Dead Reckoning Naviagtion for Walking Robots

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Abstract

Autonomous and teleoperated mobile robots require an accurate knowledge of their spatial location in order to accomplish many tasks. Many mobile robots make use of dead reckoning navigation because of its simplicity, low cost and robustness. Although dead reckoning navigation has been used for centuries for ships and wheeled vehicles, the application to a walking machine is novel. Since walking machines differ greatly from ships and wheeled vehicles, a new approach to dead reckoning was developed to solve this problem. This paper discusses the problem, a solution, preliminary test results and future goals for dead reckoning navigation. Experiments were done with the CMU Ambler, an autonomous, six-legged walking robot, but the results are general and apply to any statically stable walking robot. The current results show a systematic bias of two percent of body advance in the direction of travel. Although the cause of this bias is unknown, it is corrected in the position estimation routines.

1 Introduction

Autonomous and teleoperated mobile robots require an accurate knowledge of their spatial location in order to accomplish many tasks. Many mobile robots make use of dead reckoning navigation because of its simplicity, low cost and robustness. Although dead reckoning navigation has been used for centuries for ships and wheeled vehicles, the application to a walking machine is novel. Since walking machines differ greatly from ships and wheeled vehicles, a new approach to dead reckoning was developed to solve this problem. This paper discusses the problem, a solution, preliminary test results and future goals for dead reckoning navigation. Experiments were done with the CMU Ambler, an autonomous, six-legged walking robot, but the results are general and apply to any statically stable walking robot.

Dead reckoning is defined as:

> The determination, without the aid of external observations, of the position and orientation of a vehicle from the record of the courses travelled, the distance made, and the known estimated drift. [1]

A vehicle, starting from an initial position, P_0, travels along a path segment defined by d_1 and ϑ_1, see Figure 1.1. The new position, P_1, can be *estimated* from the course and distance travelled. The accuracy of the location is dependent on the accuracy of the data, since there is no way to *verify* the new location. Therefore, the proper representation of the new location is a position with an associated uncertainty, which is represented as a circle in Figure 1.1. Furthermore, at each subsequent location, the positional uncertainty increases.

The implementation of dead reckoning for a vehicle such as a car (travelling over smooth terrain), boat or plane is straight forward. The instrumentation required is a speedometer, a compass and a clock (or alternatively an odometer and a compass). The drift can be estimated by the known precision and accuracy of the devices and an estimation of non-measurable displacement, such as ocean currents. For a car, assuming the initial heading is known, the compass can be eliminated by keeping track of the change in heading of the vehicle. This technique has been used by

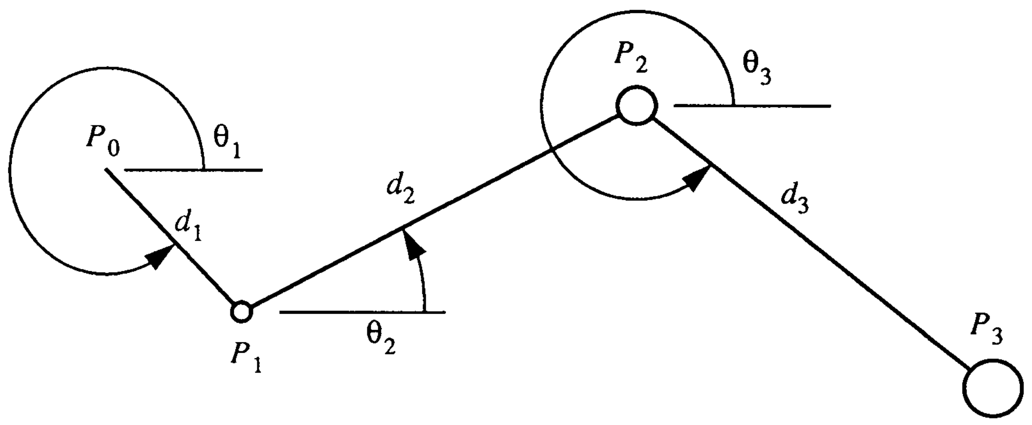

Figure 1.1. Dead reckoned course

1. Modified from Webster's Dictionary, which restricts the definition to the position of ships and planes.

mariners for centuries when stellar navigation was rendered impossible by weather conditions. The technique is used in modern times by competitors in automobile road-rallies and the sport of orienteering.

An important limitation of dead reckoning navigation, is that it can not account for changes in the vehicle's position which do not affect its measuring devices. For instance, if a vehicle were picked up from one location and placed down in another, dead reckoning would not report a change in position. This limitation does not hold for systems which require external observations, since any change in position is readily apparent.

Although prone to error and generally not as accurate as other methods of navigation, dead reckoning can be more robust than other navigators since it will continue to function when others fail. Furthermore, dead reckoning navigation is less expensive, typically by several orders of magnitude, than other navigational devices. Although in certain special instances dead reckoning navigation alone may provide sufficient navigational accuracy, in general, it should be combined with other navigational techniques to provide greater accuracy. See Section 6 for further details.

For the reasons elucidated above, dead reckoning navigation was implemented on the CMU Ambler, an autonomous, six-legged walking robot for planetary exploration. Section Figure 1.2 is a diagram of the Ambler. References [BW89],[BW90],[Man90],[Bar91] provide a description of the machine, its capabilities and dynamics.

Since wheeled vehicles maintain continuous contact with the ground, the distance the vehicle travels can be calculated from the number of wheel revolutions. (This assumes benign terrain. In rough terrain, the body displacement can be estimated by averaging the rotations of all wheels, however this is typically not very accurate.) However, when the Ambler body moves, the feet remain stationary (see discussion in Section 6), therefore, there is no direct means of measuring the Ambler's displacement. This necessitates a new methodology for dead reckoning estimation. Furthermore, the authors claim that this new methodology works equally well on benign terrain as on severe terrain, and that the dead reckoning for this class of walking machines will be more accurate than wheeled machines over severe terrain.

