Emergency Maneuver Library – Ensuring Safe Navigation in Partially Known Environments

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***Abstract*— Autonomous mobile robots are required to operate in partially known and unstructured environments. It is imperative to guarantee safety of such systems for their successful deployment. Current state of the art does not fully exploit the sensor and dynamic capabilities of a robot. Also, given the non-holonomic systems with non-linear dynamic constraints, it becomes computationally infeasible to find an optimal solution if the full dynamics are to be exploited online. In this paper we present an online algorithm to guarantee the safety of the robot through an emergency maneuver library. The maneuvers in the emergency maneuver library are optimized such that the probability of finding an emergency maneuver that lies in the known obstacle free space is maximized. We prove that the related trajectory set diversity problem is monotonic and submodular which enables one to develop an efficient trajectory set generation algorithm with bounded sub-optimality. We generate an off-line computed trajectory set that exploits the full dynamics of the robot and the known obstacle-free region. We test and validate the algorithm on a full-size autonomous helicopter flying up to speeds of $56 \mathrm{m}/\mathrm{s}$ in partially-known environments. We present results from 4 months of flight testing where the helicopter has been avoiding trees, performing autonomous landing, avoiding mountains while being guaranteed safe.**

I. Introduction

Robotic applications like cargo delivery, surveillance, people transport, reconnaissance etc. require the robots to operate in unstructured, partially known environments at high speeds. The robots should ensure safety while navigating in such environments without compromising on performance. A popular method to guarantee safety relies on limiting the vehicle speed such that it can come to a stop using longitudinal deceleration within the known obstacle-free volume [1]. This method fails to fully exploit either the vehicle's dynamics or known unoccupied volume, leading to unsatisfactory performance. Another method includes planning a trajectory such that its initial part takes the robot towards the goal while it ends in a control invariant set that lies within the known obstacle-free region [2] . Although this method fully exploits the known space it limits the planning horizon. In this work, we examine the problem of ensuring safety for mobile autonomous systems while maintaining the capability to operate the vehicle at its performance limits. The key idea is to ensure that the vehicle is always in a safe state from which it can transition to a loiter pattern or come to a stop within the known obstacle-free space. All these states are inside the control invariant set of the robot [3], which is a well-known approach to ensure feasibility for

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Fig. 1: Autonomous Unmanned Little Bird, coming in for a landing in snowy conditions. This helicopter was used in the autonomous flight experiments.

model predictive control applications [4]. Determining loiter patterns or trajectories resulting in complete stop in various environments is computationally challenging especially when the robot has non-linear dynamics. Additionally, it is required that the safety evaluation has a low worst-case response time so that it can be used for on-line motion planning at high speeds.

In order to ensure the on-line capability of the safety evaluation, the problem is decoupled in an off-line and an on-line part. The off-line part generates an optimized set of trajectories allowing the robot to reach a safe state and stay within the known obstacle free region for an infinite time horizon. The trajectory set is designed to maximize the probability of finding at least one emergency maneuver in the known unoccupied environment. The on-line part determines if the set contains a collision-free trajectory regarding the current state and environment of the robot. Thereby, the off-line generated trajectory set reduces the search space for the on-line part. This safety evaluation approach serves as a computationally tractable algorithm with bounded run time.

It can be shown, that the problem of generating this optimized trajectory set is NP-hard [5]. We present an efficient, bounded sub-optimal approximate solution that finds a trajectory set maximizing the probability of containing at least one safe trajectory given a prior obstacle distribution. The proposed novel safety assessment approach is based on an emergency maneuver library and is compared to other common known approaches such as the stopping distance. The proposed approach was evaluated through a variety of experiments conducted on the Boeing Unmanned Little Bird helicopter, as shown in Fig. 1. In all evaluated scenarios the novel safety assessment approach outperforms known common approaches by allowing for higher velocities of the rotorcraft while guaranteeing safety for the rotorcraft at all times.

Parts of this paper has been presented before with a focus on rotorcraft safety in [6]. In this paper we present the approach to guarantee safety for generic mobile robots, remove the reliance on having a fixed number of obstacles for bounded sub-optimality of the emergency maneuver library generation algorithm and present more insight into an efficient generation of the emergency maneuver library.

Our main contributions are as follows:

- Generation of an emergency maneuver library that allows for on-line safety assessment of robotic systems at high speeds in unknown environments
- Efficient algorithm for the library generation with bounded sub-optimality
- Experimental evaluation from field tests with an autonomous full-sized helicopter flying at speeds of $56\,\mathrm{m/s}$

II. Related Work

Autonomous mobile robots have matured over the years. As these systems are developed for field applications [7]–[10], the need for robust and safe autonomous robots is highlighted. Previous work on safety of autonomous robots can be broadly divided into two paradigms. One of the paradigms is to make sure that the vehicle can stop within the sensor range while applying maximum allowed longitudinal deceleration [7], [11], [12]. The stopping distance based velocity limit does not exploit the complete dynamics of the vehicle, leading to conservative velocity limits.

