Energy Utilization and Energetic Estimation of Achievable Range for Wheeled Mobile Robots Operating on a Single Battery Discharge

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***Abstract*—This paper presents an energetic model that analyzes energy utilization in mobile robot traverse and estimates maximum range achievable by wheeled mobile robots operating on a single battery discharge. After taking into account different energy utilizations, such as propulsion and steering, the model indicates that the most energy-consuming part of mobile robot is robotics functions, such as computing, sensing, communication, etc. Based on this it points out ways to improve maximum robot traverse range: increasing rover velocity, driving duty cycle (ratio of driving time to total mission time), and decreasing robotics functions' power. Considering the significant energy proportion of robotics functions, the leftover propulsive consumptions are analyzed, which directly determine the maximum range using a classic terramechanics model. The proportions of energy expended in internal robot system and external interaction with terrain are quantified with experiments using a small-sized 4-wheel robot. The maximum traverse range of wheeled-mobile robots could be significant, for example 17km with only 1.12kg battery (166 watts-hour), if the normally immense robotics consumptions are minimized or isolated from the propulsive branch. The resulting propulsion energy, which is only a small fraction of total battery energy expended, is used to estimate achievable range for wheeled mobile robots operating only on a single discharge.**

I. INTRODUCTION

Maximum robot traverse range is critical in planetary exploration, sewer inspection, mine rescue and many other applications. Short range will largely restrict robot's functionality. For example, planetary exploration would not be able to cover a large planet area, reducing the amount of collected data and mission coverage. The miners trapped in distant places in mine disasters would have less chance to get rescued. So improving maximum traverse range is always of interest in mobile robots.

Without solar panel, combustion engine generator or isotope reactor, energy carried by robots in batteries is always limited. So is the traverse range. Recharging is sometimes impossible for autonomous robots in work, especially in places and time periods lack of sunlight, such as in deep crater or night time in planetary exploration. Replacing new batteries could be difficult in places which are dangerous or unaccessible to human, for example, in exploded mines or confined sewers. Battery-powered robots without replenishment represent the generality of most mobile robots and provide the simplicity to focus on a constant energy amount while examining energy utilization. So only battery-powered wheeled robots operating on single discharge cycle are in the scope of this research.

Generally speaking, rover energy is expended for the following two purposes: (1) ancillary power for robotics functions like computing, sensing, communication, and payload which scales with duration of operations, (2) energy for motion, which predominantly scales with distance driven.

Two things that roboticist care about are how far robot goes and how fast it gets there. Curiously, total energy for driving is primarily independent on speed since driving energy is primarily related to rover mass, gravity, distance traveled and terrain resistance. By comparison, the energy for robotics functions, like sensing, computing, communication, utilizes considerable power whether driving or sitting. This ancillary power is less concerning when recharge is possible from solar, generator or radio isotope source, but this energy sink is paramount when operating from only single discharge from battery. In traditional exploration the mission energy for sensing, computing, communication and payload far exceeds driving energy. Most time is spent sitting or creeping. This traditionally requires days, weeks, months or years to drive kilometers. Energies of kilowatt hours are required because ancillary power is drained over such a long duration. The total energy expended during whole mission time could be quantified as:

$$\begin{aligned} Total\ Energy &= Robotics\ Energy + Mobility\ Energy \\ &= Robotics\ Power \times Mission\ Time \\ &+ Constant \times Rover\ mass \times Distance \end{aligned}$$

However, the significance of robotics and mobility energy could not be accurately reflected by this simple addition equation. This paper analyzes actual percentage of robotics and mobility consumption in whole mobile robot energetics and formulate a way to estimate achievable range operating on one single battery discharge.