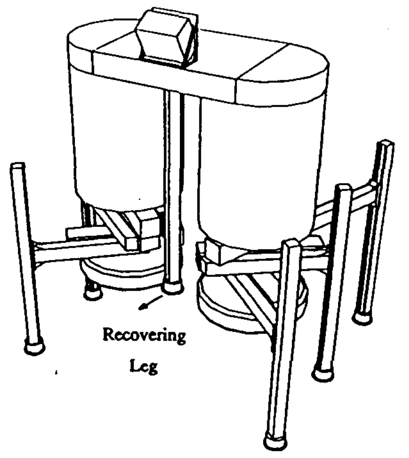

Figure 1.2. The CMU Ambler

This report is organized as follows: Section 2 describes dead reckoning navigation that has been previously implemented for other robotic systems. Section 3 presents a formal statement of the problem. Section 4 describes the approach implemented. Section 5 presents the results from the original tests of the system, improvements that were made based on these results, and a second set of test results. The final chapters discuss future work, improvements and conclusions. Appendix A discusses some of the early approaches considered and why they were discarded.

2 Previous Work

Since few mobile robots exist, there are few published articles pertaining to the use of dead reckoning navigation for this type of system. The papers by Smith *et al.* [SSC86] and [SC86], deal with a robot on a planar surface and discuss the effect of the cumulation of error. This work is conceptually important, but the authors do not appear to have tested this theory on a real robot. Furthermore, these papers assume some external sensing devices, such as acoustic sensors or cameras. Similarly, the papers by Watanabe *et al.*, [WS90], and Crowley, [Cro89], assume external sensors as well as dead reckoned position information.

The paper by Amidi, [Ami90], provides a discussion of an implementation of dead reckoning navigation for wheeled vehicles, and its actual implementation on the CMU NavLab. The paper presents equations for determining the dead reckoned position of a wheeled robot on benign terrain. The accuracy of the model was determined by comparing dead reckoned position estimates with positioning information obtained from a Global Positioning Satellite (GPS) system. In addition, there is a discussion of sensor fusion for improving overall system navigation. .

The author is not aware of any work discussing dead reckoning navigation for legged vehicles. This may be due in part to the very small number of legged vehicles in existence today.

3 Problem Statement

The problem is to determine the position and orientation of a walking robot at any given time. To do this a coordinate system, called the *body frame*, will be assigned to the robot. Another coordinate system, called the *world frame*, will be assigned to a frame of reference. Points in one frame are mapped into the other frame by a translation (by a vector **T**) and a rotation (by a matrix **R**). Figure 3.1 below shows the world frame, the body frame and a point, j, fixed in the body frame.

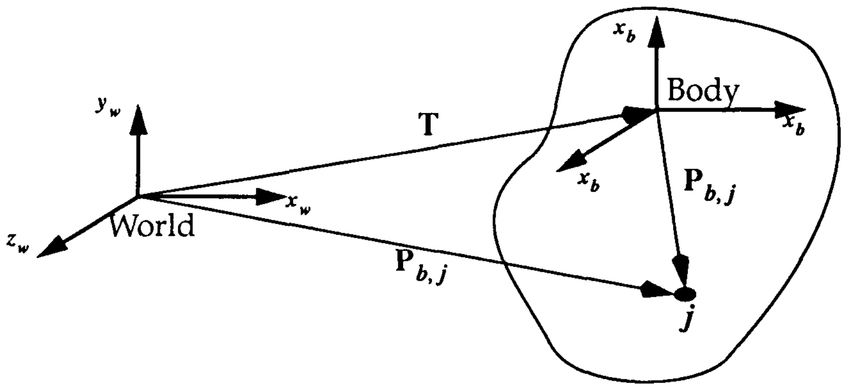

Figure 3.1. Relation between world and body frames

If the position of a point fixed on a rigid body and orientation of that body were known for all time in some frame, then the position of any other point on that body, with respect to the specified frame, is readily calculated for all time. But this will not work for the Ambler for several reasons: First, the Ambler has automatic body leveling, which permits continual reflexive changes in body orientation. Second, structural deformation will also lead to unmodelable changes in orientation. Third, the technique used to set a foot on the ground causes unintentional changes in body orientation. Fourth, small soil deformations can also cause unreported changes in body orientation as well as in foot position. (See the discussion in Section 6.) These reasons preclude the possibility of storing body orientation and using a simple scheme based solely on the addition of displacements from the initial configuration. Instead, only the positions of the feet are stored and are used as the basis for position estimation.

To solve the dead reckoning problem, it is assumed that at any given time, the Ambler can be treated as a rigid body (in this work, the effects of structural distortion are ignored, see Section 5 and Section 6 for further discussion), which means that the feet do not move unless commanded to do so. With these assumptions, the dead reckoning problem becomes one of finding the rigid body transformation (the matrix **R** and the vector **T**) which satisfies:

$$\mathbf{P}_{w,j} = \mathbf{R}\,\mathbf{P}_{b,j} + \mathbf{T} \qquad 3.1$$

where, $\mathbf{P}_{wj}$ is the position of foot j in the world frame, and $\mathbf{P}_{bj}$ is the position of foot j in the body frame. Both $\mathbf{P}_{wj}$ and $\mathbf{P}_{bj}$ are known at all times since the former are stored and the latter can be calculated from the mechanism kinematics. Since measured data is imperfect and models are not exact, a solution is sought which minimizes the squared error, $q(\mathbf{R},\mathbf{T})$, where:

$$q(\mathbf{R}, \mathbf{T}) = \sum_j w_j \mathbf{e}_j^T \mathbf{e}_j \qquad 3.2$$

with

$$\mathbf{e} = \mathbf{P}_{w,j} - \mathbf{R}\,\mathbf{P}_{b,j} - \mathbf{T} \qquad 3.3$$

where w_j are weights which are related to the observations.

To specify the position and orientation of a rigid body in some frame, the position of three, non-colinear points fixed in the rigid body must be known in that frame. If fewer than three points were known, the problem is insoluble. (If only one point were known, the body would be free to rotate about that point. If only two points were known, the body would be free to rotate about the line defined by the two points.) If more than three points are specified, the problem is over-constrained. In the case of the Ambler, six points, the positions of the feet, are known, thereby over-constraining the problem. One approach to this over-constrained problem would be to discard the extra data, but ignoring valid information does not seem productive. Another approach is to use a method which allows weighting the data according to its reliability. The weights are the w_j which appear in Equation 3.2. There are several methods for doing this which are discussed in Section 4.

