Distributional Distance Classifiers for Goal-Conditioned Reinforcement Learning

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To my grandmother, P. Sita Mahalakshmi.

Abstract

Autonomous systems are increasingly being deployed in stochastic real-world environments. Often, these agents are trying to find the shortest path to a commanded goal. But what does it mean to find the shortest path in stochastic environments, where every strategy has a non-zero probability of failing? At the core of this question is a conflict between two seemingly-natural notions of planning: maximizing the probability of reaching a goal state, and minimizing the expected number of steps to reach that goal state. Prior reinforcement learning (RL) methods based on minimizing the steps to a goal make an implicit assumption: that the goal is always reached, at least within some finite horizon. This assumption is violated in practical settings and can lead to very suboptimal strategies.

In this work, we bridge the gap between these two notions of planning by estimating the probability of reaching the goal at different future timesteps. This is not the same as estimating the distance to the goal – rather, probabilities convey uncertainty in ever reaching the goal at all. We then propose a practical goal-conditioned RL algorithm, Distributional NCE, for estimating these probabilities. Our value function will resemble that used in distributional RL, but will be used to solve (reward-free) goal-reaching tasks rather than (single) reward-maximization tasks. Not only does Distributional NCE outperform state-of-the-art contrastive RL algorithms on standard goal-reaching tasks, but it can also be used to estimate the distribution of dynamical distances to the goal. Taken together, we believe that our results provide a cogent framework for thinking about probabilities and distances in stochastic settings, along with a practical and effective algorithm for goal-conditioned RL.

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Chapter 1

Introduction

1.1 Motivation

The reinforcement learning (RL) community has seen growing excitement in goal-conditioned methods in recent years. These methods promise a way of making RL self-supervised: RL agents can learn meaningful (goal-reaching) behaviors from data or interactions without reward labels. This excitement is reinforced by the fact that goal-conditioned RL also seems to suggest effective ways of learning representations that are directly aligned with the RL objective [14, 31]. However, for a long time, there has been a sticking point in both discussion and algorithmic development of goal-conditioned RL: what is the objective?

Perhaps the most natural objective is to minimize the hitting time, the expected number of steps required to reach a goal. Indeed, this is the basis for much of the classical work in this area (often under the guise of stochastic shortest-path problems [4]), as well as more recent work based on dynamical distance learning [1, 21, 50]. However, these methods implicitly assume that the goal state is always reached; without this assumption, the expected hitting time can be infinite. Nonetheless, RL researchers have proposed a number of methods to optimize this “natural” notion of distance, often with methods that first estimate this distance and then select actions that minimize this distance, methods that often achieve excellent results.

In this work, we attempt to reconcile this tension with the steps-to-goal objective. We first lay out a few subtle issues with this objective. We show that it can lead to suboptimal behavior, both on analytic examples and on continuous-control benchmarks. What, then, is the right way to think about hitting times for goal-conditioned tasks? We advocate for taking a probabilistic approach: estimate the probability of reaching the goal after exactly t steps. We extend prior work that estimates the discounted stationary distribution of future goals via contrastive learning. We do this by learning a classifier that explicitly predicts the probability of reaching the goal at specific timesteps. By estimating the probability at different values of t, we are able to capture the local temporal structure and thereby reason about when the goal will be reached. But, importantly, these probabilities do not assume that the goal will always be reached, i.e., these probabilities remain well-defined in settings with stochastic policies and dynamics. Our analysis shows that, in deterministic environments, these two objectives are closely related.

Based on this analysis, we propose a new algorithm for goal-conditioned RL, which estimates the probability of reaching the goal for varying values of t. Our method can be viewed as a distributional extension to recent work on contrastive RL. Our experiments show that this framing of "distances as probabilities" yields substantially higher performance than simply regressing to distances. Compared to prior contrastive RL methods, our distributional approach achieves higher performance on both low-dimensional and image-based goal-reaching tasks. Finally, based on our analysis, we propose an auxiliary objective based on a self-consistency identity that these probabilities should satisfy. Augmenting our goal-conditioned methods with this auxiliary objective can further boost performance. Taken together, our analysis not only provides a better algorithm for goal-conditioned RL, but also provides a mental model to reason about "distances" in settings with uncertainty.