II. PRIOR WORK

Mobile robots can find application in a variety of fields, such as mine rescue, sewer inspection, planetary exploration. Tab. I compares prior planetary rovers in categories of mass, reduced weight (assuming $2m/s^2$ as gravitational acceleration), time for 2km traverse (based on actual data), non-propulsion power (estimated), non-propulsion energy (in the time period

TABLE I: Comparison energy expended by previous Mars and Lunar Rover missions based on their mass and speed

Rover	mass	reduced weight	time for 2,000m	non-driving power	non-driving energy	Driving energy	Total energy	mass of battery w/out recharge
Unit	kg	N	hrs	W	Whrs	Whrs	Whrs	kg
Curiosity	899.0	1762.0	4800	50	240000	489.5	240489	2405
Lunakhod	840.0	1646.4	4800	50	240000	457.3	240457	2405
Spirit & Opp.	180.0	352.8	12000	30	360000	98.0	360098	3601
Jade Rabbit	140.0	274.4	4800	30	144000	76.2	144076	1441
Sojourner	11.5	22.5	38400	10	384000	6.3	384006	3840

to accomplish 2km), driving energy (proportional to weight and resistance coefficient is assumed to be 0.5) and total energy. Assuming 100 Whrs/kg battery energy density gives a rough idea of the required battery mass without recharge. The Mars exploration rovers utilize hazard avoidance software for segments of partial autonomy, but this process is fairly slow because the software causes the rover to periodically stop, observe, and understand the terrain into which it has driven before moving on [1]. The driving energy required to travel 2km is several orders of magnitude smaller than non-driving consumptions of robotic functions for all planetary rovers listed. As mentioned above, this is due to the fact that the robotics power required for nominal operations is proportional to total operational time and not distance traveled. Assuming total energy all comes from battery, the battery mass required is tremendous. This explained why if without trickle-charging from solar panel or RTG over a long time period most state-of-art planetary rovers can hardly achieve a long distance. In this context, this research investigates the interplays among average velocity, robotic and propulsive power as they pertain to achievable range of robot traverse.

There has been a body of work in maximizing mobile robots traverse range. For example, battery researchers have predicted that the practical battery energy density will reach 1700 wh/kg [2], which would largely benefit robot range.

Battery-powered electric motor propulsion is almost identical to wheeled mobile robots, except the fact that electrical vehicles focus mainly on driving instead of those nominal operations like robotic functions. Tesla Motors has developed electric vehicle whose range reaches 265 miles (426 km) [3] in EPA 5-cycle test [4]. An range estimator is published online [5], which mainly considers driving speed and climate control. The driving speed directly influences the aerodynamics drag the vehicle is facing [6]. However, speed of autonomous mobile robots, especially when driving off-road, is slow. So the main factor that influences maximum range in electrical vehicle doesn't apply to wheeled-mobile robots. Neither does climate control. Range estimation for electrical vehicle doesn't require much information about road type due to the similar effects on vehicle wheels of different road type. The terrain type for mobile robots, however, varies significantly from paved terrain to gravel, pebble road, terrain with big rocks, or even puddle.

The world robot maximum distance record is set by Cornell University's battery-operated Ranger robot: It walked a 40.5 mile ultra-marathon on a single charge and without human touch in an indoor athletic field [7]. Ranger has a total mass of 9.9kg, which includes the 2.8kg lithium-ion battery. The 25.9V battery carries 493 Whrs energy [8]. The fraction of battery mass over total mass is 0.28. Among the 16 watts total power, only 11.3W goes to motors, while 4.7W is used for on board computing and communication [8]. So robotics energy, in contrast to walking, is 29% of total consumption. The zero ratio of stationary to moving time minimizes the consumption for computing, sensing and communication. However, legged robots only contact with the terrain in a periodical pattern and the swinging of legs helps to conserve energy, both of which don't apply to wheeled robots. So the energy consumption configuration is different.

Besides increasing range directly, in order to improve robot mobility, various research has been done in order to optimize motion planning strategies. Yongguo Mei et. al. [9] presented a new approach to find energy efficient motion plans for mobile robots which find routes and determine velocities. In another research [10], Yongguo Mei et. al. presented an energy-efficient approach to explore an unknown area, which determines the next target for the robot to visit based upon orientation information. However, they only focuses on one single source of all energy consumptions without a complete higher level energetic model. The only conclusion that can be drawn is that a certain energy consumption is minimized while the uncertainty remains about other consumption sources.

