Applications recently proposed for ultrasonic imaging, for example in "image guided surgery" [1], [2] and in the custom design of wheelchair cushions [3], demand much higher dimensional measurement accuracy than is required by any of the traditional diagnostic visualization applications of ultrasound [4]. To render dimensionally accurate ultrasonic images it is necessary either to have a priori, or to acquire as an integral part of the measurement process, accurate knowledge of the speed of sound in skin, fat, muscle, bone, and other live media whose interfaces spawn echoes. To correctly interpret the intensities of the echoes it is also necessary to have accurate knowledge of the acoustic impedances of these media.

In contrast with engineering materials, whose acoustic properties can usually be measured off-line on manufactured artifacts of known dimensions, for dimensionally accurate imaging of living tissue there is no apparent alternative to in situ calibration. In this article we pose the quantitative ultrasonic imaging problem, and we propose and model some apparently workable approaches to in situ measurement of the speed of sound and, where appropriate, its gradient. We defer discussion of acoustic impedances, and thus of signal intensities. Because a method's accuracy cannot be modeled in the absence of knowledge of the signal and noise intensities, quantitative treatment of this subject is also deferred.

Each of the results described in this article is already well known in another specialized field, e.g., medical imaging, submarine navigation, or oil prospecting. We present them here in a consistent form and hierarchical order with the hope that will be a useful tutorial and reference for students who entered the field from disciplines that did not expose them to the principles and practice of physical measurements.

The Quantitative Ultrasound Problem and Its Approach
At the literal "cutting edge" of their instruments, practitioners in the emerging field of image guided surgery would like to have navigational accuracy under 1 mm, and even better endpoint precision. Related application areas, e.g., our own collaboration with clinical practitioners who design custom seat cushions to prevent pressure sores in wheelchair-bound patients, make less stringent, but in principle quite similar, demands on the accuracy of ultrasonic dimensional measurements and image generation based on those measurements.

Unfortunately the uncertain speed of sound in an individual patient’s live skin, fat, muscle, etc., frustrates the surgeon’s (and the seat cushion designer’s) desire for these levels of accuracy and precision. The solution is to find measurement techniques that do not require a priori knowledge of the acoustic properties of the media traversed. That is, we need to find experimental techniques that measure the speed of sound in situ, through the very regions of “flesh and blood” whose dimensions we seek. These techniques must function though we are denied access to media samples in manufactured shapes, denied access to both sides, and denied any access except through whatever overlaying and underlaying strata constitute the natural structures.

We proceed by separating the general problem into three measurement “modules” for each of which we propose an apparently robust experimental solution:

1. In situ speed of sound and layer thickness measurement given multiple parallel homogeneous layers (see the subsection “For One Homogeneous Layer” and “For Several Parallel Homogeneous Layers”).

2. Coping with a speed of sound gradient in a layer (see the subsection “For Parallel Layers With a Gradient”).

3. Coping with a tapered layer (see the subsection “For Non-parallel Layers”).

We defer assembling the modules into a comprehensive system. A future integrated system, with experimental confirmation, will satisfy the medical imaging requirements posed herein, as well as corresponding requirements for nondestructive inspection of engineering structures where the materials are analogously unavailable for off-line measurement of their acoustic properties.

Assumptions and Context For Ultrasonic Imaging
Ultrasonic imaging with accurate dimensional calibration requires mechanically accurate scanning capability for the raster,
and accurately calibrated ultrasonic ranging capability to each surface of interest. The "surfaces of interest" are the interfaces between layers of different but nominally homogeneous materials, e.g., skin, fat, muscle, and bone. A viable system will also need to detect and compensate for inhomogeneities within layers.

Current mechanical, optical, magnetic, etc., tracking technologies are assumed in this article to be of sufficient accuracy and precision to meet the application's requirements for dimensional calibration of the raster.

When imaging multilayered engineering specimens, the materials comprising the individual layers can usually be characterized off-line as to, e.g., speed of sound, dispersion, attenuation vs. frequency, acoustic impedance, etc., sufficiently well that accurate gauging is straightforward. Even when there is insufficiently detailed prior knowledge of these material properties, with man-made specimens it is often sufficient to obtain relatively accurate measurements. For example, the fraction of initial aluminum thickness lost in a small corroded spot on an airplane's skin can be measured, even if the absolute thickness of the aluminum cannot be. In other words, accurate relative gauging is insensitive to the material properties.