Another paradigm is to simplify the non-linear dynamics of the UAVs and plan a path that is guaranteed to stay within the known unoccupied region. Mixed integer linear programming is used in [2] to plan paths that stay within the known region. Simplified dynamics in a sampling based graph is used in [13] while limiting the maximum planning time to ensure safety. The assumption is that the planner can always plan an obstacle free path if allowed to run until the maximum planning time. [10] uses Dubins curves to plan paths within the known space. These methods also suffer from not being able to exploit the vehicles full dynamic capabilities.

It is also important to quantify the safety of the mobile autonomous systems. [14] suggested using distance from obstacles as a metric for safety of a robot navigating through an obstacle field. This metric does not take into account the sensory and dynamic limitations of the vehicle, thus it cannot ensure safety of the vehicle.

In the next section, we present a safety metric that considers both sensory and dynamics constraints of the vehicle to evaluate the vehicle safety. We then present an emergency maneuver library based method that utilizes the true dynamics of the vehicle to find a positive control invariant set in the known unoccupied space. We formulate the problem of finding this library as a NP hard path diversity optimization [5], [15]–[17]. We prove the path diversity problem to be monotonic, sub-modular leading to an efficient, bounded sub-optimal algorithm [18], [19] to generate the trajectory set.

III. Defining Safety

The safety of a mobile autonomous system is dependent on its sensory and dynamic capabilities. In a fully-known environment a mobile system is unsafe if it enters a state for which there exists no trajectory that avoids a collision, such a state is called an Inevitable Collision State [20]. In a static partially-known environment the unknown regions may contain obstacles. Therefore, to ensure the safety of the mobile robot its state should be constrained such that it can always transition to a terminal feasible invariant set [2] that allows the robot to stay within the known obstacle-free volume for an infinite time horizon. We now formally define safety for robots operating in uncertain environments. Let, $\mathbf{x}(t)$ be the state of the robot at time t in the state space $\mathcal{X}$ which is in a manifold $\mathcal{X} \subset \mathbb{R}^n$. The workspace of the robot is defined as $\mathcal{W}$ and the occupancy of the robot system in the workspace at a certain state is given as $\mathcal{A}(\mathbf{x}(t)) \subset \mathcal{W}$. The known space of the workspace at a given time t is denoted as $\mathcal{K}_t \subset \mathcal{W}$. The occupancy of the known obstacles at time t is given by $\mathcal{O}_t \subset \mathcal{K}_t \subset \mathcal{W}$. Let $\Phi_{\mathrm{F}}(\mathbf{x})$ be the search space of trajectories for a given state $\mathbf{x}$, that end in a terminal feasible invariant set. Let $\phi(\mathbf{x})$ be such a trajectory and let $\phi(\mathbf{x}, \tau)$ be the state of the vehicle at time τ, along the trajectory $\phi(\mathbf{x})$, which is by definition rooted at state $\mathbf{x}$. Then any trajectory followed by the vehicle can be considered safe if for all states on the trajectory there exists a trajectory $\phi(\mathbf{x})$ which completely lies inside the known obstacle-free space at that time. Equation (1) presents this definition formally:

Definition 1 (Motion Safety):

$$\forall t, \forall \tau, \exists \phi(\mathbf{x}) : \mathcal{A}(\phi(\mathbf{x}, \tau)) \subset (\mathcal{K}_t \setminus \mathcal{O}_t) \tag{1}$$

In the next section, we discuss how this safety definition can be enforced on mobile autonomous vehicles in real-time with the use of an emergency maneuver library.

IV. Approach

Finding a trajectory that satisfies (1) online is non-trivial due to computation costs involved, especially if the robot's dynamics are non-linear. The current methods in use, lead to a very conservative behavior, with robots acting well below their dynamics and sensory capabilities. We provide a method that allows for guaranteeing the safety of the vehicle in static environments, while exploiting the limits of vehicle dynamics and considering the available known obstacle-free space. We split the problem into two parts, first, we efficiently compute a reduced set of terminal invariant trajectories offline then use this reduced set to search for safe terminal invariant trajectories online. In the next section we discuss how to compute the terminal invariant trajectory set resulting in the emergency maneuver library. Before discussing how this emergency maneuver library can be used online to ensure safety of the mobile robot.

A. Emergency Maneuver Library

Instead of solving for dynamically feasible trajectories that end in a terminal invariant set on-line, we approximate the search space by a finite set of such trajectories. The trajectory set is designed such that the probability that at least one of the trajectories stays collision-free, given a prior on the obstacle configuration for a given state is maximized for a static environments. We show that this problem is NP hard [5], [16] and then prove that it is monotonic sub-modular, providing a sub-optimality bound for a greedy algorithm.