In summary:

> The Ambler dead reckoning problem is to determine the position and orientation of the Ambler in the world based on the positions of the feet in the world frame. The solution is the rigid body transformation which minimizes the squared error defined by Equation 3.2. This is an over-constrained problem.

4 Approach

The implementation of dead reckoning on the Ambler is decomposed into three components: the elimination from consideration of slipped feet, the solution of the rigid body transformation problem and the update of the foot positions. The first step is used to remove erroneous data points so they do not affect the accuracy of the solution. The second step is the actual position determination. The third step is used to update the positions of the feet based on the newly calculated body position.

4.1 Checking for "slipped feet"

It is possible that a foot may make an uncommanded move due to some external force; this would violate our assumptions. To achieve the best position estimate, these positions should be eliminated so they do not corrupt the solution. For any rigid body, the distance between two points fixed on that rigid body is a constant. Using this fact, the foot positions are preprocessed to prune any suspect ones. Using the assumption that the Ambler is a rigid body, the distance between each pair of feet is calculated in the world frame and the body frame, and then compared. In predicate form, this can be expressed as

$$\text{slip(i,j)} = \begin{cases} 1 & \text{if } \left| \|\mathbf{P}_{w,i} - \mathbf{P}_{w,j}\| - \|\mathbf{P}_{b,i} - \mathbf{P}_{b,j}\| \right| > \Delta \\ 0 & \text{otherwise.} \end{cases} \qquad 4.1$$

If a foot slips, the rigid body assumption is violated, and the difference between the distance calculated between two feet in the world frame and the distance calculated between two feet in the body frame will exceed some allowed value Δ. The position of a foot which is found to have slipped is not used for the solution of the rigid body transformation.

4.2 Solution of rigid body transformation

The determination of a rigid body transform is a common problem in computer vision, arising frequently in techniques such as motion analysis and camera calibration. Three different techniques are commonly used for the solution of this problem. The first method ignores the fact that **R** is an orthonormal matrix and treats it instead as a matrix of nine independent unknowns. Once a solution is obtained, **R** is orthonormalized. The other methods treat **R** as an orthonormal matrix from the start, but derive the result by different representations of **R**. The second method leaves the problem in the form of vectors and matrices, while the third makes use of quaternions.

4.2.1 R as nine independent unknowns

This technique is conceptually the simplest, but probably yields the worst results. The fact that **R** is a rotation matrix is ignored, instead it is treated as a matrix of nine independent unknown quantities. This allows rewriting the right-hand side of Equation 3.1 as a matrix of constants multiplied by a vector of 12 unknowns (9 from **R** and 3 from **T**). Since each foot position consists of three values (the Cartesian coordinates in the world system), a minimum of four foot positions are required to find a solution using this method. A solution which minimizes the squared error is applied to obtain **R** and **T**, [Gel74]. Once an **R** matrix is obtained, Gram-Schmidt orthonormalization, or equivalent, [PFTW88], is applied to make **R** an orthonormal matrix.

This method seems appealing because of the simplicity of the solution. However, the solution may require more computer time than the other methods due to the inversion of a 12×12 matrix and the orthonormalization procedure. In addition, poor results are expected because there are *not* 12 independent unknowns, but rather only six.

4.2.2 R as a rotation matrix

Two techniques are presented which find the solution to Equation 3.2. The first represents **R** as a matrix, the second represents **R** as a quaternion. Since these techniques make use of the fact that rotation matrices are orthonormal, only three foot positions are required for the solution (see page 4).

For both of these techniques, auxiliary equations are needed in addition to Equation 3.1 to ensure the orthogonality of **R**. In the singular value decomposition scheme, these are expressed as additional constraint equations forcing the orthogonality and unit norm of the columns of the matrix. For the quaternion approach, this condition is enforced by normalizing the resultant quaternions. It is important to note, that in both of these cases, as well as the previous case, the solution is obtained without iteration.

The Singular Value Decomposition Technique

The derivations for this method are found in [Mat89] in Appendix B.1. Given a set of j measurements with associated weights, w_j, the best estimate for **R** and **T** to minimize Equation 3.2 is given by:

$$w = \sum_j w_j \qquad (4.1)$$

$$\mathbf{P}_1 = \sum_j w_j \mathbf{P}_{w,j} \qquad (4.2)$$

$$\mathbf{P}_2 = \sum_j w_j \mathbf{P}_{b,j} \qquad (4.3)$$

$$\mathbf{A} = \sum_j w_j \mathbf{P}_{w,j} \mathbf{P}^{\mathbf{T}}{}_{b,j} \qquad (4.4)$$

$$\mathbf{E} = \mathbf{A} - \frac{1}{w}\mathbf{P}_1 \mathbf{P}_2^T \qquad (4.5)$$

and let

$$\mathbf{E} = \mathbf{U}\mathbf{S}\mathbf{V}^T \qquad 4.6$$

be the singular value decomposition (SVD) of **E**. Then

$$\hat{\mathbf{R}} = \mathbf{U}\mathbf{V}^T \qquad 4.7$$

$$\hat{\mathbf{T}} = \frac{1}{w}[\mathbf{P}_1 - \hat{\mathbf{R}}\mathbf{P}_2]. \qquad 4.8$$

One important implementational note is that the **U** and **V** matrices returned by the SVD routine must have the same "handedness", that is they must *both* be right-handed or left-handed (that is, the determinants have the same sign), otherwise, the result of Equation 4.7 will be incorrect.