1.2 Thesis Outline

This thesis starts with a brief primer on related works from the literature in Chapter 2. Next, we describe the goal-conditioned reinforcement learning problem setting in Chapter 3. Having laid down the fundamentals for the study, we begin by outlining a problem with dynamical distance learning approaches in Chapter 4 that can result in very suboptimal behavior. Subsequently, in Chapter 5, we propose our fix: Distributional NCE, a principled goal-conditioned RL algorithm with convergence guarantees. In Chapter 6, we describe (i) the goal-reaching tasks (Sec. 6.1) that were used in our study to test our hypotheses and (ii) the implementation details (Sec. 6.2) for running the proposed Distributional NCE algorithm in practice. Chapter 7 describes the hypotheses that were tested in this study, the results of our experiments, and the corresponding analysis. Lastly, Chapter 8 concludes with a summary of our work.

Chapter 2

Background & Related Work

2.1 Noise Constrastive Estimation

Noise Contrastive Estimation (NCE) [19] is a simple idea to model an unknown probability density function up to proportionality, using samples from this density. It relies on the fact that a good density model needs to be able to distinguish positive samples under the distribution against noise examples. Imagine training a binary classifier to distinguish samples under the true distribution $p_+(x)$ against negative examples drawn from a noise distribution $p_-(x)$ using the cross-entropy loss:

$$\mathbb{E}_{x_+ \sim p_+, x_- \sim p_-} \left[\log C(x_+) + \log(1 - C(x_-)) \right]$$

It can be seen that the Bayes optimal classifier for the above-mentioned training objective is as follows:

$$C^*(x) = \frac{p_+(x)}{p_+(x) + p_-(x)}; \qquad \frac{C^*(x)}{1 - C^*(x)} = \frac{p_+(x)}{p_-(x)}.$$

Thus, the learned classifier approximates the $p_+(x)$ distribution up to a proportionality defined by the noise distribution $p_-(x)$.

2.2 Goal-Conditioned Reinforcement Learning

Goal-conditioned RL (GCRL) is one of the long-standing problems in RL, with roots back to the early days of AI [38]. GCRL can be viewed as a multi-task RL problem, where the objective is to learn a policy that is capable of reaching multiple goals. In the recent decade, researchers have proposed a wide array of successful approaches for goal-conditioned RL, including those based on conditional imitation learning [17, 30, 47], temporal difference learning [11], quasimetric learning [28, 52], contrastive learning [13, 14] and planning [32, 49]. More recent approaches employ a form of hindsight relabeling [2] to improve sample efficiency, or even as a basis for the entire algorithm [14].

2.2.1 Hindsight Experience Replay

Imagine a goal-reaching task with sparse 0-1 rewards, where the agent only receives a reward of 1 when it reaches the commanded goal. With such low supervision, naive reinforcement learning approaches either completely fail or end up requiring a large number of samples to solve the task. However, even when the RL agent fails to reach the commanded goal, it still ends up at a different goal. This information can be used to effectively train an off-policy algorithm by hindsight relabeling a subset of unsuccessful trajectories with reached goals. Hindsight Experience Replay (HER) [2] has emerged as a very effective technique for increasing the supervision in goal-reaching tasks, and can significantly reduce the number of samples required to solve the task. Hindsight relabeling can be combined with any off-policy algorithm [2] and often provides a significant boost in sample efficiency. Moreover, simple supervised learning methods such as goal-conditioned behavior cloning (GCBC) [9, 27] that are based on imitating suboptimal hindsight-relabelled goals can be very successful for some goal-reaching tasks.

2.2.2 Contrastive Goal-Conditioned RL

In most RL problems, the Q-function is trained to minimize the squared Bellman error objective. Interestingly, in sparse 0-1 reward goal-reaching tasks, the Q-function is simply the discounted future state occupancy distribution, i.e., $Q(s, a, g) = p^{\pi(\cdot|\cdot,g)}(s_+ =$

$g|s,a)$. As a result, the Q-function can also be approximated using density estimation techniques. Contrastive NCE [14], a prior contrastive RL approaches uses this insight to train a binary classifier using noise contrastive estimation (NCE) objective (Sec. 2.1, [19]) as follows:

$$\max_C \mathbb{E}_{p(s_t,a_t)} \left[\mathbb{E}_{g\sim p^\pi(s_{t+}|s_t,a_t)}[\log C(s_t,a_t,g)] + \mathbb{E}_{p(g)}[\log(1-C(s_t,a_t,g))]\right].$$