B. Problem Definition

We want to find a set of trajectories that are control invariant and maximize the probability that the set contains at least one collision-free trajectory. The trajectory set is optimized for a spatial stochastic process defined in $r \in \mathcal{W}$, that captures the distribution of obstacle configurations that the robot is likely to encounter during its lifetime. We assume this spatial field is give by $\zeta(u, r)$, where u is the event of point r being unoccupied/free. The probability density function defined by this process is then given by $p_\zeta(u, r)$. Probability that there is no obstacle inside a volume $V \subset \mathcal{W}$ is thus given $P_\mathrm{u}(V) = \int_V p_\zeta(u, r)dr$. The volume which is swept by the robot following a certain trajectory ϕ is expressed as $V(\mathcal{A}(\phi)) = \{\mathbf{r} | \mathbf{r} \in \mathcal{W}, \exists \tau \quad \mathbf{r} \in \mathcal{A}(\phi(\tau))\}$, [8]. In the rest of the paper we use the shorter notation $V_\phi = V(\mathcal{A}(\phi))$ to denote the volume swept by the robot. The probability of a path being safe is given by the probability of the swept volume being free of obstacles.

$$P_\mathrm{u}(\phi) \qquad = P_\mathrm{u}(V_\phi)$$

which allows to determine the probability of path ϕ_1 or ϕ_2 being unoccupied

$$P_\mathrm{u}(\phi_1 \cup \phi_2) = P_\mathrm{u}(\phi_1) + P_\mathrm{u}(\phi_2) - P_\mathrm{u}(\phi_1 \cap \phi_2)$$

where,

$$P_\mathrm{u}(\phi_1 \cap \phi_2) = P_\mathrm{u}(V_{\phi_1} \ \cap \ V_{\phi_2}).$$

Using the inclusion-exclusion principle, we get the probability of a path set $\Phi = \{\phi_1, \phi_2, ..., \phi_n\}$ having an obstacle free path as

$$P_\mathrm{u}(\Phi) = \sum_{k=1}^{n} (-1)^{k-1} \Big(\sum_{1 \le i_1 \le ... \le i_k \le n}^{n} P_\mathrm{u}(\phi_{i_1} \cap ... \cap \phi_{i_k}) \Big). \tag{2}$$

In order to maximize the safety of the robot, a finite path set Φ must be determined maximizing the probability that at least one path is collision-free. This is formulated as the path diversity problem:

Problem 1 (Path Survivability): *The desired trajectory set Φ_d maximizes the probability of finding at least one obstacle-free path.*

$$\begin{aligned} &\Phi_\mathrm{d} := \arg\max P_\mathrm{u}(\Phi) \\ &\textit{subject to } \|\Phi_\mathrm{d}\| < N_\Phi \\ &\textit{where, } \Phi_\mathrm{d} \subseteq \Phi \subseteq \Phi_\mathrm{F} \end{aligned} \tag{3}$$

Φ_F is the search space of trajectories. Since the path diversity problem is known to be NP-hard, we present a greedy method to optimize (3). But before that we prove that greedily optimizing equation (3) is bounded sub-optimal. To prove bounded sub-optimality we prove the $P_\mathrm{u}(\Phi)$ is sub-modular and monotonically increasing in the cardinality of Φ.

C. Monotonicity Proof

In the following we show that the probability of at least one path in a path set being collision-free is monotonically increasing by the cardinality of the path set.

Proposition 1 (Monotonicity of Path Sets)**.** *Given a path set Φ_A, a path ϕ_a and a path set $\Phi_B = \{\Phi_A, \{\phi_a\}\}$ the probability that the set Φ_B contains at least one collision-free path is bigger or equal than for the set Φ_A*

$$P_\mathrm{u}(\Phi_B) - P_\mathrm{u}(\Phi_A) \ge 0.$$

Proof. The probability of the path set Φ_b having at least one obstacle-free path is given by

$$P_\mathrm{u}(\Phi_B) \qquad = P_\mathrm{u}(\Phi_A \cup \{\phi_a\}).$$

Using inclusion exclusion principle lead to

$$\begin{aligned} P_\mathrm{u}(\Phi_B) \qquad &= P_\mathrm{u}(\Phi_A) + P_\mathrm{u}(\phi_a) - P_\mathrm{u}(\Phi_A \cap \phi_a) \\ P_\mathrm{u}(\Phi_B) - P_\mathrm{u}(\Phi_A) &= P_\mathrm{u}(\phi_a) - P_\mathrm{u}(\Phi_A \cap \phi_a). \end{aligned} \tag{4}$$

For all ϕ_a, $P_\mathrm{u}(\phi_a) \ge 0$ and $\max[P_\mathrm{u}(\Phi_A \cap \phi_a)] = P_\mathrm{u}(\phi_a)$, which is the case when all the volume covered by path ϕ_a is already covered by Φ_A. This implies that

$$P_\mathrm{u}(\phi_a) - P_\mathrm{u}(\Phi_A \cap \phi_a) \ge 0$$

and inserted in (4) leads to $P_\mathrm{u}(\Phi_B) - P_\mathrm{u}(\Phi_A) \ge 0$ □

Prop. 1 shows that adding more trajectories to a path set, cannot decrease the probability of finding an obstacle-free path in the set. In other words, $P_\mathrm{u}(\Phi)$ is monotonically increasing in the cardinality Φ.