In robotics, not much work has been done to analyze the influence exerted by average velocity on energetics and achievable range, to scrutinize the complete actual energy consumption in wheeled mobile robots, especially in propulsion. Almost no research aims to obtain analytical energy model based on existing mobile robots to estimate achievable range.

The model in this paper analyzes energy utilization for wheeled mobile robots. It pointed out the most important energy usage is by ancillary consumptions, especially for those rovers with high sitting to driving ratio. After ruling out this part, the amount of propulsion energy is quantified, with which the traverse range can be estimated. In addition to estimating maximum range achievable by battery-powered wheeled mobile robots on a single charge, the model can also estimate necessary battery mass on board according to specific planned traverse.

III. Ideal Terramechanics Propulsion and Mobility Energetic Model

The ideal terramechanics propulsion is derived from Bekker's Derived Terramechanic Model (BDTM) [11], which is a simple, linear 1 DOF model, assuming that in a perfection cohesionless or frictional soil, soil thrust is a linear function of vehicular weight [12]. So the resistance is approximated by $R = C_{rr}mg$, where C_{rr} is determined by terrain type. The propulsive energy can be approximated by an ideal model that equates propulsion work (product of resistance force R and distance d) to total battery energy. On the other side, achievable range is dependent on specific energy, which is battery capacity divided by weight. In most ideal case, battery mass composes total robot weight. The total energy is $E = \mu m$, where μ is energy density. So the ideally achievable distance of battery-powered wheeled mobile robots on one single charge is

$$d = \frac{u}{C_{rr} g} \tag{1}$$

In later sections we will discuss an effective approach to correct these unrealistic assumptions and extend this ideal model into realistic robotics world.

IV. Mobile Robot Energetic: Robotics vs. Mobility

The energetic of mobile robots could be divided into two categories: mobility and robotics consumption.

The energy required for mobility is dependent on rover drive train, terrain type, traverse distance and rover mass. Given a rover on a certain terrain, energy required for mobility is approximately constant for a certain range, although this may vary slightly due to different steering activity. The mobility consumption doesn't depend on rover velocity or traveling time, since the mobility power increases with velocity, while traveling time decreases accordingly, which keeps the final mobility energy unchanged.

However, the energy for sensing, computing, and communication is expended at all time whether moving or sitting. So energy consumption of robotic functions is mainly determined by total mission time, therefore rover speed, if given a certain range. Some rovers don't move all the time during traverse. They have to stop intermittently for reasons like navigation, planning, teleoperation, data collection, etc. So they have so called "driving duty cycle", which refers to the percentage of time that the rover is actually driving at payload operation in the total traverse time. In the time period when rover stops and doesn't have mobility consumption, the robotic functions still continuously consume energy.

A. Rover Velocity and Driving Duty Cycle

The amount of mobility energy is approximately constant given a certain range d. The robotics energy is the product of power (P) and traverse time t, and the average rover speed is dependent on both actual rover velocity (v) and driving duty cycle (D). That is

$$E_{total} = \frac{C_{rr} \cdot m \cdot g \cdot d}{\eta} + \frac{d}{v \cdot D} \cdot P \tag{2}$$

Assuming a 10-kilogram (m) rover is supposed to achieve a 2 km (d) traverse on earth ($g = 9.81 m/s^2$). The terrain resistance coefficient (C_{rr}) is 0.15. The effective propulsion energy consists only 30% (η) of total mobility energy. The robotics power is assumed to be 30 watts. The effect of rover velocity and driving duty cycle on required battery energy can be seen in Fig. 1a. So with a certain driving duty cycle value, total required energy increases with decreasing rover velocity.

Fig. 2: Influence of Different Rover Parameters on Total Required Battery Energy to Accomplish a 2km Traverse

Especially in the low rover velocity range, the required energy increases more dramatically. Greater driving duty cycle value reduces required battery energy. Another way to express this is the required rover velocity given certain amount of energy under different duty cycles. Fig. 1b shows that with certain total battery energy, required rover velocity decreases with increasing driving duty cycle. Larger battery capacity allows slower rover velocity. It could be concluded that given a rover and terrain, the only way to achieve more range with limited battery energy is driving fast.