The above-mentioned objective is very straightforward to implement in practice - Pick a random state-action tuple (s_t, a_t) from the buffer, a future state s_{t+} and a random state (not conditioned on s_t, a_t), train a classifier to distinguish future states from random states when conditioned on the current state-action tuple. The Bayes optimal classifier for this is as follows:

$$\frac{C^\pi(s,a,g)}{1-C^\pi(s,a,g)} = \frac{p^\pi(s_{t+}=g \mid s_t,a_t)}{p(g)}$$

Since the transformation $\frac{C(s,a,g)}{1-C(s,a,g)}$ is monotonic in $C(s,a,g)$, i.e.:

$$\operatorname*{argmax}_a \frac{C(s,a,g)}{1-C(s,a,g)} = \operatorname*{argmax}_a C(s,a,g)$$

one can directly maximize $C(s,a,g)$ w.r.t the actions to learn the optimal goal-reaching policy. Note that this derivation holds for any negative goal distribution as long as it does not depend on the action a_t. C-learning [13] extends the Contrastive NCE RL objective into a recursive classification problem akin to temporal difference learning, that can be trained in off-policy and offline RL settings. Our work directly builds on top of these contrastive RL methods. Our key contribution will be to show how such methods can be extended to give finer-grain predictions: predicting the probability of arriving at a goal state at specific future timesteps.

2.2.3 Model-based RL and Planning

Model-based approaches in RL learn a dynamics model of the world, which can then be used for online planning [35, 51] and policy optimization [23]. In goal-conditioned settings, these methods have been used for planning intermediate waypoints to distant goals [36, 37]. Akin to hindsight relabeling, a learned dynamics model can also be used for foresight relabeling [55], i.e., overwriting the goals with future states from imagined rollouts. Further, curiosity-driven approaches [33] alternate between using an ensemble of learned dynamics models to discover unseen goal states and training the policy to achieve these goals. Instead of predicting the single-step future, one can also train a γ-model [24], i.e., a generative model fit to the discounted state occupancy measure. A goal-reaching policy is then obtained by maximizing the likelihood of the goal under the γ-model.

2.3 Distances in Reinforcement Learning

Shortest path planning algorithms are the workhorse behind many successful robotic applications, such as transportation and logistics [6, 15, 25, 40]. Many reinforcement learning methods have built upon these ideas, such as devising methods for estimating the distances between two states [1, 12, 21, 50]. The key point of our analysis is to highlight some subtle but important details in how these distances are learned and what they represent. More precisely, we show that distances can be ill-defined in situations when the commanded goals are actually unreachable under the current policy distribution. As a result, using distances for selecting actions can yield poor performance. The fix we propose is simple, take a probabilistic perspective - estimate the probability for each distance, and reason jointly in this space of probability over distances.

2.4 Distributional Reinforcement Learning

Our proposed method will be reminiscent of distributional approaches to RL [3, 7, 46]: rather than estimating a single scalar value, they estimate a full distribution over possible future returns. In the goal-reaching setting, it is natural to think about this distribution over future values as a distribution over distances [12]. However, as we will show, distances are not well defined in many stochastic settings, yet a probabilistic analogue does make theoretical sense and achieves superior empirical performance. While our proposed method does not employ temporal difference updates, Sec. 5.2 will introduce an auxiliary objective that resembles TD updates. This auxiliary objective boost performance, perhaps in a similar way that the distributional RL loss enjoys stable gradients and smoothness characteristics [48].

Chapter 3

Problem Statement

3.1 MDP Formulation

We consider the reward-free goal-conditioned RL framework, which is defined by a state-space $\mathcal{S}$, action-space $\mathcal{A}$, a transition dynamics function $p(s_{t+1} \mid s_t, a_t)$, an initial state distribution ρ_0 and a goal distribution $p(g)$. The goal-conditioned RL objective is to simply find the goal-conditioned policy $\pi(a|s, g)$ that maximizes the following objective:

$$\pi^* = \underset{\pi}{\operatorname{argmax}} \ \mathbb{E}_{s_0 \sim \rho_0, p(g)} \left[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t, g) \right], \tag{3.1}$$
$$\text{where } a_t \sim \pi(.|s_t, g), s_{t+1} \sim p(. \mid s_t, a_t)$$