D. Sub-Modularity Proof

In order to show that a greedy algorithm for the path diversity problem (Prop. 1) is bounded sub-optimal, we will show that $P_\mathrm{u}(\Phi)$ is a submodular set function. This means, that the difference in the probability P_u that a single trajectory makes when added to the path set decreases as the size of the path set increases.

Proposition 2. *Let there be a path set $\Phi_\Gamma \subseteq \Phi_\Upsilon \subseteq V$, where $P_\mathrm{u} : 2^V \rightarrow R$. Now, assume a path ϕ_e, such that $\phi_e \subseteq V \backslash \Upsilon$. Define $\Phi_{\Gamma+e} = \{\Phi_\Gamma, \phi_e\}$, $\Phi_{\Upsilon+e} = \{\Phi_\Upsilon, \phi_e\}$. For sub-modularity*

$$\triangle(e|\Upsilon) < \triangle(e|\Gamma)$$

where, $\triangle(\cdot)$ is the discrete derivative.

Proof. The discrete derivative of $\triangle(e|\Gamma)$ is defined as

$$\begin{aligned}\triangle(e|\Gamma) &= P_{\mathrm{u}}(\Phi_{\Gamma+e}) - P_{\mathrm{u}}(\Phi_\Gamma)\\ &= P_{\mathrm{u}}(\Phi_\Gamma \cup \phi_e) - P_{\mathrm{u}}(\Phi_\Gamma)\\ &= P_{\mathrm{u}}(\Phi_\Gamma) + P_{\mathrm{u}}(\phi_e) - P_{\mathrm{u}}(\Phi_\Gamma \cap \phi_e) - P_{\mathrm{u}}(\Phi_\Gamma)\\ &= P_{\mathrm{u}}(\phi_e) - P_{\mathrm{u}}(\Phi_\Gamma \cap \phi_e)\end{aligned}$$

and similarly

$$\triangle(e|\Upsilon) = P_{\mathrm{u}}(\phi_e) - P_{\mathrm{u}}(\Phi_\Upsilon \cap \phi_e).$$

Taking the difference of the discrete derivatives

$$\begin{aligned}&\triangle(e|\Gamma) - \triangle(e|\Upsilon) =\\ &\quad= P_{\mathrm{u}}(\phi_e) - P_{\mathrm{u}}(\Phi_\Gamma \cap \phi_e) - P_{\mathrm{u}}(\phi_e) + P_{\mathrm{u}}(\Phi_\Upsilon \cap \phi_e)\\ &\quad= P_{\mathrm{u}}(\Phi_\Upsilon \cap \phi_e) - P_{\mathrm{u}}(\Phi_\Gamma \cap \phi_e)\\ &\quad= P_{\mathrm{u}}((\Phi_\Gamma \cup \Phi_{\Upsilon/\Gamma}) \cap \phi_e) - P_{\mathrm{u}}(\Phi_\Gamma \cap \phi_e)\\ &\quad= P_{\mathrm{u}}((\Phi_\Gamma \cap \phi_e) \cup (\Phi_{\Upsilon/\Gamma} \cap \phi_e)) - P_{\mathrm{u}}(\Phi_\Gamma \cap \phi_e)\\ &\quad= P_{\mathrm{u}}(\Phi_\Gamma \cap \phi_e) + P_{\mathrm{u}}(\Phi_{\Upsilon/\Gamma} \cap \phi_e)\\ &\qquad - P_{\mathrm{u}}(\Phi_\Gamma \cap \phi_e \cap \Phi_{\Upsilon/\Gamma}) - P_{\mathrm{u}}(\Phi_\Gamma \cap \phi_e)\\ &\quad= P_{\mathrm{u}}(\Phi_{\Upsilon/\Gamma} \cap \phi_e) - P_{\mathrm{u}}(\Phi_\Gamma \cap \phi_e \cap \Phi_{\Upsilon/\Gamma})\end{aligned}$$

Applying Baye's Rule
$P_{\mathrm{u}}(\Phi_\Gamma \cap \phi_e \cap \Phi_{\Upsilon/\Gamma}) = P_{\mathrm{u}}(\Phi_\Gamma|(\phi_e \cap \Phi_{\Upsilon/\Gamma}))P_{\mathrm{u}}(\Phi_{\Upsilon/\Gamma} \cap \phi_e)$
it follows that

$$\triangle(e|\Gamma) - \triangle(e|\Upsilon) = P_{\mathrm{u}}(\Phi_{\Upsilon/\Gamma} \cap \phi_e)(1 - P_{\mathrm{u}}(\Phi_\Gamma|\phi_e \cap \Phi_{\Upsilon/\Gamma})).$$

With $P_{\mathrm{u}}(\Phi_{\Upsilon/\Gamma}\cap\phi_e)(1-P_{\mathrm{u}}(\Phi_\Gamma|\phi_e\cap\Phi_{\Upsilon/\Gamma})) \geq 0$ the equation can be rewritten as

$$\triangle(e|\Gamma) - \triangle(e|\Upsilon) \geq 0.$$

□

E. Greedy Algorithm

Since, $P_{\mathrm{u}}(\Phi)$ is monotonic sub-modular, the path diversity problem (Prop. 1) can be greedily optimized while maintaining a sub-optimality bound of $(1 - 1/e) \approx 63\%$ [19], [21]. We describe the greedy algorithm in Alg. 1. We start with an empty trajectory set and search through Φ_F to find the trajectory that maximizes P_{u}. This trajectory is saved in Φ_G and in the next step, the search for trajectory that maximizes P_{u} is conducted in Φ_F/Φ_G, and added to Φ_G. The process of greedily selecting trajectories from Φ_F/Φ_G and adding them to Φ_G is repeated till the desired number of trajectories N_ϕ have been added. In the next section we explain how to use this greedily generated set to guarantee safety.