The Eigenvalue Technique

This solution differs from the singular value decomposition approach in the mathematics only. The underlying principles and assumptions are the same. Faugeras and Hebert first published this solution in [FH86]. First, Equation 3.2 is written in the form:

$$q = \sum_j |\mathbf{q}*\mathbf{x}_j - \mathbf{x}'_j*\mathbf{q} - \mathbf{t}*\mathbf{q}|^2 \qquad 4.9$$

where **q** is a quaternion representing the rigid body rotation, **t** is a quaternion representing the rigid body displacement, and **x** is a quaternion representing the data points. It can be shown that

$$\mathbf{q}*\mathbf{x}_j - \mathbf{x}'_j*\mathbf{q} = \mathbf{A}_j\mathbf{q} \qquad 4.10$$

where $\mathbf{A}_i$ is a 4×4 matrix and * is the quaternion multiplication operator. Defining

$$\mathbf{A} = \sum_j \mathbf{A}_j\mathbf{A}, \qquad 4.11$$

$$\mathbf{C} = \sum_j \mathbf{A}_j, \qquad 4.12$$

the solution is found to be

$$\mathbf{t} = \mathbf{C}\mathbf{q}_{\min}*\bar{\mathbf{q}}/N \qquad 4.13$$

where $\mathbf{q}_{\min}$ is the eigenvector associated with the minimum eigenvalue of the matrix $\mathbf{A} - \mathbf{C}^T\mathbf{C}/N$, where N is the number of data points. The rotation matrix can be extracted directly from $\mathbf{q}_{\min}$. For further details, refer to [Hor87] and [Ham69].

Since a rigid body rotation can be described by three numbers, storing the entire **R** matrix is superfluous. Routines are used to uniquely extract three numbers from a rotation matrix, [Cra86], [KLL83].

4.3 Update positions of slipped feet

After the position of the body has been calculated, the positions of the feet in the world are calculated based on the new body position and compared to the stored values. Ideally, there should be no difference, but since the algorithms used are based on minimizing the squared error, it is not expected that the difference will be identically zero for any foot. To minimize the accumulation of error, the position of a foot is updated only if the difference exceeds some tolerance value. This can be done since the primary assumption of the dead reckoning technique is that the feet do not move unless commanded to do so.

4.4 Determination of rotations

A clinometer measures the angle of the gravity vector relative to itself. To understand the need for clinometers for the determination of the tilt angles, the following thought experiment is helpful. Imagine the Ambler standing in a normal stance. If one leg were to fully extend vertically, the Ambler would be at a severe angle, with only three feet making contact with the ground. This violates the assumption that feet which are not commanded to move, do not move. However, in this case, the dead reckoning would erroneously report that the Ambler had placed the foot into a deep hole and that body position had not changed.

To solve this problem, clinometers are used to detect the body tilt angles. After the dead reckoning algorithm has computed the new body position, the clinometers are read, and the values are used in the rotation matrix. Experiments conducted before the installation of the clinometers showed that the tilt angles were never changed by the dead reckoning algorithm. The effect of this was that the dead reckoning reported that the Ambler was walking on a slope, depending on the initial value of the slope that was supplied to the system. The use of clinometers dramatically improves the accuracy of the dead reckoning. Further improvements can be made by using additional sensing information. This will be discussed in more detail in Section 6.

5 Test Results

This section first describes the current implementation, then presents the results from several tests. The term "slipped foot", see Section 4.1, is actually a misnomer, which was devised before the Ambler had been constructed. It was originally believed that an external force could backdrive a leg, resulting in a difference between the commanded leg position and the actual leg position. Experience with the Ambler has shown that this is highly unlikely, however, the check for slipped feet has not been eliminated because the inclusion of an erroneous data point would adversely affect the results of the position calculations.

To date, the only approach implemented on the Ambler for the solution of the rigid body transformation problem is the singular value decomposition technique. Several tests of the dead reckoning accuracy have been conducted. To test the real-world accuracy of the Ambler, a system was devised which measures the position of the body in the world directly. This is used as the ground-truth against which the accuracy of the dead reckoning is compared. As the Ambler moves along a path, the commanded move, the dead reckoned position and the directly measured position are recorded for each step.

To measure the ground truth position of the Ambler, retro-reflectors were mounted on the Ambler in several locations. The positions of these locations in the body frame are known from direct measurement. To determine the position of the Ambler, a surveying instrument is used to locate these points in the world frame. By using the same technique as the dead reckoning algorithm, to find the solution of a rigid body transformation, the position of the Ambler is calculated.

This method has a certain degree of inaccuracy, due to errors in measured location of the retro-reflectors, operator inconsistency in taking the measurements, the use of a least-squares technique, etc.; however, each measurement is independent of the previous measurement and the error sources are independent, so there will be no accumulation of ground-truth measurement error.

5.1 Test Prior to Comprehensive Calibration

The two figures below are the results of a dead reckoning test in which the Ambler took 6 steps following an approximately straight trajectory. Figure 5.4 shows the difference between the commanded and dead reckoned position. The error is on the order of several millimeters per step. Figure 5.2 shows the difference between the dead reckoned position and the ground truth position. This shows a cumulative error on the order of 3.5 cm per step (seven percent of the body advance).

As seen in the figures, the dead reckoning calculation is consistent with the commanded body moves. This is because both the dead reckoning and commanded move routines use the same kinematic routines, which are based on a set of nominal leg parameters. These leg parameters were derived from the blue-prints for the Ambler and do not accurately represent the as-built configuration of the machine. Kinematic inaccuracies are a large contributor to the difference between the dead reckoned position and the ground truth position.

It is important to note, that even though the results from this test were quite poor, the seven percent error is systematic, not random. That is, the dead reckoned position always fell short of the actual position. This indicated that a bias existed in the system that had not been properly modelled.

5.2 Kinematic Improvements

To perform the basic task of walking, an error of 3.5 cm/step is too large. To reduce this error, some calibration work was done on the Ambler. This resulted in improvement in two areas: a good representation of the kinematics of the extensional links and a model of the structural deflection.

As noted in Section 5.1, the parameters used for the calculation of the commanded body moves and the dead reckoning were based on as-drawn values. As expected, the as-built mechanism differs from the as-drawn mechanism, and a method is needed to determine the actual dimensions. This is a robot calibration problem and has been solved previously, for example [HTR88]. The basic idea of this calibration scheme is that each of the Denavit-Hartenberg parameters, [DH55], is perturbed slightly from its nominal position. The perturbed values can be solved for by accurately measuring the end point of the robot and comparing this position to the robot controller's estimation of its position (based on the nominal kinematic parameters).

Since the largest error occurred in the plane of motion, only the inner and outer links were calibrated. This significantly reduced the scope of the calibration problem, since the these links form a planar mechanism. The parameters of interest are the outer link offsets from the shoulder joint in both planar dimensions, see Figure 5.3. The distance from the foot to the shoulder joint was measured directly and the outer link extension is known exactly as a function of the gearing ratio. Using a least-squares algorithm, the values of the two parameters of interest were updated to reflect the as-built configuration.