In the reward-free RL framework, the reward function is implicitly defined by the transition dynamics and a discount factor $\gamma \in [0, 1) : r_g(s_t, a_t) = (1 - \gamma)p(s_{t+1} = g \mid s_t, a_t)$. For this reward function, the corresponding action-value function of a goal-conditioned policy $\pi_g(a|s) = \pi(a \mid s, g)$ takes the form of the discounted future density $p^{\pi_g}(s_+ = g \mid s, a)$ over the goal states:

$$Q^{\pi_g}(s_t, a_t) = (1 - \gamma)\mathbb{E}_\pi \left[\sum_{\Delta=0}^{\infty} \gamma^\Delta p(s_{t+\Delta+1} = g \mid s_{t+\Delta}, a_{t+\Delta}) \right] = p^{\pi_g}(s_+ = g \mid s_t, a_t). \tag{3.2}$$

By using this Q-function to score actions, the policy directly maximizes the chance of reaching the goal in the future. To estimate the Q-function, we will use contrastive RL [13, 14] (Sec. 2.1), which trains a binary classifier with cross-entropy objective to represent this Q function:

$$\underset{C}{\operatorname{argmin}}\, \mathbb{E}_{g \sim p^{\pi}(g|s,a)}[\log C(s,a,g)] + \mathbb{E}_{g \sim p(g)}[\log(1 - C(s,a,g))]$$

The resulting Bayes' optimal classifier C^{π} for a policy π is proportional to its Q function:

$$\frac{C^{\pi}(s,a,g)}{1 - C^{\pi}(s,a,g)} = \frac{p^{\pi_g}(s_+ = g \mid s,a)}{p(g)}$$

Since the noise distribution $p(g)$ is independent of the actions, we can then optimize a policy with respect to the classifier by $\operatorname{argmax}_a C^{\pi}(s,a,g)$.

$$\pi(s,g) = \underset{a}{\operatorname{argmax}}\, C^{\pi}(s,a,g)$$

Chapter 4

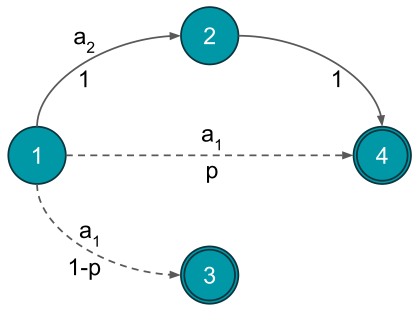
The Perils of Monte Carlo Distance Functions

A common strategy in prior work is to predict the number of steps that elapse between one observation and another [45, 49]. This estimate is then used as a distance function, either for greedy action selection [45], planning [49], or reward shaping [21]. We will call this approach "Monte Carlo distance regression."

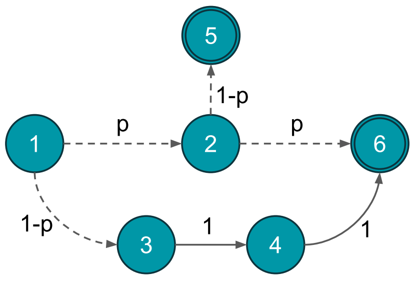
Intuitively, it seems like such an approach is performing reinforcement learning with the reward function that is -1 at every step until the goal is reached. Prior work thus interprets the distances as a Q function. However, it turns out that this distance function is not a Q function. In this section, we show that these distance functions do not (in general) correspond to a Q function, and their predictions can be misleading.

4.1 Toy example illustrating pathological behavior

Consider the toy MDP example in Fig. 4.1(a) with a goal state **4** and an absorbing state **3**. From state **1**, the agent can choose an action a_1 to directly reach the goal state **4** in a single step with a probability of p, but risks getting trapped in state **3** with $1-p$ odds. On the other hand, the agent can choose an action a_2 to deterministically reach the goal **4** in 2 steps. The agent receives a reward of -1 at every timestep it is not at the goal.



(a)



(b)

Figure 4.1: Toy MDPs to illustrate the pathological behaviors exhibited by MC distance regression. Solid lines and dashed lines denote deterministic and stochastic state transitions, respectively.

Proposition 1. *Relative to the reward-maximizing policy, MC regression can incur regret that is arbitrarily large.*

Assuming a discount factor γ, we can compute the optimal Q function analytically: $Q(\mathbf{1}, a_1, \mathbf{4}) = -\frac{(1-\gamma p)}{(1-\gamma)}$ and $Q(\mathbf{1}, a_2, \mathbf{4}) = -(1+\gamma)$. This suggests that for transition probability $p < \gamma$, choosing the action a_1 is suboptimal with a linear regret of $Q(\mathbf{1}, a^* = a_2, \mathbf{4}) - Q(\mathbf{1}, a_1, \mathbf{4}) = \frac{\gamma(\gamma-p)}{(1-\gamma)}$. In the limit $\gamma \to 1$, this regret becomes unboundedly large for any $p \in [0, 1)$, suggesting that even a slight risk of getting indefinitely stuck in a trap state is much more costly than the reward of reaching the goal a few steps early.

