MECHANICS OF TENSEGRITY PRISMS

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ABSTRACT

Tensegrity prisms are three-dimensional self-stressing cable systems with a relatively small number of disjoint compression members, invented by Buckminster Fuller. They form novel structural geometries and they constitute a class of mechanisms which have not been previously studied for possible application as variable geometry truss (VGT) manipulators. They have a number of seemingly advantageous properties -- they are self-erecting, in that tensioning the final cable transforms them from a compact group of members into a large three-dimensional volume, and they are predominately tension systems, in that they can function as a VGT manipulator while actuating members only in tension. These properties have not been explored but could be broadly useful, for applications ranging from temporary terrestrial construction to large on-orbit space stuctures. However, they have a number of properties which make them seemingly inappropriate for use -- they are not conventionally rigid, they exist only under specific conditions of geometry with a corresponding prestress state, and the governing equations that do exist include singular (non-invertible) matrices. In our opinion the advantages and application potential justify the study and discussion of tensegrity behavior. The full mathematics of tensegrity geometry, statics, and kinematics have not been formulated, and such mathematical results must be developed and assembled before applications can be undertaken. This paper describes the physical behavior of a basic family of tensegrity prisms, presents the most useful available mathematical results, and outlines a preliminary simulation study of such a prism used as a VGT manipulator.

INTRODUCTION

A tensegrity structure is a three-dimensional truss with numerous tension members and a smaller number of disjoint compression members, achieving a prestressed state at some funicular geometry. While such structures can be formed with many general patterns of connectivity, we restrict this paper to a particular fundamental form, in which three tension members and one compression member meet at each node and in which no two compression members meet. The least-complex such spatial structure is a prism possessing nine (tension) cables and three (compression) bars, which we refer to as a T-3 prism, and our specific results will be offered for that geometry; there exists a related T-4 prism with 12 cables and 4 bars, and so on. Intriguing characteristics of such a prism include the fact that it transforms from a compact bundle of bars into a full three-dimensional framework as the last cable is pulled in tension, that it is a form-finding structure, that it becomes a prestressed system, that our conventional simple use of Maxwell's rule does not properly describe the structural type, that our conventional definitions of statical stability and statical determinacy do not directly apply, that our conventional model of structural stiffness does not apply, and so on. Such prisms have been constructed for artistic purposes and as educational toys, and it has long been understood that by suitably changing the cable lengths the prism can be made to change its nodal geometry, and it is for this behavior that the prisms are likened here to variable geometry truss (VGT) manipulator behavior.

We note at the outset that there is no general way to dictate the nodal positions that define a tensegrity state. However, a starting state can be defined for the case of a right regular T-prism by symmetry and can be proven by simple statics. The T-3 consists of two equilateral triangular end faces, parallel to one another and normal to the axis joining their centers; in such a geometry there is a relative twist angle between the end faces of 30 degrees. In principle, from that recongizable starting state every achievable T-3 geometry can be reached by suitably changing member lengths. However, we will show that identifying "suitable" (permissible) changes in member lengths is a challenging problem in itself.

Several distinctive characteristics of tensegrity structures motivate us to study their suitability for robotic applications, including the following:

- Tensegrity prisms can be totally self-erecting from a low-volume bundle by tensioning the last cable. Such a capability would be useful for constructing temporary structures. It could also be used for constructing a conventionally framed tower; a tensegrity tower could be self-erected, strong enough and stiff enough to support the weight of workers, who could then climb the tower and add additional structural members to create a conventional tower framing. Similarly, a self-erecting prism could be remotely maneuvered to some location within a piping system in its collapsed bundle state, and then self-erected to form an internal framework.
- Noting the large number of cables in any typical tensegrity structure, it is likely that they will frame a geometry with less total material weight than a conventional three-dimensional truss. The self-erecting property, together with the anticipated weight efficiency, suggests its applicability for on-orbit space robotics and space structures.
- When member lengths can be varied or when large elastic elongations are admitted tensegrity structures present a new family of machine kinematics and a new family of shape-adaptable structures, with the advantage that actuation is required only for tension members, requiring a far simpler technology than general bi-directional translational actuators.

