appear in that chapter. Figure 54 to Figure 58 show $U(\theta_j)$ for each image. The values of the $W$ for each sample appear in Table 8. As before, the samples are ordered correctly. The differences between the values of $W$ seem significant, and this would make the method useful for roughness differentiation. We must perform more experiments to determine the uncertainty in these calculations, to see if the differentiation is valid.

5.5.4. New Images

We have seen that the width measure $W$ can order the surfaces by roughness. In order to determine the usefulness of the method, however, we must determine the uncertainty in these calculations and see if the method can work for different configurations of source, reflective surface, and camera. Our next set of experiments uses only the 1.5 microinches roughness sample. The experi-
Figure 54: The function $U(\theta)$ for the 1.5 microinches roughness sample.

Figure 55: The function $U(\theta)$ for the 3.0 microinches roughness sample.
Figure 56: The function $U(\theta)$ for the 4.5 microinches roughness sample.

Figure 57: The function $U(\theta)$ for the 7.5 microinches roughness sample.
Figure 58: The function $U(\theta)$ for the 9.5 microinches roughness sample.

<table>
<thead>
<tr>
<th>sample (microinches roughness)</th>
<th>$W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.0983</td>
</tr>
<tr>
<td>3.0</td>
<td>3.29</td>
</tr>
<tr>
<td>4.5</td>
<td>25.5</td>
</tr>
<tr>
<td>7.5</td>
<td>16.1</td>
</tr>
<tr>
<td>9.5</td>
<td>31.9</td>
</tr>
</tbody>
</table>

Table 8: The value of $W$ for the five samples. The images used are the same as those in Chapter 3.
mental configuration is the same as that in the previous set of experiments, except that now the source-object distance, $D$, and viewing angle, $\theta_{r_0}$, are allowed to vary. Figure 59 to Figure 67 show the calculated function $U(\theta_f)$ for each image. For all nine of the images, the graph of $U(\theta_f)$ looks almost the same as that shown in Figure 54, although the range of $\theta_f$ for which we have calculated the function varies. Table 9 shows the values of $D$, $\theta_r$, and $W$ for the nine images.

Now, let us analyze these results. Although the calculated values of $W$ do vary, if we consider all the results for the 1.5 microinches sample, including that from the previous experiment, we see that the calculated value of $W$ is $-0.0294 \pm 0.146$. This range does not overlap the value found for the 3.0 microinches roughness in the previous experiment, and we may conclude that the differentiation of the samples by roughness is legitimate. It would be preferable to be able to assign error bars to the values of $W$ found for all the samples, but we were unable to obtain the samples for such an experiment.

![Figure 59: The function $U(\theta_f)$ for image 1.](image-url)
Figure 60: The function $U(\theta)$ for image 2.

Figure 61: The function $U(\theta)$ for image 3.
Figure 62: The function $U(\theta_j)$ for image 4.

Figure 63: The function $U(\theta_j)$ for image 5.
Figure 64: The function $U(\theta)$ for image 6.

Figure 65: The function $U(\theta)$ for image 7.
Figure 66: The function $U(\theta)$ for image 8.

Figure 67: The function $U(\theta)$ for image 9.
<table>
<thead>
<tr>
<th>image</th>
<th>$D$ (meters)</th>
<th>$\theta_{r0}$ (degrees)</th>
<th>$W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.322</td>
<td>8.00</td>
<td>0.0397</td>
</tr>
<tr>
<td>2</td>
<td>0.368</td>
<td>20.00</td>
<td>0.0133</td>
</tr>
<tr>
<td>3</td>
<td>1.423</td>
<td>20.00</td>
<td>0.000845</td>
</tr>
<tr>
<td>4</td>
<td>0.320</td>
<td>45.00</td>
<td>-0.000718</td>
</tr>
<tr>
<td>5</td>
<td>0.598</td>
<td>30.00</td>
<td>0.00217</td>
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<td>6</td>
<td>0.223</td>
<td>30.00</td>
<td>0.00947</td>
</tr>
<tr>
<td>7</td>
<td>0.071</td>
<td>30.00</td>
<td>-0.459</td>
</tr>
<tr>
<td>8</td>
<td>0.714</td>
<td>30.00</td>
<td>0.00119</td>
</tr>
<tr>
<td>9</td>
<td>1.016</td>
<td>30.00</td>
<td>0.000853</td>
</tr>
</tbody>
</table>

Table 9: The width measure $W$ for the images of the 1.5 microinches roughness sample.