Algorithm 1: Greedy Optimization for a Emergency Maneuver Trajectory Set

```
Initialize: Φ_G = ∅

while |Φ_G| < N_φ do
    φ_s = arg max_{φ∈Φ_F/Φ_G} P_u(Φ_G ∪ {φ})
    Φ_G = {Φ_G ∪ {φ_s}}
end
```

F. Safety Algorithm

We ensure the safety of the mobile autonomous system by using the emergency maneuver library to enforce the constraint that the current and next state of the system always lies in the positive invariant set, which does not intersect the obstacles and stays within the known volume. The algorithm to ensure safety is explained in Alg. 2. Let $\sigma : [0, T] \rightarrow \mathcal{X}$ be the nominal trajectory that the vehicle is following to reach the goal. Let, Φ_{S_t} be the emergency maneuver library for state at time t in σ and Δt be the time interval between safety checks.

Algorithm 2: Emergency Maneuver Trajectory Set Application for Reactive Safety

```
Initialize: t = 0
Φ_Previous = Φ_{S_t}

while mission active do
    Φ_New = {∅}
    for ∀φ_c ∈ Φ(s_{t+Δt}) do
        if ∀τ  A(φ_c(τ)) ⊆ (K_t \ O_t) then
            Φ_New = {Φ_New, {φ_c}}
        end
    end
    if Φ_New = {∅} then
        execute φ_e ∈ Φ_Previous
    else
        Φ_Previous = Φ_New
        follow σ
    end
end
```

The algorithm queries the emergency maneuver library at a future state of the system, and ensures it can transition to an emergency maneuver which lies in known obstacle free space. If there are no such maneuvers, one of the emergency maneuvers computed at the previous step (for the current state) are executed. Otherwise the vehicle carries on its nominal trajectory. This algorithm has a maximum response time and is guaranteed to keep the vehicle safe. The algorithm is explained with a working example in Fig. 2.

V. Results

We generated the emergency maneuver library to ensure the safety of the autonomous Boeing Unmanned Little Bird Helicopter, equipped with a large field of view range sensor. The dynamic constraints of the helicopter are given in Tab. I.

Given these constraints we approximate Φ_F, by five hundred trajectories each forming a positive control invariant set. The trajectories for this application end in a hover and can trivially be extended to end in a loiter if desired. Each trajectory slows down the helicopter using the maximum allowed deceleration. The trajectories are generated by sampling the roll rate and z acceleration uniformly. Once the helicopter has made a $180°$ coordinated turn the radius of the turn is fixed and the vertical velocity is forced to be