In contrast, with living anatomical specimens the subject-to-subject variation in material properties is problematic, yet individual off-line characterization of these properties is obviously impossible. Even for an individual subject, when dimensional accuracy is a critical issue, the possibility needs to be considered that a nominal tissue type in fact has locally inhomogeneous acoustical properties, it has globally different acoustical properties in different parts of the body, and it has temporally variable acoustical properties due to diet, muscle tone, etc. Furthermore the possibility must be considered that the vital state of tissue, e.g., whether muscles are tense or relaxed, whether limbs or buttocks are mechanically loaded or unloaded, etc., may affect acoustical properties and thus the dimensional accuracy of ultrasonic images.

In the past these difficulties-in-principle have rarely been a practical impediment because the applications of anatomical ultrasonic imaging have been primarily diagnostic, requiring only qualitative or semi-quantitative dimensional accuracy, i.e., enough to allow the physician to assess the normality of anatomical structures, to observe the approximate location, size, and shape of organs, etc. Recently, however, the applications mentioned have been hampered by the need for dimensional accuracy and precision beyond any that can be expected from only "generic" speed of sound estimates.

**Review of Basic Ultrasonic Measurement Techniques**

The basic single-sided ultrasonic measurement is a recording, versus time, of multiple echo amplitudes. Each interface between two layers of different acoustic impedances spawns an echo. The time delays between the transmitted pulse and the first echo, and between successive echoes, combined with a priori knowledge of the speed of sound in each layer, gives the layer thicknesses. This is illustrated for one layer in Fig. 1. Multiple layers are handled straightforwardly providing the number and nature of up and down segments constituting each echo can be surmised. (Important signal processing issues, such as how to define time-of-flight when dispersion and frequency-dependent attenuation distort the reflected pulse shape, are omitted from the present discussion.) For measured time \( t_i \) in layer \( i \) with speed of sound \( c_i \), the layer thickness is

\[
z_i = c_i \frac{t_i}{2}.
\]

Images are built up by raster scanning of pencil sensors, by linear scanning of one dimensional array sensors, or in areal patches by two dimensional array sensors.

If modest dimensional distortions can be tolerated then nominal values can be used for the speed of sound; these are tabulated in standard reference books for common engineering materials and for typical human tissue types [5]. However, individual differences, and additional fine details (such as the inhomogeneity and geometry issues discussed in subsequent sections, and the signal-processing complications of the sort mentioned previously) all frustrate rendering highly accurate images.

**Differential Methods**

Given a homogeneous layer of well defined mechanical properties (perfectly fluid, or perfectly elastic, or otherwise precisely characterized), simple differential measurements suffice, at least in principle, to measure both the thickness of the layer and the speed of sound in it. As illustrated in Fig. 2, changing the thickness of a layer slightly, by \( \Delta z \), and measuring the change in echo time-of-flight, \( \Delta t \), suffices to find the speed \( c_i = 2 \Delta z / \Delta t \) and thus the thickness \( z_i = c_i / 2 \).

It is, however, doubtful that this approach will be adequate in applications of interest to us. While skin, fat, and muscle layers can be depressed enough to make the method at first seem feasible, in the absence of complete mechanical models for these media we are at a loss to predict how the total compression is distributed among the various layers, how the densities and bulk moduli of the various layers may change (and thus how the speed of sound in them may change) due to incomplete fluidity, how compression may introduce directional dependencies into the speed of sound, etc.

It is thus not immediately obvious, given multilayered subjects of complex and incompletely known mechanical properties, that ultrasonic imaging with the accuracy demanded by the anticipated applications is actually possible. In the next section we outline approaches that appear to have sufficient promise that
external soft tissue shape can be obtained. This, in turn, will lead to the development of better preventative measures for wheelchair-bound individuals.

The CASS is a good example of the difficulties researchers face in making quantitative ultrasonic measurements. Multiple, uncharacterized layers of soft tissue lie between the surface of the buttocks and the pelvis. Furthermore, both the mechanical and biological (healthy vs. deteriorating) state of these tissues affects the velocity of sound within them. While an average velocity of 1540 m/s could be used to compute tissue thicknesses, much more accurate and meaningful measurements could be obtained if in situ calibration of the velocity of sound in different tissue layers could be performed. This would not only lead to more accurate tissue thickness measurements, but would also aid in identifying different tissue types and their states of health.