Now, imagine if we regressed a distance function $d(s_1, s_2, a)$ to the Monte-Carlo rollouts from state 1. The distance function suggests that the $d(1, 4, a_1) = 1$, since all the trajectories that start from **1** and end up in **4** after taking an action of a_1 are of unit length. Similarly, $d(1, 4, a_2) = 2$. Notice that the optimal MC distances do not depend on the transition probability p, suggesting that MC distances offer an optimistic distance estimate by ignoring the stochasticity in dynamics. Acting greedily with an MC distance function results in a policy that takes the shortest path on the graph by treating stochastic edges as being deterministic, which can be very suboptimal in stochastic settings. For instance, Fig. 4.2 shows that if the transition probability $p = 0.1$ for $\mathbf{1} \to \mathbf{4}$, MC distance suggests the suboptimal action a_1 which incurs a significantly higher regret that the optimal action a_2, as suggested by the optimal Q-function. This demonstrates a fundamental disconnect between

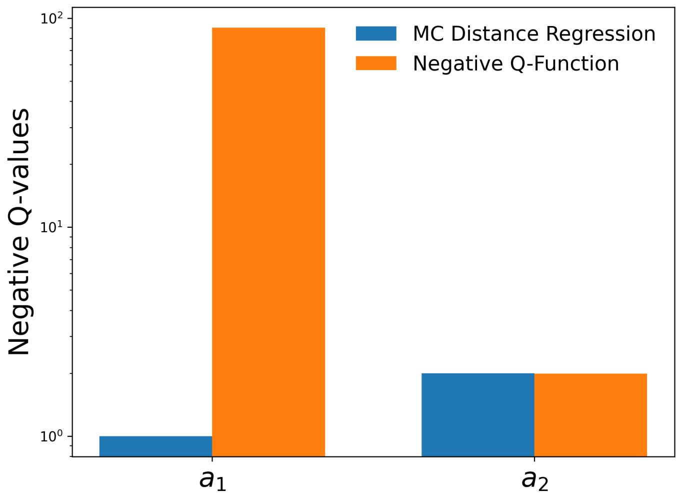


Figure 4.2: MC distances and optimal negative Q-values at disagreement for $\mathbf{1} \rightarrow \mathbf{4}$ on the toy MDP in Fig. 4.1(a) with $\gamma = 0.99$ and $p = 0.1$. The y-axis has a logarithmic scale.

shortest-path solutions and reasoning about the likelihood of reaching a goal state in the future.

Proposition 2. *Monte-Carlo distance functions are generally not a valid distance under any metric (or quasimetric).*

To show this, consider the MDP in Fig. 4.1b, where the agent has no control over state transitions through actions. The MC distance function $d(s, g)$ answers the following question: *if the agent traveled from s to g, how many steps would elapse (on average)?*. For example, $d(3, 4) = 1$ because this state 4 always occurs one step after state 3. But, perhaps strangely, $d(1, 2) = 1$: even though it may be unlikely that state 2 occurs after state 1, *if* state 2 occurs, it would occur after a single step. Similarly, $d(2, 6) = 1$. However, these distances violate the triangle inequality. Even though $d(1, 2) + d(2, 6) = 2$, the estimated distance directly from 1 to 6 is $d(\mathbf{1}, \mathbf{6}) = \frac{2p^2+3(1-p)}{p^2+(1-p)}$, which is greater than 2 for all $p \in [0, 1)$. Thus, these MC distances are not a valid distance metric (nor quasimetric).

Proposition 3. *Monte-Carlo distance functions do not obey the quasimetric property and hence do not represent the optimal goal-conditioned value function for any reward function.*

An optimal goal-conditioned value function is a quasimetric: it always has to obey the triangle inequality [52]. This is enforced because of the optimality and Markov property in MDPs: the optimal reward going from s_1 to s_3 should be atleast as high as the sum of optimal sub-paths from s_1 to an intermediate state s_2, followed by s_2 to s_3, i.e., $V^\star(s_1, s_2) + V^\star(s_2, s_3) \leq V^\star(s_1, s_3) \; \forall \; s_1, s_2, s_3$. Proposition 3 directly follows from Proposition 2, since MC distance functions violate the triangle inequality.