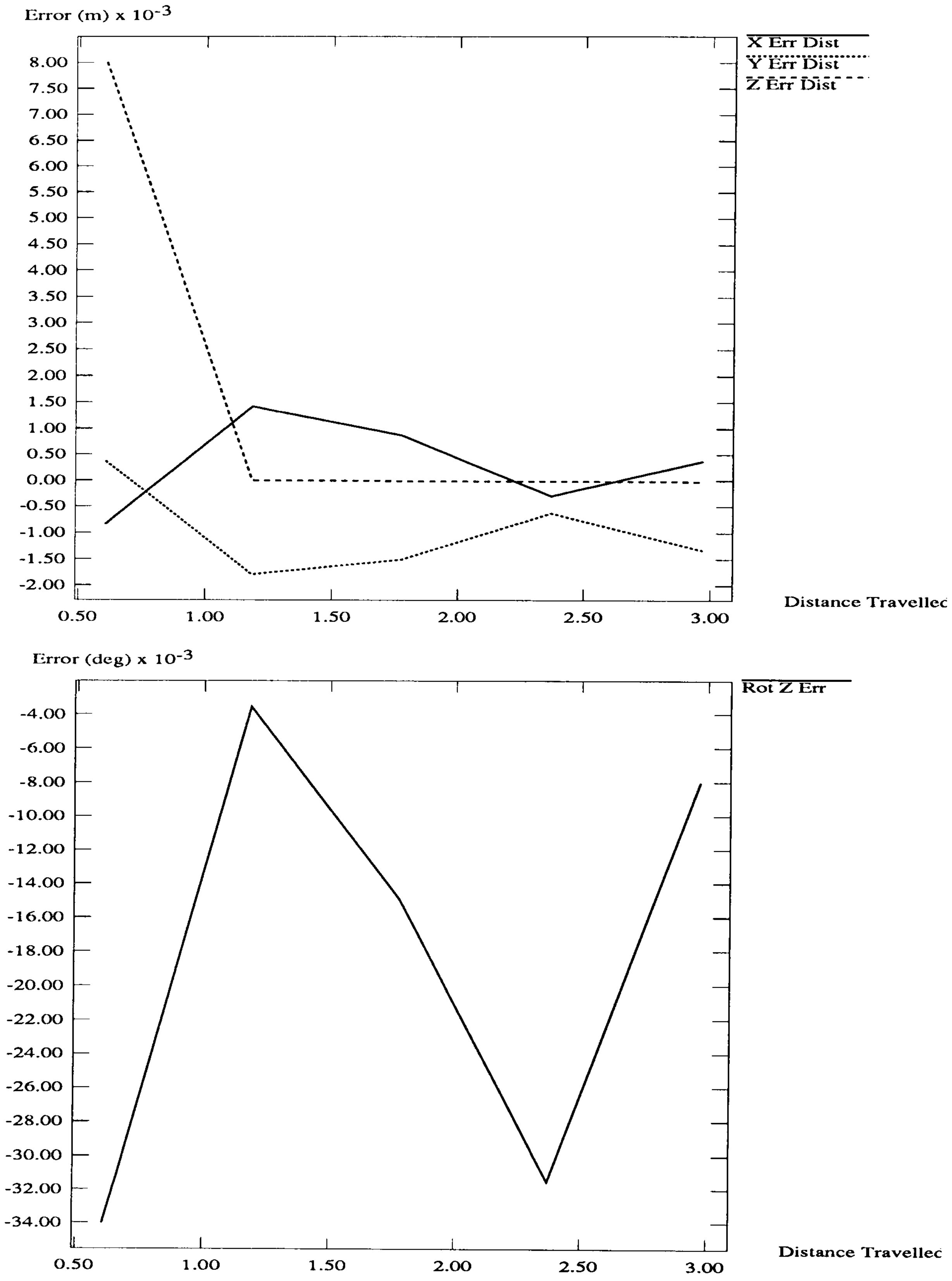

Figure 5.1. Difference between dead reckoned position and commanded move, uncalibrated