B. Achievable Range

Given battery energy capacity, achievable range can be analytically calculated, where mobility energy is no longer constant. The relation between rover velocity and achievable distance is illustrated in Fig. 1c, assuming 100 Whrs battery energy (about 1kg) on board. It proves that greater range requires greater rover velocity. By taking the limit of rover velocity v, the maximum asymptotic range can be derived:

$$d_{asymptotic} = \lim_{v \to \infty} \frac{v \cdot E \cdot D \cdot \eta}{v \cdot C_{rr} \cdot m \cdot g \cdot D + P \cdot \eta} = \frac{E\eta}{C_{rr} m g} \tag{3}$$

C. Sensitivity Chart

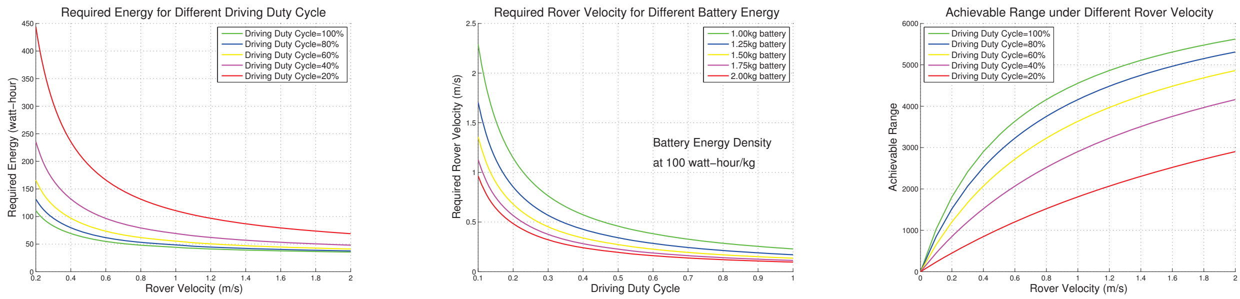
After including the most dominant factors, Fig. 2 shows that rover velocity and driving duty cycle, which together determine average rover velocity, have the most significant influence on energy required to finish 2km traverse. Robotics power is the third important factor. The mass and terrain resistance coefficient, which play a role in mobility energy, don't exert much influence on total energy.

V. Wheeled Mobile Robots Energetic Model

The general energy utilization model for wheeled mobile robots is illustrated in Fig. 3a, where rectangle boxes represent system components, while ellipses show related energy consumption.

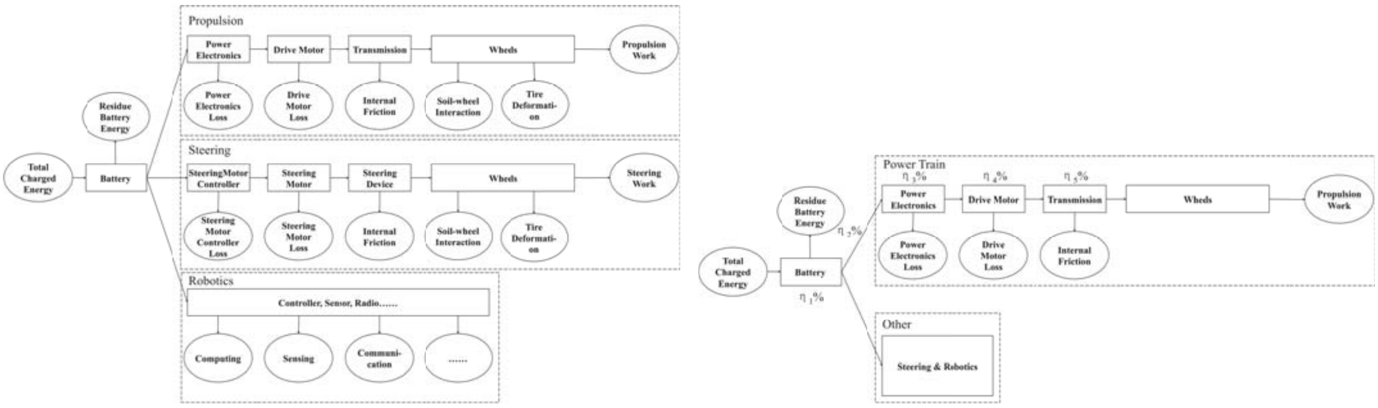
A. Propulsion

A fraction of available battery energy is used by power train, where energy is output in mechanical form. Most of the energy consumption and losses are illustrated in the first branch in Fig. 3a.