There are several things we can say about these results. First, note that $W$ decreases with increasing viewing angle and with increasing source-object distance. If we look at the graphs for $U(\theta_f)$ we can see why this is so. For both of these situations, the range of values of $\theta_f$ decreases as we increase these variables. As a result, the calculated value of $W$ will be lower, since there are no terms at high values of $\theta_f$ to contribute to the sum. The presence of noise will obviously drive up the value of $W$, since it will provide nonzero values of $U(\theta_f)$ at high values of $\theta_f$.

Note also that the plots of $U(\theta_f)$ seem more ragged at higher values of $\theta_f$. This is probably the cause of the negative value of $W$ for the fourth image. This raggedness is most likely due to the fact that this part of the curve has the most noise and is solved for last. Noise may cause one value of $U(\theta_f)$ to be higher or lower than its correct value. In order to compensate for this incorrect
value, the next point solved for will either lower or higher than its correct value. This may cause oscillations that build up over time. There are possible ways to solve this problem and improve the method. For example, we may solve for the function $U(\theta_f)$ twice by starting from both positive and negative values of $\theta_f$ and then averaging these results. We may also smooth the average reflected edge profile prior to solution for $U(\theta_f)$. We leave the pursuit of such methods to other researchers.

Perhaps the most important thing that can be said about the second set of experiments is that results were obtained, and they were fairly consistent and close to that found in the first series of experiments. This cannot be said of the iterative method of roughness estimation. Remember, these images did not have complete edge profiles, and the roughness values found by the iterative method varied widely. Although the results found by this method are not as precise nor as easily understood as those of the iterative method, the calculation of our width measure seems to be a more robust method of ordering surfaces by roughness.

5.6. Conclusions

In this chapter, we have shown that although the iterative method of roughness estimation produces accurate values, it requires most of the reflected edge profile in order to work correctly. In order to produce a new method which does not require the full edge, we modified the reflected radiance equation slightly and cast it in the form of a first kind Volterra integral equation. We showed how to solve the equation numerically and introduced a derived quantity, $W$, which increases monotonically with increasing roughness. We calculated $W$ for the images of Chapter 4., and showed that it orders the surfaces correctly. We next calculated $W$ for the images for which the iterative method would not work. The new method worked, and yielded values close to that
found for the image of the 1.5 microinches roughness sample from Chapter 4.

Although it is difficult to relate physical values such as the root-mean-square height or root-mean-square slope of the surface to this new width measure, the function $W$ does seem to be a possibly useful function for the differentiation of surface roughness, and should be explored further. Perhaps the accuracy of the solution for $U(\theta)$ could be increased to allow calculation of the full FADF of the surface. This would allow us to gain more information about surfaces which do not follow the Beckmann and Spizzichino model of reflection.
6. Summary and Conclusions

6.1. Summary

In this thesis we have shown that it is possible to estimate the roughness of a surface by studying a single image of step edges reflected in the surface. We derived an equation describing the appearance of a reflected edge, and have shown that in order to determine the roughness, we must have knowledge of the relative positions of the light source, the reflective object, and the camera. Two techniques for estimating the roughness of the surface were presented. The first technique fits a curve described by the model of the reflected edge to an image of a reflected edge and finds the root-mean-square slope of the surface. This algorithm could be useful for accurate measurement of surface roughness. The second technique analyzes the reflected edge profile and calculates a quantity which we have derived that increases monotonically for increasing roughness and thus allows us to order surfaces by roughness. This is a more robust method, since it requires less of the reflected edge image than does the first method, and would be more useful for general purpose vision systems.

6.2. Contributions

There are several contributions this thesis makes to our understanding of the appearance of rough reflective surfaces.

- This thesis presents a model of the appearance of reflected step edges.

Although many researchers within and without the computer vision field have studied the reflection of light from rough surfaces, they have usually focused on situations with
point light sources. This thesis addresses the effect of roughness on the reflected images of step edges, since edges are an important low-level visual feature and are used in many physics-based vision algorithms. It describes how the reflected images of edges change with respect to roughness.