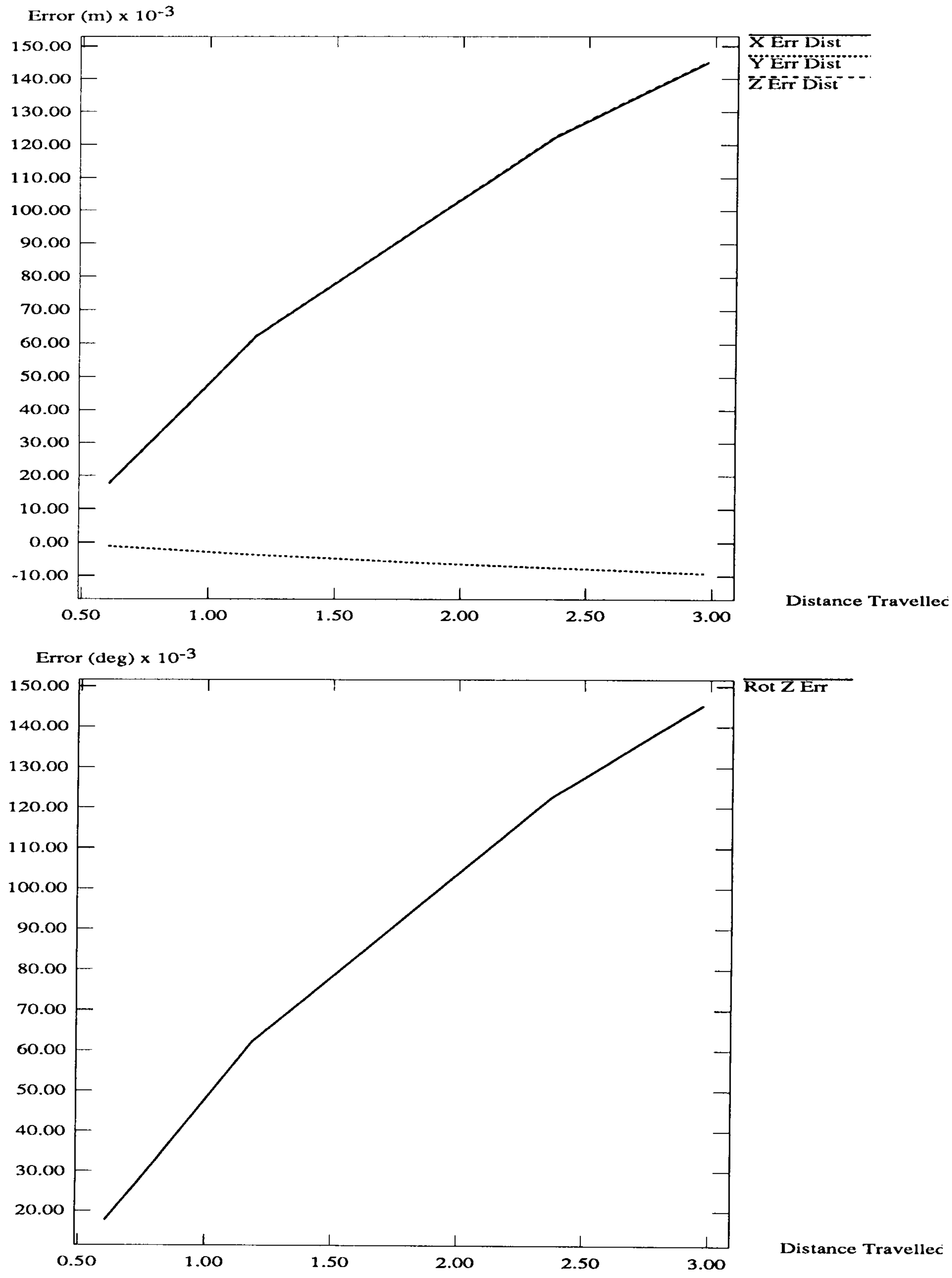

Figure 5.2. Difference between dead reckoned and ground truth positions, uncalibrated

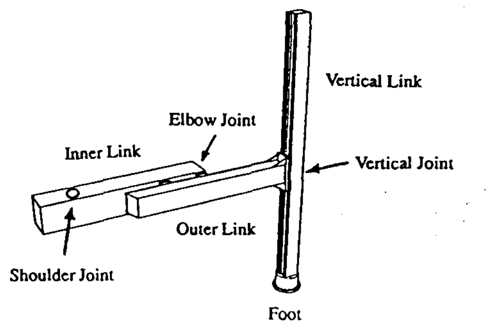

Figure 5.3. Ambler leg detail

The structure of the Ambler deflects due to the weight of its body. This deflection causes two phenomena: a point on the body is lower than expected (sag) and the feet move apart from each other (spread). To improve the accuracy of the dead reckoning, sag and spread were measured for a variety of tread widths. Using this data, a routine was written which can approximate the sag and spread as a function of the configuration.

5.3 Test after Calibration

After the incorporation of the kinematic improvements detailed in Section 5.2, another dead reckoning test similar to the first was conducted. Again, as in the first test, the dead reckoned position and the commanded move were consistent. Figure 5.4 and Figure 5.4 present the same information as Figure 5.4 and Figure 5.2.

After these improvements, the error per step has been reduced to about 1.2 cm/body advance (two percent of body advance). As in the previous test, the dead reckoned position always lags the actual position. There are two possible conclusions that can be drawn: there is still an unmodelled systematic error and/or the Ambler slips as it walks. Instead of trying to determine the source of this bias, it is corrected by simply adding two percent of the body advance in the direction of travel. (See Figure 5.6.)