(a) Required Battery Energy (b) Required Rover Velocity (c) Achievable Range

Fig. 1: Interplay among battery energy, rover velocity, driving duty cycle, and achievable range for a 10kg rover to achieve a 2km traverse



(a) Rover Energy Utilization Model (b) Simplified Model

Fig. 3: For quantification purpose a simplified model on the right is proposed based on the complete model on the left.

B. Steering

For motor-powered steering systems, the configuration of energy chain is almost the same as that of propulsion. This part of consumption is very difficult to quantify since it depends on a variety of factors, such as terrain type, traverse path, steering activity, and so on. For skid-steer rovers without separate steering system this consumption is zero.

C. Robotics

In all the energy actually extracted from battery, a large percentage is consumed by robotics functions. Unlike propulsion and steering energy which are mainly dependent on distance, robotics consumption is determined by mission time. It is impossible to generalize all consumption types in one model since they vary significantly from rover to rover.

VI. Simplified Model

In order to get rid of the negligible and unquantifiable energy consumptions, a simplified model is proposed (Fig. 3b). In this model, only the residual battery energy, steering and robotics consumption, power electronics loss, motor loss, and internal friction are considered. Five efficiencies for the intermediate steps in the energy chain are defined to indicate the series of bypasses and losses before reaching the propulsion work, which directly determines the maximum traverse distance: Battery Efficiency η_1, Propulsion Branch Efficiency η_2, Power Electronics Efficiency η_3, Motor Efficiency η_4, and Mechanical Efficiency η_5.

A. Comparison with the Ideal Model

Through these five intermediate efficiencies, the energy left is the effective fraction used directly for propulsion. This partially resemble the ideal model in Eqn. 1. Here, the term f is introduced to describe the fraction of battery mass over total rover mass, in sight of the fact that no mobile robot can carry zero non-battery mass on board. In realistic case, the reduced energy used for propulsion through η_1 to η_5 should be $E_p = \prod_{i=1}^{5} \eta_i E = \prod_{i=1}^{5} \eta_i f m u$. In a simplified homogeneous terrain with a constant resistance coefficient C_{rr}, the achievable range should be

$$d = \frac{\prod_{i=1}^{5} \eta_i f u}{C_{rr} g} \tag{4}$$

When compared with Eqn. 1 we have an extra term $\prod_{i=1}^{5} \eta_i f$, which takes into account realistic energy distribution in robot system and battery mass fraction, therefore extends our ideal model into real world.

B. Propulsion Branch Efficiency

The most variable propulsion branch efficiency caused by the great variance in robotics consumptions, as discussed in Section IV, could be calculated by:

$$\eta_2 = \frac{P_{propulsion} \cdot D}{P_{propulsion} \cdot D + P_{robotics}} \tag{5}$$

C. Asymptotic Maximum Traverse Distance

Maximum distance for a certain rover is asymptotic, since increasing total energy with more batteries also increases rovers mass, which has a counter effect on range. Assuming battery number to be n, energy of a single battery is e, rover mass without battery is m_r and single battery mass is m_b, the asymptotic maximum distance is

$$d_{asymp} = \prod_{i=1}^{5} \eta_i \lim_{n \to \infty} \frac{ne}{C_{rr}(m_r + nm_b)g} = \prod_{i=1}^{5} \eta_i \frac{u}{C_{rr}g} \tag{6}$$

Compared with Eqn. 4, Eqn. 6 sets f to 1, which is the asymptotic value of battery mass fraction. This asymptotic maxima will be shortened by rover payload capacity. That is, if too many batteries are placed on rover, it will finally lead to stall of drive motors. This also aligns with Eqn. 3 since assuming infinite number of batteries means ignoring rover mass and thus equaling $\frac{E}{m}$ to energy density u.