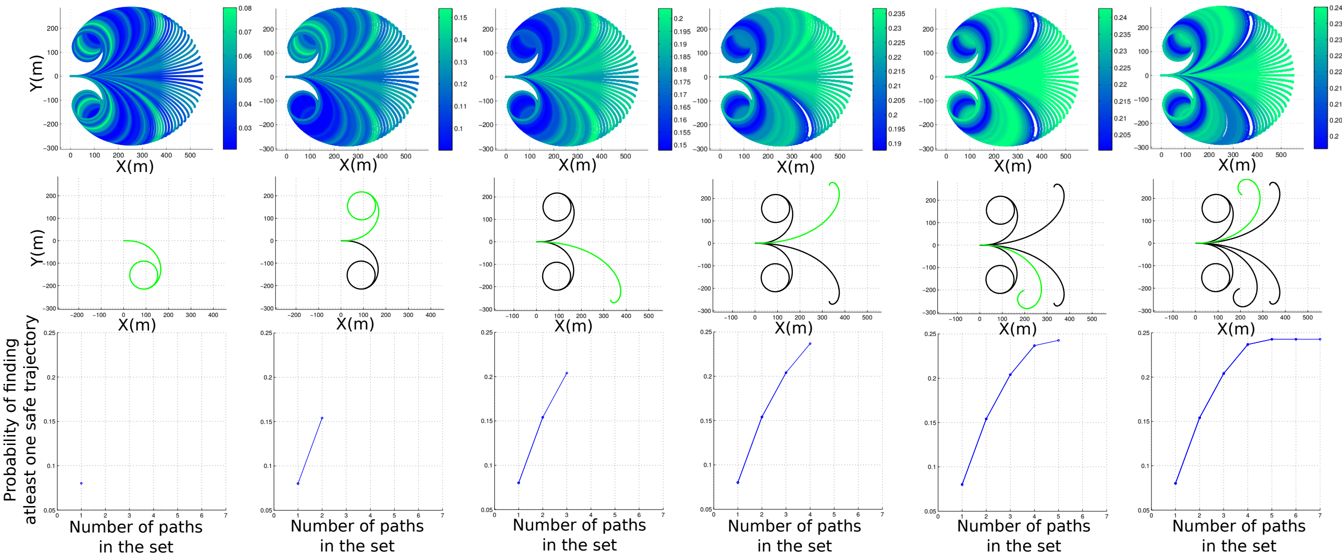

Fig. 3: Generation of emergency maneuver library for one state. From left to right the plots step through the generation of emergency maneuver library for 6 iterations. The top row displays the search space from which the current trajectory is picked, where each trajectory is colored according to the probability of not passing through an obstacle in the set. The middle row shows the greedily selected maneuver in the current step in green and existing maneuvers in the set in black. The bottom row shows the total probability of finding at least one maneuver in the set not passing through an obstacle. The robot starts at 25m/s longitudinal velocity for all the maneuvers and for illustration purposes, is restricted to move in the xy plane.The benefit of adding new maneuvers diminishes as more trajectories are added and almost levels out after 5 trajectories.

TABLE I: Constraints on trajectory

Constraint	**Velocity** $\|v(t)\|$	
	$\geq 20\,\mathrm{m/s}$	$< 20\,\mathrm{m/s}$
Roll [°]	25.00	28.50
Roll rate [°/s]	15.00	–
Heading rate [°/s]	–	28.50
Longitudinal vel. [m/s]	60.00	20.00
Vertical vel. [m/s]	5.00	5.00
Longitudinal accel. [m/s²]	0.75	0.75
Vertical accel. [m/s²]	1.00	1.00

$0\,\mathrm{m/s}$. We use a constant resolution three dimensional grid as our representation and assume uniform probability of occupation of each voxel. The probability of a trajectory set containing at least one unoccupied trajectory is calculated using inclusion-exclusion principle as suggested in [5]. Fig. 3 steps through the emergency maneuver library generation process for the robot motion restricted to a plane starting at $25\mathrm{m/s}$ forward longitudinal velocity. The probability of at least one maneuver in the set surviving reduces with each trajectory being added and almost levels off at about 5 trajectories. Given a trajectory set we can calculate the sensor range required for different velocities. Given an emergency maneuver library, the minimum sensor range required for a certain velocity is calculated as

$$\text{range} = \min_{\phi_c}(\max(\xi(\phi_c))). \tag{5}$$

The function ξ returns a vector of the euclidean distances between starting state $\mathbf{x}$ and all the states in $\phi_c \in \Phi_G(\mathbf{x})$.

The best case sensor range required while using the emergency maneuver library is given by (5). The worst case is the same as the stopping distance. Hence, the emergency maneuver library is guaranteed to provide at least as much performance as using only the stopping distance for the safety evaluation. In Fig. 4 the different requirements on the sensor range for stopping distance and the emergency maneuver library are illustrated. We can quantify the performance of an emergency maneuver trajectory by calculating the maximum safe velocity it allows the helicopter during a mission and the planning time it allows the planner before it becomes imperative for the helicopter to execute the emergency maneuver library. Fig. 5 shows the maximum safe velocity and allowed planning times for a flight test conducted in Quantico, Virginia. The red line shows the path where the helicopter is turning towards the landing zone. The orange part of the path corresponds to the part of the mission for which the sensor on the helicopter focuses on the landing zone for its evaluation. This implies, when the helicopter is moving through the path in orange the sensor stops looking for obstacles and the helicopter comes increasingly close to the known/unknown volume boundary, leading to a drop in maximum safe velocity and allowed planning time. The red part of the path corresponds to turns, it should be noted how the maximum safe velocity according to the stopping distance decreases as the vehicle turns. This happens due to a reduction in effective range of the sensor because of the sparsity of observations in front of the vehicle while turning. The maximum safe speed by the emergency maneuver library

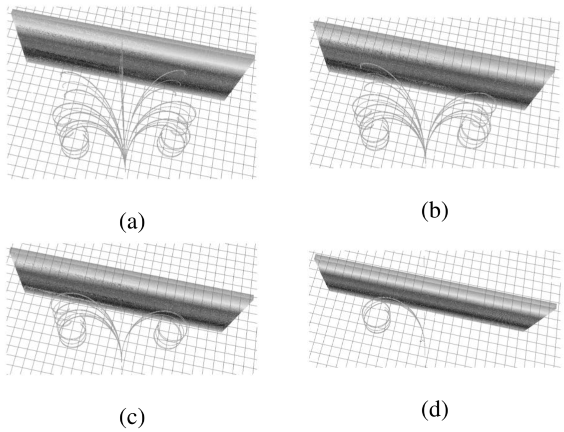

Fig. 2: Data from a flight test conducted on 18th December 2013 in Manassas,Virgina. a) Helicopter approaches a large simulated wall with the emergency trajectory libraries wih no emergency maneuver in contact with the wall. b) As the helicopter gets closer to the wall, the emergency maneuvers intersect the wall and become invalid. Only valid maneuvers are displayed. c) More emergency maneuvers are pruned away as they come in contact with the wall d) An emergency maneuver is executed as the future state is no longer safe.