What is the cause for MC distance functions to exhibit these pathological behaviors? In the examples from Fig. 4.1, the MC distance estimates do not account for the transitions that could result in getting stuck in a trap state (**3** and **5** in Fig. 4.1(a) and (b) respectively). More generally, the pathological behaviors of MC distances can be attributed to their optimism bias, wherein they are computed assuming the agent will inevitably reach the goal without considering the associated risks.

However, these pathologies are not exhibited when using the Q-function corresponding to the sparse reward function $r_g(s_t, a_t) = (1-\gamma)p(s_{t+1} = g \mid s_t, a_t)$ (Eq. 3.2). For instance, consider the toy MDP in Fig. 4.1(a), where the $Q(s = 1, a_1, g = 4) = p^\pi(s_+ = 4|s = 1, a_1) = (1-\gamma)p$ and $Q(s = 1, a_2, g = 4) = p^\pi(s_+ = 4|s = 1, a_2) = (1-\gamma)\gamma$. As a result, this Q-function suggests picking the optimal action a_2 when $p < \gamma$, as $Q(s = 1, a_1, g = 4) < Q(s = 1, a_2, g = 4)$.

4.2 Distance Regression is secretly learning a Normalized Classifier

Monte-Carlo distance regression is equivalent in the limit to learning a *normalized* distance classifier over the observed horizon, followed by using the bin probabilities to obtain the mean distance. More precisely, let $H \in \{0, 1, \cdots B-1\}$ be a random variable denoting how far ahead to look to sample the future states (a.k.a goals). The distance classifier represented by $C(s, a, g) \in \mathcal{P}^B$ can then be learned using a categorical cross-entropy loss:

$$\mathbb{E}_{p(H), s_t, a_t \sim p(s,a), g \sim p^\pi(s_{t+H}|s_t,a_t)} \left[\log C(s_t, a_t, g)[H]\right].$$

Obtaining distances from this classifier is straightforward:

$$d(s, a, g) = \sum_H H \; C(s, a, g)[H].$$

Using Bayes' Rule, we can express the Bayes optimal classifier as

$$C^\pi(s_t, a_t, g)[H] = P^{\pi_g}(H \mid s_t, a_t, g) = \frac{p^{\pi_g}(s_{t+H} = g \mid s_t, a_t)p(H)}{p^{\pi_g}(s_+ = g \mid s, a)}.$$

This expression reveals a subtle nuance with distance regression. The distance classifier predicts normalized probabilities, which implicitly assume that the goal can be reached within a finite horizon. Consider this example: say that action a_1 has $p^{\pi_g}(g \mid s, a_1, H = 1, 2, 3, ...) = [0.01, 0, 0, \cdots]$ while a_2 has $p^{\pi_g}(g \mid s, a_2, H = 1, 2, 3, ...) = [1, 1, 1, \cdots]$. Then, distance classifier prefers action a_1 over a_2 since $d(s, a_1, g) = 1$ and $d(s, a_2, g) > 1$, despite it succeeding in reaching the goal with $100\times$ lower probability.

4.3 Connection between Maximizing Likelihood and Stochastic Shortest Path Methods

The overall objective of contrastive RL methods [13, 14] is to maximize the likelihood of the goal state under the discounted state occupancy measure; it is about maximizing a probability (density). This stands in contrast to prior work on stochastic shortest path problems, where the aim is to minimize the expected distance to the goal. Our motivation for using probabilities, rather than distances, is that distances can be ill-defined in settings where there is some probability of never reaching the goal. However, in settings where the goal is reached with probability one, we can directly relate these two objectives; this section explains this connection.

Let's consider a policy π for reaching a goal and define Δ as the number of timesteps required to reach the goal from the start state. Note that Δ is a discrete random variable that takes integer values. Formally, we define $\pi(\Delta)$ as the distribution over the number of timesteps required to reach the goal under the policy. Then, a policy that tries to reach the goal as soon as possible is trying to optimize the following objective:

$$\max_{\pi} -\mathbb{E}_{\Delta\sim\pi}[\Delta] \tag{4.1}$$

Alternatively, consider an MDP where the episode does not terminate upon reaching the goal. In this setting, the reward-free goal-conditioned RL agent is incentivized to maximize its time at the goal:

$$\begin{aligned}
\max_{\pi}(1-\gamma)\mathbb{E}_{\pi}\left[\sum_{t=0}^{\infty}\gamma^t r(s_t,a_t,g)\right] &= \max_{\pi}(1-\gamma)\mathbb{E}_{\pi}\left[\sum_{t=0}^{\infty}\gamma^t \delta(s_t==g)\right] \\
&= \max_{\pi}(1-\gamma)\mathbb{E}_{\Delta\sim\pi}\left[\sum_{t=0}^{\infty}\gamma^{t+\Delta}\right] \\
&= \max_{\pi}(1-\gamma)\mathbb{E}_{\Delta\sim\pi}\left[\frac{\gamma^{\Delta}}{1-\gamma}\right] \\
&= \max_{\pi}\mathbb{E}_{\Delta\sim\pi}\left[\gamma^{\Delta}\right].
\end{aligned}$$

By applying a log transformation on both sides of the equation, followed by Jensen's inequality, we get:

$$\begin{aligned}
\max_{\pi}\ \log\left((1-\gamma)\mathbb{E}_{\pi}\left[\sum_{t=0}^{\infty}\gamma^t r(s_t,a_t,g)\right]\right) &= \max_{\pi}\ \log\left(\mathbb{E}_{\Delta\sim\pi}\left[\gamma^{\Delta}\right]\right) \\
&\geq \max_{\pi}\ \mathbb{E}_{\Delta\sim\pi}\left[\log\left(\gamma^{\Delta}\right)\right] \\
&= \max_{\pi}\ -\mathbb{E}_{\Delta\sim\pi}\left[\Delta\right]\log\left(\frac{1}{\gamma}\right).
\end{aligned} \tag{4.2}$$

The final RHS expression can be interpreted as minimizing the expected time to the goal under the policy (Eq. 4.1), which corresponds to the shortest-path planning objective. Thus, optimizing the shortest-path planning objective is a lower bound of the max likelihood objective. If the policy always takes the same number of steps to reach the goal, i.e. $\pi(\Delta)$ is a Dirac distribution, then the lower bound becomes an equality and maximizing the probability of reaching the goal (LHS) is equivalent to minimizing the expected steps to reach the goal (RHS). One setting where this always happens is deterministic MDPs with deterministic policies.

If both maximizing likelihood and shortest-path planning seem closely related in theory, why do shortest-path methods suffer from pathological behaviors? The answer lies in the logarithmic transformation that gets applied to the likelihood. In simple words, the likelihood of success while failing to reach the goal is 0, which is a well-defined number, whereas the corresponding expected distance to the goal is unboundedly large (the negative logarithm of 0). More formally, the problem with optimizing the shortest-path objective in RHS is that it remains unclear how to correctly train a distance function in stochastic settings, when every strategy has a non-zero chance of failing to reach the goal. For instance, training a distance function via MC regression [21, 49] provides optimistic distance estimates because the training goals are always reached within some finite horizon, which can result in very sub-optimal behaviors as shown in Sec. 4.1. A naive approach to fixing this optimism bias is to train the distance function on unreachable goals as well. However, this poses two practical problems:

1. *Sampling from the distribution of unreachable goals under a policy is non-trivial*: One can sample from the set of easily reachable goals (positive examples) under the policy by simply rolling it out in the environment for a short duration. However, sampling far-away goals (hard to reach under the current policy) requires one to run long policy rollouts, making such far-away goals sparser than easily reachable goals in the collected dataset. Extending this idea to the limit, one can simply never know if a state is unreachable from the policy even after a large number of steps through Monte-Carlo policy rollouts alone. But even if one could sample these negative goals,
2. *optimal distance functions become ill-defined when the regression targets are unboundedly large*: An unreachable state has an unboundedly large target distance (infinity). This makes it numerically unstable to perform direct MC regression since a part of the dataset involves regressing to infinite target distances. Alternatively, one can learn a *normalized* distance classifier (Sec. 4.2) with a catch-all bin to handle hard-to-reach and unreachable goals. However, converting such a distance classifier into an MC distance function by computing the expected distance $d(s, a, g) = \sum_H H\ C(s, a, g)[H]$ is again ill-defined since the upper-bound of the catch-all bin is unboundedly large (infinity).