D. Cost Of Transport

With the model described above, "Cost of Transport" can be calculated analytically, which helps to estimate approximate maximum traverse range or back solve the required amount of batteries given a targeted range, since the changes of most efficiencies are minuscule within normal payload range (except mission-dependent η_2). Cost of Transport is defined as total energy used per unit weight per unit distance travelled:

$$COT \triangleq \frac{E}{mgd} \Rightarrow d = \frac{fu}{COTg} \Rightarrow COT = \frac{C_{rr}}{\prod_{i=1}^{5} \eta_i} \tag{7}$$

First arrow comes from assuming energy comes from battery and second arrow is achieved by comparing with Eqn. 4.

VII. Model Verification

In order to verify the model, a series of tests were done on a crawler robot: Killer Krawler 2 (KK2), a wheeled robot that uses an extremely light and flexible chassis to surmount extreme obstacles relative to its size. KK2 employs a 2-motor 4-wheel drive configuration. Two independent motors drive the front and rear axles respectively and in the test 4-wheel-drive mode is selected. KK2 has steering systems on both front and rear axles, which further expands its capacity to negotiate with rough terrains. They are, however, deliberately isolated from the power train along with the robotic power, in order to exclude the most variable source of energy consumption. This sets the propulsion branch efficiency η_2 to be 100%. One single element of the primary power source is a 11.1V 5000mAh 3-cell LiPo battery and the ancillary battery pack comprises four 7.6V 2200mAh 2-cell LiPo batteries. The energy distribution schematic is shown in Fig. 4.

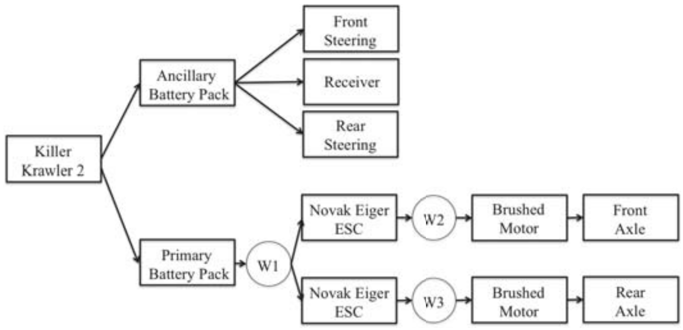


Fig. 4: KK2 Energy Distribution Schematic

TABLE II: Wheels-up Current and Voltage

	Averaged Current (A)	Averaged Voltage (V)
Front	3.0	4.0
Rear	3.1	4.1

The verification mainly focuses on propulsion branch, since after all it is the propulsion branch that directly influences the maximum traverse range. The robotics consumption varies significantly from rover to rover and can be included into the model by Eqn. 5. In order to achieve generality for all wheeled mobile robots with different payload, the verification is done only on the common parts: propulsion.

In order to determine other efficiencies other than η_2, three current voltage monitors are plugged into the rover. They are shown by W in Fig. 4 and help to measure the power in different positions in energy chain.

A. Experiment and Data

In the energy model described in Fig. 3b, the residue battery energy is the difference between energy measured by W1 and primary pack capacity. The power electronics loss is the difference between W1 and the sum of W2 plus W3. One term that can not be measured directly is the internal friction loss. So the "wheels-up test" is designed. In the wheels-up test, the rover was suspended and run in the air to simulate all the situations in real robot traverse, but excluding the necessary energy to overcome the resistance from external terrain. In this context, all the mechanical energy that comes out of the motors are purely consumed by internal friction, which can give us an idea of the energy consumption inside the rover itself after the motor. In the field test, KK2 was driven on a plastic runway of an athletic field.

The data of the 2-hour wheels-up test and 1.25-hour field test are shown in Tab. II and Tab. III.