- This thesis identifies the information necessary to determine the roughness of a surface from a single image containing reflected edges.

In this thesis, we have shown that many different configurations of source, reflective surface, camera, and roughness can produce images which are indistinguishable to within noise levels of the camera. Therefore, we conclude that in order to calculate the roughness of a surface from a single image of a reflected step edge, we must know the relative positions of the source, object, and camera. This knowledge is incorporated in all our methods of roughness estimation.

- The experiments discussed in this thesis indicate that the width of the specular spike component of reflection increases with increasing roughness.

The change in the width of the specular spike component of reflection with respect to roughness is not predicted by Beckmann and Spizzichino's theory of the reflection of light from rough surfaces. Knowledge of this effect allowed us to produce a realistic model of the profile of a reflected step edge.

- The most important contribution of this thesis is the presentation of two roughness estimation algorithms.
The first algorithm iteratively fits a curve predicted by our six-parameter model to a reflected edge profile and calculates the rms slope of the surface. It produces good fits to the edge profiles and orders surfaces correctly by roughness. This method has the potential to be used as part of an accurate roughness estimation system.

The second algorithm demonstrates that it is possible to order surfaces by roughness without having the full reflected edge profile, and without solving for the roughness directly. The program models the reflected radiance equation as a first-kind Volterra integral equation, and solves it to obtain the weighted sum of the Facet Area Distribution Function for the surface and a Gaussian which is related to the width of the specular spike component. From this function, it calculates a quantity which we define and which varies monotonically with respect to roughness. Since this algorithm does not compute the roughness directly, it probably will not find use as part of a mensuration scheme. Because it orders surfaces correctly by roughness even when only part of the reflected edge profile is present, however, it may be useful as part of a general purpose vision system.

We have not exhaustively studied the topic of light reflection from rough surfaces, or even the effects of surface roughness on the reflected images of step edges, but these findings provide the necessary first information and methods which will aid any future computer vision work in this area. There remains much research to be done on this topic.
6.3. Directions for future research

In all of our work, we assumed that the surfaces were one-dimensionally rough, and that they had Gaussian distributions of height values. However, there exist many other types of rough surfaces. As we showed in this thesis, the appearance of reflected edges in some other types of surfaces, such as those with non-Gaussian height distributions, can differ from that in surfaces with one-dimensional, Gaussian roughnesses. Future research should expand the applicability of the roughness estimation methods, so that they will work for surfaces with non-Gaussian distributions of heights and two-dimensional roughnesses, both directional and isotropic. There has been much research recently in the optical scattering literature on the reflection of light from rough surfaces with non-Gaussian height distributions, such as surfaces with fractal height distributions. The results from research such as this will obviously provide the theoretical framework for these new methods.

Other ideas for research would remove or modify the assumptions in our algorithms. One such idea would be to work around the assumption of known source, object, and camera positions. For example, if we allow multiple images of the reflective surface, it might be possible to use stereo on the reflected image of the edge in order to calculate the source-object distance. We could also modify our analyses to study the interaction of several edges, and the profiles of non-step edges. These changes would allow us to estimate roughness under many more circumstances than do our present methods.

The analysis of edge profiles and roughness by frequency space methods is another area of possibly fruitful research. Fourier, wavelet, and related frequency-domain techniques have shown great promise in many areas of physics-based vision, from texture analysis to segmentation to ste-
reo. Such a method of roughness estimation could be a part of a larger vision system which uses frequency space based methods as the unifying representation. The convolution form of the reflected radiance equation presented in Appendix 2 is a first step in this direction.