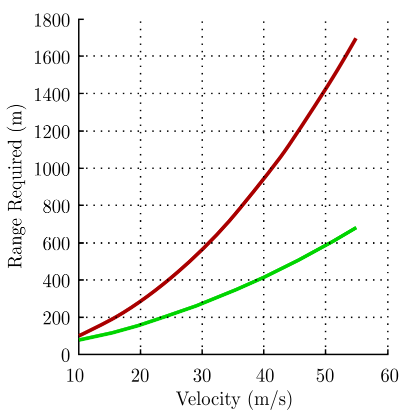

Fig. 4: Changes in Sensor Requirements. The sensor range required for safe operation of the vehicle when using stopping distance for safety is displayed in red, in green is the sensor range required for safe operation of the vehicle when using emergency maneuver library for helicopter safety.

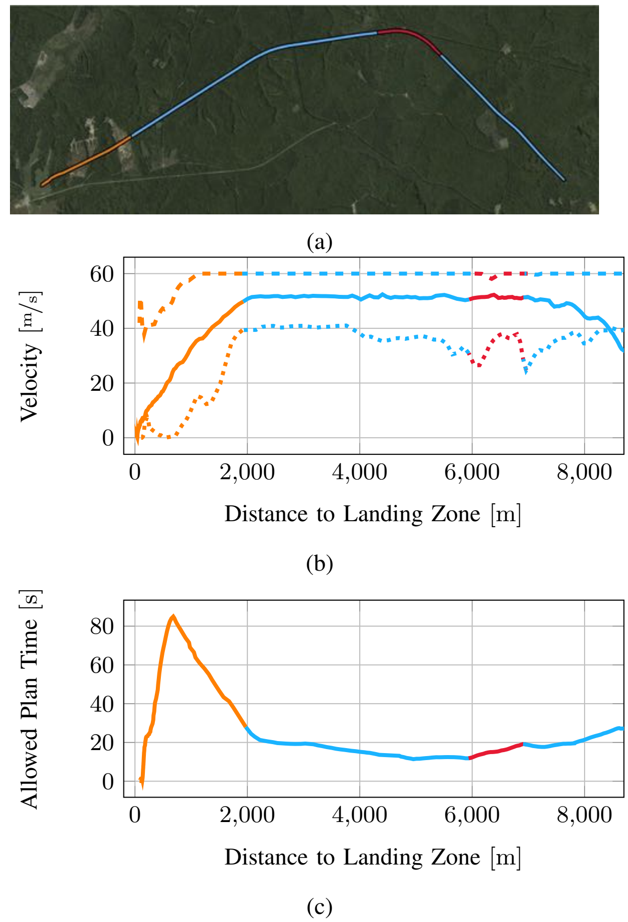

Fig. 5: Safety Quantization: Flight test in Quantico, Virginia. a) Shows an autonomous landing mission conducted in Quantico, Virginia on Unmanned Little Bird. b) Shows the safe velocity of the helicopter with the emergency maneuver library during the flight tests in dashed line, the executed velocity in solid line and the safe velocity if stopping distance is used in dotted line. c) This figure shows planning time available to the planner, before the vehicle will reach the edge of known space and execute one of the emergency maneuvers. The planning time calculated assuming the helicopter will follow the current planned trajectory.

is unaffected, as it efficiently utilizes the known space.

Fig. 7 shows the maximum safe velocity and allowed planning times for seven flight tests conducted in Quantico, Virginia which are shown in Fig. 6. As can be seen in Fig. 7, the maximum safe speed is always greater than the helicopter speed, which means the helicopter is always safe. Furthermore, the stopping distance based safe velocity limit is always considerably below the executed velocity which shows that the emergency maneuver library approach is less conservative than the stopping distance approach. The use of the emergency maneuver library also allows higher available planning times allowing for a better overall performance of the motion planning approach due to longer available

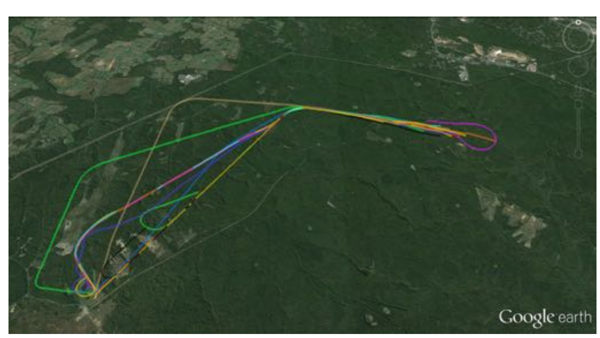

Fig. 6: Paths taken by the autonomous Unmanned Little Bird during landing and wave-off missions conducted in Quantico, on 26 Feb. 2014.