B. Analysis

1) Internal Friction Power: In the 2-hour wheels-up test, the energy supplied to the motor is calculated by summing

TABLE III: Field Test Current and Voltage

	Averaged Current (A)	Averaged Voltage (V)
Front	3.6	5.7
Rear	3.5	5.7
Total	4.6	9.6

TABLE IV: Efficiencies Calibration

η_1	η_2	η_3	η_4	η_5	η_{total}
99.5%	100%	91.6%	68.9%	44.7%	28.1%

the product of front and rear currents, voltages, and time: $E_{motor} = 1.78 \times 10^5 J$. Assuming all motor loss is heat loss due to resistance enables us to equal the total motor loss to the product of squared current, motor resistance (0.5Ω), and time according to Joule's First Law, which is $6.70 \times 10^4 J$. This makes the efficiency of the two motors in the wheels-up test about 62%. So all leftover energy is transformed into mechanical energy, which is only consumed to overcome the internal friction to rotate wheels in air. So internal friction power is the ratio of the leftover energy divided by wheels-up time: 15.4W.

2) Field Power: The actually extracted energy from the primary battery pack is $1.99 \times 10^5 J$. By the same token, the energy goes into the motor is $1.82 \times 10^5 J$. The motor current increases in field test due to the increasing load caused by terrain interaction. The increasing load in normal range would increase the motor efficiency as well, which is now about 68.9%. The out-coming mechanical energy from the two motors is $1.25 \times 10^5 J$. However, mechanical energy is now divided into two parts, internal friction and propulsion.

3) Propulsion Energy: The energy actually used for propulsion in the field test is the total mechanical energy subtracted by internal friction loss: $5.61 \times 10^4 J$. This is at the bottom of the energy chain and is directly used for propulsion. So at this point it is reasonable to apply the modified ideal propulsion model $d = \frac{E_{propulsion}}{C_{rr} mg}$.

With the data from the wheels-up and field test (not including the measured distance), all the efficiencies η_1 to η_5 can be derived (Tab. IV).

C. Range Estimation and COT Verification

According to Eqn. 7, Cost of Transport is $C = 0.25$ with a certain terrain and certain rover, say KK2 on plastic runway. Several distance tests are done to see if the constant can give a sufficiently accurate estimation of maximum traverse distance. The mass of the rover including the ancillary battery pack is 12.90kg and one primary element is 0.37kg. The resistance coefficient C_{rr} is 0.07. And the energy density of the LiPo battery is $u = \frac{5000mAh \times 11.1V}{0.37kg} = 150Wh/kg$. We changed the battery mass fraction f with multiple number of batteries. 1, 2, 3, and 18 [1] batteries were used in four tests, the results are shown in Tab. V

[1]The 4th test with 18 batteries was done by placing the weight of 17 batteries and only 1 actual battery on the rover. This is to simulate the situation when the rover is driving with 18 batteries on board while avoid to make the actual traverse. The achieved range is multiplied by 18.

TABLE V: Comparison of Actual and Estimated Range

Battery #	Battery Mass Fraction f	Estimated Range (C=0.25)	Actual Range	Error Rate
1	2.8%	6300m	6031m	4.5%
2	5.4%	12150m	12623m	3.7%
3	7.9%	17775m	17029m	4.4%
18	34.0%	75238m	79974m	5.9%

VIII. Conclusion

In conclusion, an effective energetic model is generated and verified in order to extend the usage of an ideal model into real world application. It shows the importance of rover velocity and driving duty cycle in the maximum traverse range problem. One conclusion that can be drawn from the model is that in order to achieve greater range the rover should drive faster and increase driving duty cycle. The analytically calculated Cost of Transport by this model also helps to estimate maximum range of rovers based on battery mass fraction and energy density. The energetic model indicates that in propulsion chain internal friction is the most significant consumption. Only 28% of the total energy in propulsion chain can reach the bottom and be used directly for propulsion on KK2. Without deliberate energy chain isolation, effective propulsion work percentage will be further reduced, especially with a low rover velocity and driving duty cycle value. This explains why the range of battery-powered mobile robots is not satisfactory.

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