6.4. Concluding Remarks

Obviously, the study of the reflection of light from rough surfaces for computer vision is a rich area for research. As our understanding of how to measure roughness in both constrained and unconstrained environments grows, we will be able to use these methods, and new ones, to calculate roughness for inspection and quality control, and for general purpose robots.
7. Bibliography

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[Healey and Binford 87] Glenn Healey and Thomas O. Binford, "Local Shape from Specularity", 


[Nayar et al. 90] Shree K. Nayar, Katsushi Ikeuchi, and Takeo Kanade, “Determining Shape and Reflectance of Hybrid Surfaces by Photometric Sampling”, IEEE Transactions on Robotics and


8. Appendix 1

We wish to determine if the surfaces actually exhibit a Gaussian distribution of height values. Therefore, for each surface, we made a histogram of the number of occurrences of each height value in the ten profiles for each surface. These histograms appeared in Figure 25 to Figure 29. As the number of measured points increases, these histograms should approximate a Gaussian distribution more and more closely. The true form of the histograms can be derived, however. We first consider the case of no measurement errors. Consider a discretely sampled rough surface with a Gaussian distribution of heights. The probability of a given sample being in the height range \( h \) to \( h + dh \) is

\[
P = \frac{(h - \mu)^2}{2\sigma^2} \frac{1}{\sqrt{2\pi}\sigma_h} dh
\]

where \( \sigma_h \) is the standard deviation of the distribution. Now, if we have a scan of the surface with \( S \) sample points, the probability that \( n \) of them will be in the range from \( h \) to \( h + dh \) is given by the binomial distribution, or

\[
\text{probability} = \binom{S}{n} P^n (1 - P)^{S - n}
\]

The mean and variance of the binomial distribution are known to be \( SP \) and \( SP(1 - P) \), respectively.

If we now allow the existence of measurement errors, we must change the value of \( P \). The probability that a measured point has value \( h_i \) is, as before,
The probability that the measuring instrument will record the value of this point as \( h \) is

\[
q_i = e^{-\frac{(h-h_i)^2}{2\sigma_m^2}} dh
\]

where we have assumed that the measurement errors are Gaussian distributed with standard deviation \( \sigma_m \). The probability that the point has value \( h_i \), but is recorded as \( h \) is then \( p_i q_i \). We sum these probabilities over all possible values of \( h_i \) to determine the probability of a point having measured value in the range from \( h \) to \( h + dh \). This probability is

\[
P = \sum p_i q_i = \sum \left( e^{-\frac{(h_i-\mu)^2}{2\sigma_h^2}} \right) \left( e^{-\frac{(h-h_i)^2}{2\sigma_m^2}} \right) dh_i
\]

If \( \sigma_m < \sigma_h \), i.e. if the measurement errors are much smaller than the standard deviation of the height distribution, as is our case, the second term in the sum is appreciable only for a small range of values of \( i \), and the first term is fairly constant over this range. Therefore, we may approximate \( h_i \) in the first term by \( h \), and factor out the first term to find

\[
P = e^{-\frac{(h-\mu)^2}{2\sigma_h^2}} \sum \left( e^{-\frac{(h-h)^2}{2\sigma_m^2}} \right) dh
\]
The sum is approximately equal to one, so that

\[
P = \frac{(h - \mu)^2}{2\sigma_h^2} \int_{-\infty}^{\infty} \frac{e^{-h^2/(2\sigma_h^2)}}{\sqrt{2\pi}\sigma_h} dh
\]

and we may use the results we derived for the case with no measurement errors.
9. Appendix 2

Let us show that we may approximate the reflected radiance equation by a convolution. It is easiest to begin from equation (40)

\[ L_r(x_{obj}) = \int_{\theta_{p(x_{obj})}}^{\theta_{f(x_{obj})}} \left( A e^{-\left(\frac{\theta_j}{\alpha}\right)^2} + B \frac{\cos (\theta_f - \theta_r) \cos \theta_r}{\cos \theta_r} \right) d\theta_j \]  

(58)

As in Chapter 5, we will multiply the first term of the integrand by

\[ \frac{\cos (\theta_f - \theta_r)}{\cos \theta_r} \]  

(59)

to obtain

\[ L_r(x_{obj}) = \int_{\theta_{p(x_{obj})}}^{\theta_{f(x_{obj})}} \left( A e^{-\left(\frac{\theta_j}{\alpha}\right)^2} + B S(\theta_j) \left( \frac{\cos (\theta_f - \theta_r)}{\cos \theta_r} \right) \right) d\theta_j \]