Our Approaches to the Real Problem

For One Homogeneous Layer

The approach hypothesized in the previous section and illustrated in Fig. 2 attempts to measure the unknown speed of sound by measuring time-of-flight over a path that is under the experimenter's control; we reject it because we are not confident that its application leaves unchanged the material properties, and thus the speed of sound we are trying to measure. A differential approach that is free of this concern is illustrated in Fig. 3. It involves two or more measurements over different oblique paths. (While diffuse reflection is also reported in the medical imaging literature, in the present article we consider only specular reflection.) In each measurement the speed of sound c and the layer thickness z are intertwined in

\[ c^2 \cdot \frac{z^2}{4} = z^2 + x^2. \]

Their simultaneous solution when \( i = (1,2) \) yields
Fig. 2. (a) Differential time of flight method for one layer. (b) Differential time of flight method for multiple layers. The stiffnesses of the individual layers must be known.

\[
c = \frac{\sqrt{a^2 - b^2}}{t^2 - t_1^2}
\]

\[
z = \frac{\sqrt{x_2^2 - x_1^2 - t_2^2}}{t_2^2 - t_1^2}
\]

If \(n > 2\), a least-squares solution will optimize accuracy.

**For Several Parallel Homogeneous Layers**

The single-layer method of Fig. 3 is easily extended to multiple layers, as illustrated in Fig. 4. Selecting two values of \(x_1\) and measuring the two corresponding values of \(x_2\), the two speeds of sound \(c_1\) and \(c_2\), and the two depths \(z_1\) and \(z_2\) are measured. (It is presumed that the signals at receivers \(R_1\) and \(R_2\) can be distinguished by their relative amplitudes.) The approach can be applied to an arbitrary number of layers. This technique is the mainstay of geoaoustics [6], where, in oil prospecting, for example, it is routinely necessary to characterize multiple complex rock layers (strata).

**For Nonparallel Layers**

When layers are tapered, as illustrated (in a two-dimensional cross-section) in Fig. 5, the acoustic time-of-flight defines an elliptical locus to which the reflecting discontinuity is tangent. For each transmitter \(T_i\) - receiver \(R_i\) separation \(2x_i\), we thus have the equation of an ellipse:

\[
\frac{x^2}{a_i^2} + \frac{z^2}{b_i^2} = 1
\]

where \(4a_i^2 = c_i^2 t_i^2\) and \(b_i^2 = a_i^2 - x_i^2\).

Usually only one physically reasonable line will be tangent to two such ellipses. Thus if \(c\) is known, two \((x_i, t_i)\) pairs fix the depth and slope of the reflecting plane. If necessary, an additional pair will resolve any ambiguity. When \(c\) is not known in advance, an additional pair is sufficient to find both \(c\) and the correct reflecting plane.

**For Parallel Layers With A Gradient**

Snell's Law of refraction in the form \(\sin \theta/c = k\), where \(k\) is a constant, holds even if the speed of sound \(c\) is a function of position within a medium. The angle \(\theta\), measured between the local tangent to the trajectory and the local gradient of \(c\), is then also a function of position, therefore the trajectory is curved. (If \(c\) is continuous there is refraction but no reflection.) A linear gradient, \(c = c_0 + \alpha z\), where \(c_0\) is the baseline speed, \(\alpha\) is its gradient, and \(z\) is the depth, is illustrated in Fig. 6. The linear approximation to arbitrary gradients is particularly useful because the trajectories...
The large differences in scale notwithstanding, the analogies between seismology and quantitative medical imaging are strong, and the seismology literature has been our best source of intuition-building about the kind of problems that are likely to arise (e.g., inaccessibility, multipath effects, scattering from inclusions), and about the measurement protocols and signal processing techniques that can best be brought to bear on the resulting potentially complex and confusing signals.

are then simply circular arcs; this result is well known in underwater acoustics [7]. Four transmitter/receiver separations suffice to measure $c_0$, $\alpha$, $\omega$, and the four launch angles [$\theta_b$, $\theta_p$, $\theta_p$, $\theta_\theta$] corresponding to $(x_1, x_2, x_3, x_4)$. The radius of the circular arc is given by

$$R = \frac{c_i}{\alpha \sin \theta}$$

where the index $i$ emphasizes that the radius $R_i$, the speed of sound $c_i$, and the angle $\theta_i$ are all measured at the same point. The center of the arc is at distance $R_i$ along the perpendicular to the trajectory on its concave side.

Conclusions and Future Work

We have considered the necessity of integral in situ calibration of acoustic properties for precision dimensional measurements and image rendering using ultrasound echo time-of-flight methods on living subjects. We have shown that by combining time-of-flight measurements over several paths with external measurements of transmitter and receiver locations generating those paths the relevant acoustic parameters and dimensional measurements can be extracted. It remains to be shown, theoretically and experimentally, that these methods can be successfully combined in a practical system that addresses natural geometries combining the impediments that we have herein addressed only separately.

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Fig. 6. Trajectories when the speed of sound is not constant.

References


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