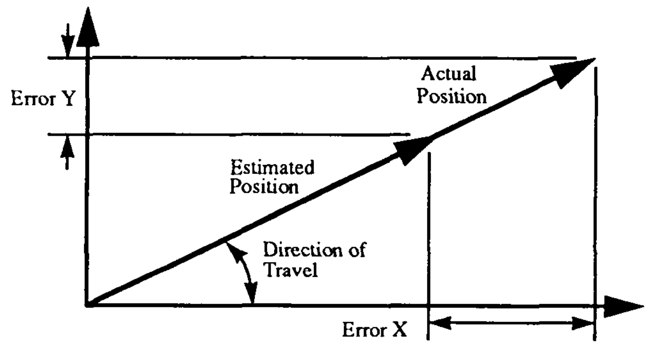

Figure 5.6. Bias correction

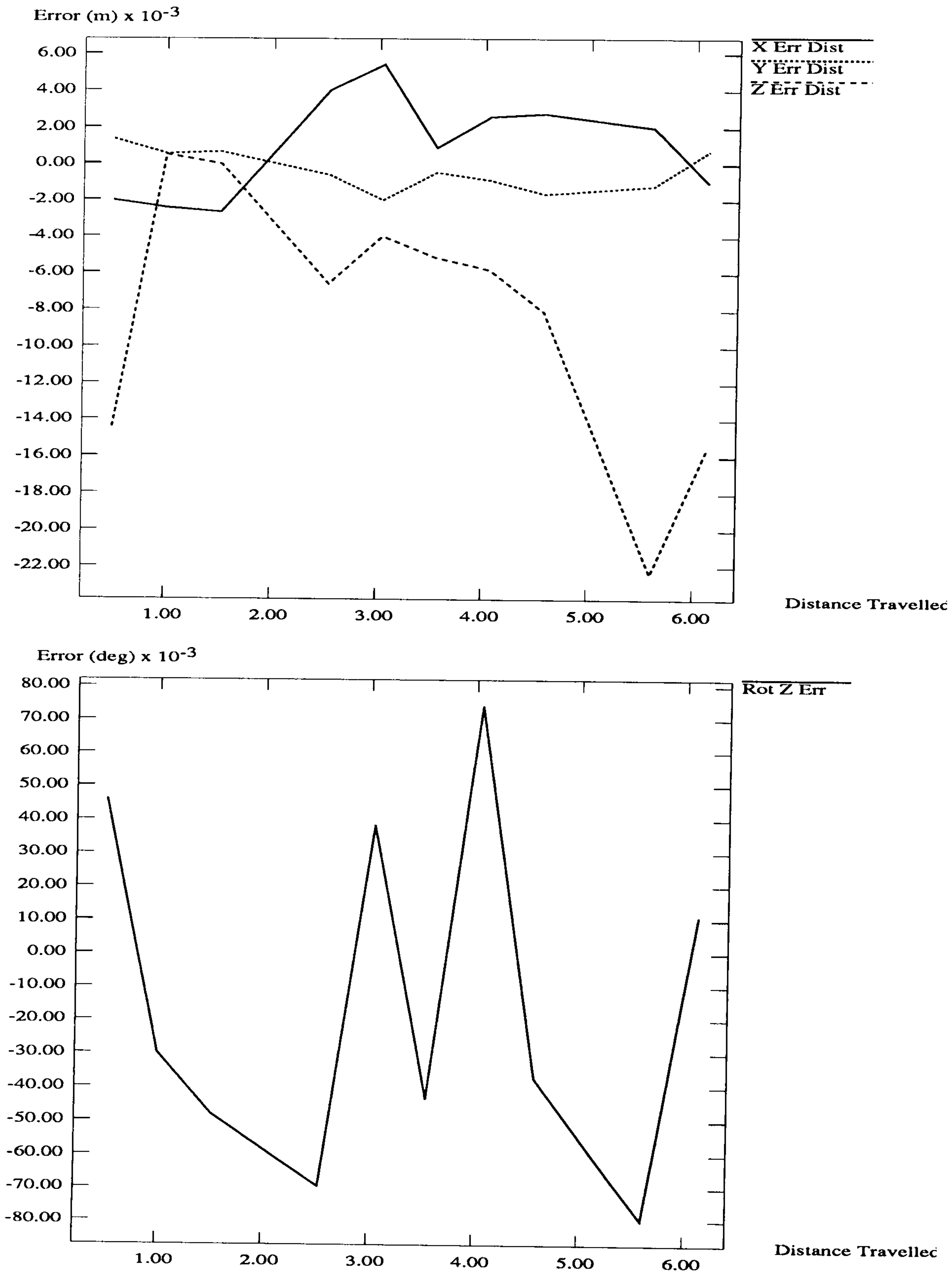

Figure 5.4. Difference between dead reckoned position and commanded move, calibrated

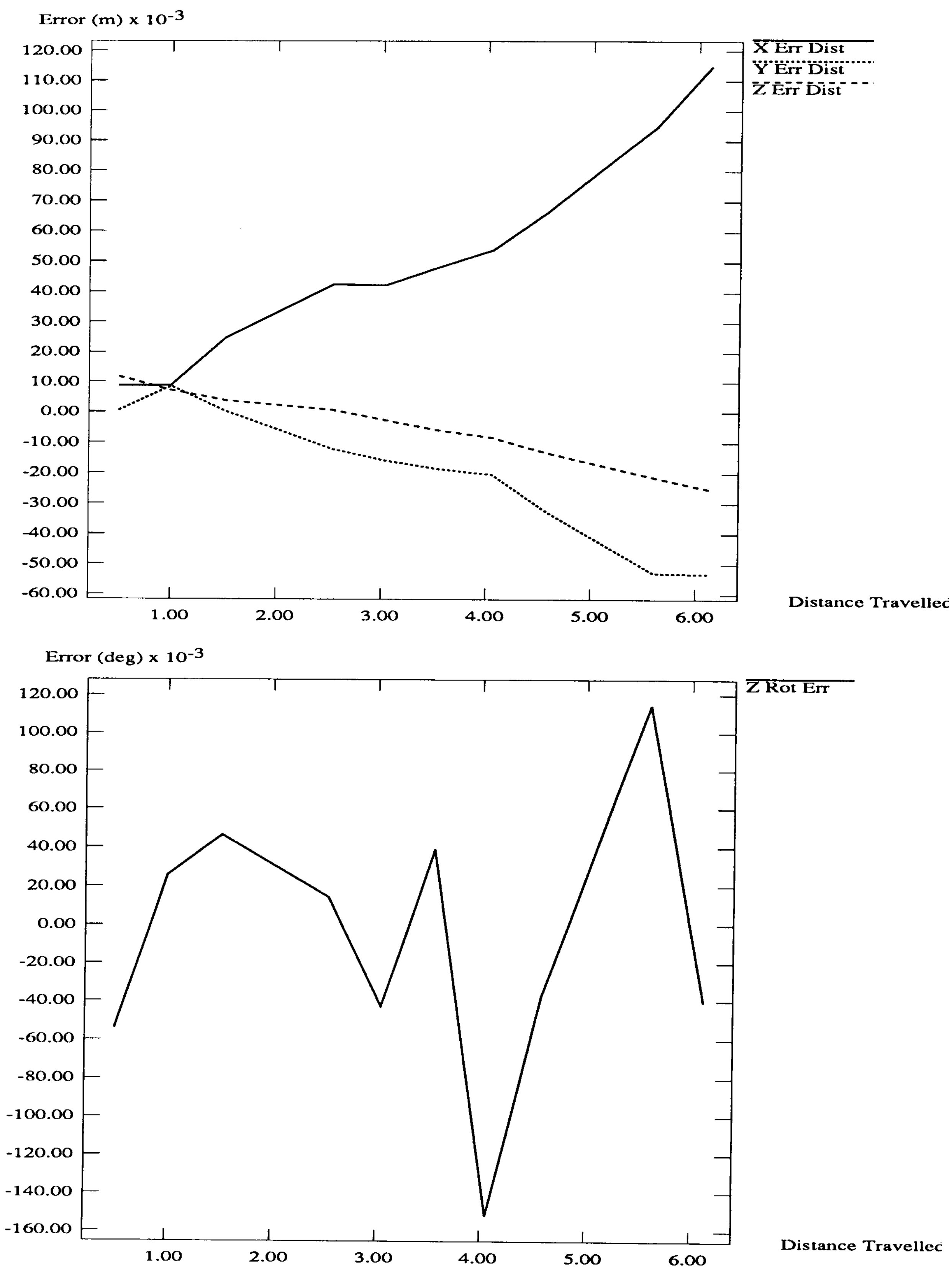

Figure 5.5. Difference between dead reckoned and ground truth positions, calibrated

The inaccuracy is the direction perpendicular to the line of travel, the world Z axis, is probably due to the unmodelled effects of the foot terrain interaction. This inaccuracy is caused by the compression of the soil. The Ambler sinks into the soil with each step, but this sinkage is not modelled. Examination of the data shows that the Ambler position estimates are always higher than the ground truth position. The error in angle of the rotation in the plane shows what appears to be a random error, on the order of 0.1 degrees. A larger number of points would have to be studied to determine if the error is truly random.