We could also write this as

\[ L_r(x_{obj}) = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} L_i(\theta_i) \left( A e^{-\left(\frac{\theta_j}{\alpha}\right)^2} + B S(\theta_j) \left( \frac{\cos (\theta_f - \theta_r)}{\cos \theta_r} \right) \right) d\theta_j \]

where \( \theta_i = 2 \theta_j - \theta_r \) is the angle of the incident light ray, and \( L_i(\theta_i) \) describes the configuration of the source. \( L_i(\theta_i) \) is equal to 1 for \( \theta_i \) in the range

\[ 2 \theta_{f(x_{obj})} - \theta_r < \theta_i < 2 \theta_{f(x_{obj})} - \theta_r \]

and zero otherwise. This is not yet a true convolution, partly because the range of non-zero values
for $L_i(\theta_i)$ changes from point to point on the object. Let us now assume that the source function, $L_i(\theta_i)$, is the same for all points on the surface. This implies that the source is very far from the object. It is no longer necessary to assume that the source is a step edge. We then have that

$$L_r(\mathbf{x}_{\text{obj}}) = \frac{\pi}{2} \int_{\frac{\pi}{2}}^{\pi} L_i(2\theta_f - \theta_r) \left( Ae^{-\frac{(\theta_f/\alpha)^2}{2}} + BS(\theta_f) \right) \frac{\cos(\theta_f - \theta_r)}{\cos \theta_r} d\theta_f$$  

(60)

We may write equation (59) as

$$\frac{\cos(\theta_f - \theta_r)}{\cos \theta_r} = \frac{\cos \theta_r \cos \theta_f + \sin \theta_r \sin \theta_f}{\cos \theta_r}$$

and if the viewing angle, $\theta_r$, is small, we may make the approximation

$$\frac{\cos(\theta_f - \theta_r)}{\cos \theta_r} \approx \cos \theta_f$$

and equation (60) becomes

$$L_r(\theta_r) = \frac{\pi}{2} \int_{\frac{\pi}{2}}^{\pi} L_i(2\theta_f - \theta_r) \left( Ae^{-\frac{(\theta_f/\alpha)^2}{2}} + BS(\theta_f) \right) \cos \theta_f d\theta_f$$

If we switch our variable of integration to $u = 2\theta_f$, this becomes

$$L_r(\theta_r) = \frac{1}{2} \int_{-\pi}^{\pi} L_i(u - \theta_r) \left( Ae^{-\frac{(u/2\pi)^2}{2}} + BS(\frac{u}{2\pi}) \right) \cos \frac{u}{2} du$$

which is a convolution.
Convolution is a commonly used operation in the physical sciences and signal processing. In order to solve a convolution equation, we may move into the Fourier domain by taking the Fourier transform of both sides of the equation to yield

$$3 \{ L_r(\theta) \} = 3 \left[ A e^{-\left( \frac{u}{2\alpha} \right)^2} + B S(u) \right] \cos \frac{u}{2} 3 \{ L_i(u - \theta) \}$$

where $3 \{ f(x) \}$ denotes the Fourier transform of the function $f(x)$. We then solve for the weighted sum by dividing by the transform of $L_i(u - \theta)$ and taking the inverse Fourier transform to yield

$$\left( A e^{-\left( \frac{u}{2\alpha} \right)^2} + B S(u) \right) \cos \frac{u}{2} = 3^{-1} \left| \frac{3 \{ L_r(\theta) \}}{3 \{ L_i(u - \theta) \}} \right|$$

Obviously, the form of the source function must be known.

There are a few problems with this method, none of which is insurmountable. First, if the Fourier transform in the denominator takes on values with small or zero magnitude, the division becomes unstable or even undefined. We may solve this problem by solving in the forward direction rather than by division, or by simply avoiding division for the small values and interpolating the result from the rest of the function. Another problem with solving this type of equation is that for computer applications, the Fourier transform is found with the Fast Fourier Transform algorithm. This algorithm imposes periodic boundary conditions on the data; in other words, it wraps the data so that its beginning and end are the same value. This can cause problems, but there are common techniques used in numerical analysis which circumvent any difficulties.

The convolution is one of the most used types of equations in image analysis. Therefore, it should be easy to solve the reflected radiance equation with this method. We decided to use the
Volterra equation method of solution, however, since it did not involve the assumption that the source is far from the surface, as does the convolution equation. We feel that it will be a fruitful area of research for future investigators, however.