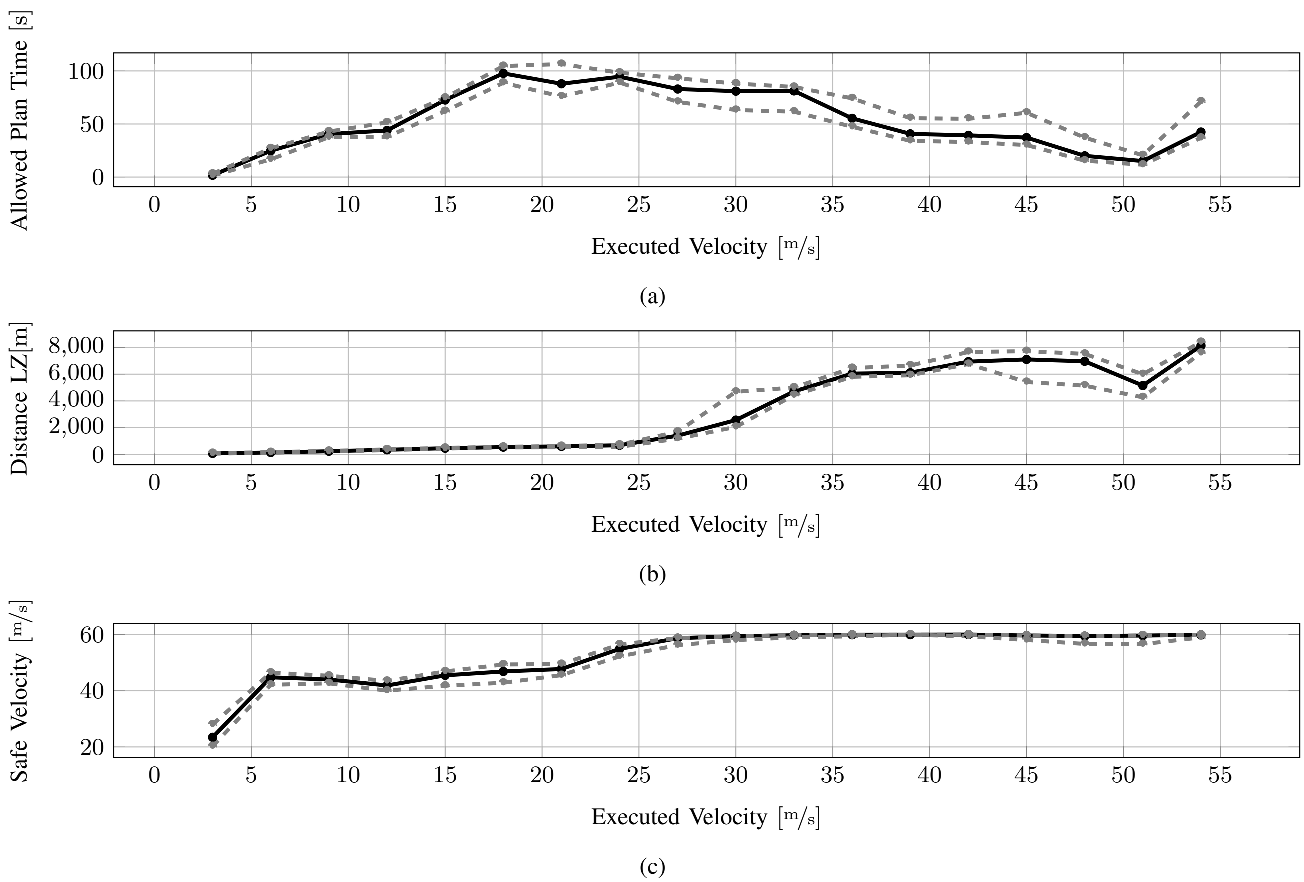

Fig. 7: The figures show the allowed plan time, distance to the landing zone (LZ) and safe velocity relative to the executed velocity of the helicopter. The black line shows the mean and the gray dashed line illustrates the upper and the lower bound of the measurements of all considered flight tests.

computation time.

VI. Conclusions

The main contribution of this paper is the development and evaluation of emergency maneuver libraries to guarantee safety of high speed mobile autonomous systems online in unknown environments. The algorithm determines the maximum velocity for which safety can be ensured, given a future path of the robot. Therefore, it takes into account the constraints of the perception system as well as the dynamics of the rotorcraft. An off-line optimized set of control invariant trajectories (the emergency maneuver library) represents the core part of the presented approach. As a result, the approach is independent of the planner guiding the rotorcraft to the goal and the dynamics of the rotorcraft. Furthermore, the off-line generated trajectories enable real-time evaluation of the safety of the rotorcraft.

The experimental evaluation of the proposed approach shows, that it is always less conservative than the stopping distance based safety evaluation. Thus, the helicopter is able to operates at its performance limits while the presented maneuver based approach still ensures safety at all time.

In the future, we want to further decrease the conservativeness of the safety evaluation by considering a closer coupling between the emergency maneuvers and the motion planner. Therefore, the planner pro-actively responds to potentially unsafe situations rather than just react to them. Another focus of our current research is to consider wind disturbances and obstacle distributions to further improve the robustness of the emergency maneuver library.

VII. Acknowledgement

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