PREVIOUS WORK

Tensegrity structures were demonstrated by Buckminster Fuller [1] and Snelson with a patent date of 1962. The famed literary critic Hugh Kenner [2] provided an insightful quantitative analysis of regular tensegrity prisms and spherical tensegrities as introductory chapters in his 1974 book on geodesic dome geometry. A related book by Pugh [3] is esssentially an instruction manual for building hundreds of tensegrity structures, most of them regular. The fundamental contributions in mechanics are found in papers by Calladine [4-9] and his colleagues (Tarnai, Pellegrino, and others), and that work is the basis for most of the results reported here.

Mathematical studies of tensegrity frameworks have focused on characterization of rigid configurations, generalizing classical results on rigidity of pinned frameworks. Recent contributions are well characterized in the work of Roth and Whiteley [10], Connelly [11], and Conneley and Whiteley [12]. There is even a body of work in cell biology, described by Ingber [13], examining the mechanical behavior of the tensegrity prism as the basis of the ctyoskeleton. However, our discussion in this paper will focus on analytical results for engineering applications to construction robotics.

ANALYSIS; EXAMPLE PROBLEMS

The analytical results are best described with reference to examples, and without loss of generality the T-3 and T-4 prisms will be used. Examining these prisms by the conventional interpretation of Maxwell's rule we offer the following provocative observations:

- The T-3 prism yields a determinacy measure of 0, which conventionally describes a determinate truss. However, the T-3 prism supports a prestress state, while a conventional determinate truss cannot do so.
- The T-4 prism yields a determinacy measure of -2, which conventionally describes an unstable structure, namely a machine with two independent (finite) mechanisms. However, the T-4 prism supports a prestress state, while conventional (finite) mechanisms cannot do so. Moreover, the T-4 prism is structurally stable, while (finite) mechanisms are by unstable by definition.

The explanation of these observations and the useful mathematical tools for analysis are obtained from Calladine's findings [4,8]. They apply to a tensegrity prism only after it has been established in its tensegrity geometry, and about that equilibrium state they derive from linear algebra for small nodal displacements. The results derive from the well-known statics equilibrium matrix [A} and from its transpose, the compatibility matrix [B]. The results are summarized as follows:

- There is a proper, generalized interpretation of Maxwell's rule establishing that the determinacy measure actually expresses a quantity (s-m), where (s) is the number of possible independent states of prestress, and (m) is the number of possible independent (infinitesimal) kinematic mechanisms. For tensegrity structures these conditions supersede the more conventional definitions of necessary conditions for structural stability, determinacy, etc. In our example the T-3 prism has 1 prestress state and 1 (infinitesimal) mechanism, equalling the resulting measure of 0. Similarly, the T-4 prism has 1 prestress state and 3 (infinitesimal) mechanisms, equalling the resulting measure of -2.
- The number of prestress states and the number of (infinitesimal) mechanisms is found from the rank of [A]; in a tensegrity geometry that matrix is seen to be rank-deficient. The prestress states and the mechanisms themselves are then expressed in the fundamental subspaces of [A].
- Applied loads can be decomposed into those which are resisted within the equilibrium geometry and those which are resisted by (infinitesimal) mechanism motion.
- In the tensegrity prisms of interest, the mechanisms are infinitesimal rather than finite, such that they display a geometric stiffness (cubic force-displacement relationship) to resist applied loads.
- The prestress condition establishes in each mechanism an initial constant stiffness (linear force-displacement relationship) to resist applied loads.

Most significantly for robotics, the compatibility matrix [B] multiplied by a vector describing a (small) change in nodal positions yields a vector describing the (small) change in member lengths needed to reach those new nodal positions. We believe that sequential application of this process, together with the determination of the tensegrity status of the new nodal positions, can trace a path through neighboring tensegrity geometries by which the prism can be actuated like a manipulator.

These results have been verified additionally by experimental study of a T-3 prism, by geometrically non-linear finite element analysis using the ABAQUS application package, and by trial applications of the Working Model 3-D software package. Experimental results confirm the basic role of the compatibility matrix [B], derived as the transpose of the statics matrix [A].

We note that previous work has not addressed the form-finding problem itself; we cannot state a priori that a particular set of nodal co-ordinates is a tensegrity geometry. In all instances the analyses have been posed for a structure in a valid tensegrity geometry, or (equivalently) in an equilibrium prestress state, and for this reason it is useful to identify a starting state such as the right regular prisms pictured here.

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