The current results are sufficiently accurate to be used for near-term testing of the Ambler. However, as the mission duration increases, greater accuracy will be required from the navigation system. This will be achieved by combining the dead reckoned position with other position estimates. To do this, the uncertainty of the dead reckoning must be modelled. The technique presented above for removing the bias is a first step towards this model, but is insufficient. However, if the error in the angle of rotation in the plane is shown to be a random error, then its uncertainty model would be based on the magnitude of the error.

6 Future Work

There are several areas in which work still needs to be completed. The first is to implement alternative techniques for the solution of the rigid body transformation problem. The second is to perform more tests of the dead reckoning to determine system accuracy and to help determine an error model. A third is to account for small positional perturbations of the foot positions which are not currently modelled. A fourth is to continue testing the system, including turns, point turns, longer distances, different soils, etc.

The implementation of the other techniques is already in progress. Since the bulk of the dead reckoning code is in place, all that is required are routines which perform the actual calculations; the rest of the code remains unchanged. Unfortunately, testing the dead reckoning code offline is limited to simple checks of a static model to determine if the code functions properly. The current dead reckoning simulator can not effectively account for foot slippage on the soil, terrain deformation, drive-train over shoot and backlash, and hence lacks rigorous fidelity. However, experience indicates that if the routines function on the static model, they will function on the Ambler. In addition, the accuracy of and computational time for each of the three techniques for the solution of the rigid body problem will be compared. Although, computational efficiency is not of extreme importance, if two techniques are equally accurate, the more efficient one may be preferred.

These techniques will be tested to see which provides the greatest accuracy and which executes the most quickly. Although not yet implemented, it is expected that the quaternion approach will produce results of the same order of accuracy as the singular value decomposition approach, but will do so more quickly because the number of calculations necessary to find the eigenvectors of a 4×4 symmetric matrix is fewer than the number of calculations to find the singular value decomposition of a matrix. (Actually, only the eigenvector associated with the smallest eigenvalue is needed, and this can be extracted very quickly.) In addition, several implementational issues, such as matrix handedness, are avoided.

Although the current tests have provided a quantification of the positioning error, further tests are required to see if this error is a function of length of a body move and how it is affected by turns. This information will be used to combine the dead reckoned position with the position estimates from other navigational devices.

As stated several times, a primary assumption of the dead reckoning is that the feet do not move unless commanded to do so. However, this is not the case. In the current mode of operation, the Ambler walks by recovering its rear-most foot, placing it in front of the body, loading it to some pre-determined force, then moving the body forward. For dead reckoning purposes, the position of the foot in the world is updated after the foot has placed down, but before the body moves forward. Since most terrains are compliant, as the body moves forward, a greater load is applied to the front legs, causing them to sink into the terrain. Currently, this sinkage is not accounted for and may prove to be a source of inaccuracy. These effects must be studied, and if the results are significant, they should be incorporated into the model. This study should also provide useful information that can be used in other aspects of the project other than navigation, e.g., determining the appropriate amount of force to apply when a foot is set down.

7 Conclusion

As previously stated, Section 1, dead reckoning alone is usually not sufficient for precision navigation. Other sensors are needed to improve the overall system accuracy. These sensors include gyroscopic inertial navigators, magnetic compasses, global positioning system (GPS), terrain matching using vision, celestial navigation, horizon navigation, etc. To maximize the accuracy of the system, the output from each of these sensors is combined in an appropriate manner to maximize system accuracy. One technique for combining these positions and uncertainties is Kalman filtering, [Gel74]. The basic concept is that each measurement has an associated uncertainty and the contribution by each constituent to the final result is weighted by its uncertainty.

Testing of the Ambler has shown that absolute positioning accuracy is on the order of several millimeters. In Section 5, the best dead reckoning results presented show an error of two percent of the body advance. The question must be raised is whether more time should be spent improving the accuracy of the dead reckoning, perhaps by further mechanism calibration? The authors feel that this is not necessary, because the error can be quantified and used to combine the results of dead reckoning with other navigational schemes.

The final acceptance tests will be to determine the accuracy of the Ambler as it walks outdoors over large distances, elevation changes and obstacles. Even though the floor of the current test area has been sculpted to provide varying terrain and compliance, the range is quite limited.

The dead reckoning system implemented thus far is the first step towards the goal of a robust, autonomous navigation system. A key development in the work to date was the recognition that the location of the Ambler can be determined by the solution of the rigid body transform problem, which permits using previously developed methodologies. Initial tests of the system have shown that the system works well and have indicated those areas where improvements are required. The navigation problem takes on increased importance as the goals for the Ambler become more ambitious. The work presented in Section 4 is only the beginning of the solution to the navigation problem, a problem which may eventually encompass such topics as soil mechanics and integration of additional navigational devices.

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A Discarded Approaches

The first approach used the notion that the shoulder joints for each of the legs, on either stack, lies along a line in space. The plan was to perform inverse kinematics from each of the foot positions to locate each of the shoulders in space. A line drawn through these points would represent the two central shafts of the Ambler, and the position and orientation could then be calculated. This approach was rejected because it appeared to require an iterative technique to determine the orientation angles and because it seemed that it would give poor results if the Ambler was not perfectly vertical.

The next approach was based on the idea that three, non-coincident, overlapping spheres intersect at exactly two points in space. This approach, although robust, was rejected for two reasons: it requires an iterative solution and there was no way to extend this technique to solve for the rotation angles.

Once it was realized that the dead reckoning problem is the determination of a rigid body transform, the utility of certain concepts and mathematical approaches became apparent. Particularly, techniques developed for camera calibration could be adapted since that too is a problem of determining a rigid body transformation. These techniques were used since the image calibration problem has extensively studied.