Prior works in contrastive RL [13, 14] are closely related to the former idea of sampling negative goal examples with subtle modifications: (1) instead of sampling from the distribution of unreachable goal states, we simply sample from a noise distribution, and (2) replace regression objective with the NCE classification objective [18] to differentiate between samples drawn from the positive and negative goal distributions. However, these methods directly estimate the likelihood of reaching the goal without providing any information about the dynamical distance, i.e., the expected timesteps to reach the goal. Our work proposes a distributional variant of contrastive NCE algorithm [14], which can: (1) estimate the likelihood of reaching the goal, and (2) reason about the dynamical distance via normalization using Bayes rule (Eq. 5.4).

Chapter 5

The Fix: Estimate Probabilities, not Distances

In this section, we propose a method that directly estimates the probabilities of reaching goals at different horizons. We describe our method and provide analysis in Sec. 5.1. As we will show in our experiments, this method can already achieve excellent results in its own right. Sec. 5.2 proposes a regularization term based on an identity that our probabilities should satisfy. Our experiments will demonstrate that adding this regularization term can further boost performance.

5.1 Our Method: Distributional NCE

The underlying issue with distance classifiers (discussed in Sec. 4.2) is that they are normalized across the horizon; they have a softmax activation. Replacing that softmax activation with a sigmoid activation resolves this issue and opens the door to new algorithms that resemble distributional RL.

The connection with distributional RL is interesting because it motivates distributional RL in a different way than before. Usually, distributional RL is motivated as capturing aleatoric uncertainty, providing information that can disambiguate between a strategy that always gets +50 returns and a strategy that gets +100 returns 50% of the time. Here, we instead show that distributional RL emerges as a computationally efficient way of learning distances, not because it gives us any particular notion of uncertainty. This is also interesting in light of prior work that distributional RL does not necessarily produce more accurate value estimates [3].

We start by introducing an MC method to learn a distance classifier $C(s, a, g) \in [0, 1]^B$; note that each element of this vector is a probability, but they need not sum up 1. This distance classifier can be learned via *binary* classification:

$$\max_C \mathbb{E}_{p(H)p(s_t,a_t)} \left[\mathbb{E}_{g\sim p^\pi(s_{t+H}|s_t,a_t)}[\log C(s_t, a_t, g)[H]] + \mathbb{E}_{p(g)}[\log(1 - C(s_t, a_t, g)[H])] \right]. \tag{5.1}$$

The Bayes' optimal classifier satisfies

$$\frac{C^\pi(s_t, a_t, g)[H]}{1 - C^\pi(s_t, a_t, g)[H]} = \frac{p^{\pi_g}(s_{t+H} = g \mid s_t, a_t)}{p(g)}. \tag{5.2}$$

On the RHS, note that actions only appear in the numerator. This means that selecting the actions using the LHS is equivalent to selecting the actions that maximize the probability of getting to the goal in exactly H steps. While this notion of success is non-Markovian, this same classifier can be used to maximize the (Markovian) RL objective with $r(s, a, g) = \mathbb{1}(s = g)$ using the following:

$$\sum_{\Delta=1}^{\infty} \gamma^{\Delta-1} \frac{C^\pi(s_t, a_t, g)[\Delta]}{1 - C^\pi(s_t, a_t, g)[\Delta]} = \sum_{\Delta=1}^{\infty} \gamma^{\Delta-1} \frac{p^{\pi_g}(s_{t+\Delta} = g \mid s_t, a_t)}{p(g)} = \frac{p^{\pi_g}(s_+ = g \mid s_t, a_t)}{(1-\gamma)p(g)}. \tag{5.3}$$

The expression on the RHS is the same as the objective in Contrastive NCE [14], which corresponds to maximizing the likelihood of the goal state under the discounted state occupancy measure.

In practice, we use the last bin of the distributional NCE classifier as a catch-all bin. This modification avoids ill-defined Q-values due to a finite number of bins, by accounting for the future states from the trajectory that are at least h steps away, where h is the number of classifier bins in the distributional NCE algorithm. We discuss more details about using the catch-all bin in Sec 5.3.1, but on a high-level, implementing the distributional NCE fix is easy: (1) change the final activation of the distance classifier from a softmax to a sigmoid; (2) change the loss for the distance classifier from a categorical cross-entropy to an (elementwise) binary cross entropy.

Algorithm 1 DISTRIBUTIONAL NCE: `h` is the number of bins in the classifier output, which may be less than the task horizon. Comments denote the shapes of tensors.

```
def critic_loss(states, actions, future_states, dt):
  logits = classifier(states, actions, future_states)  # (batch_size, batch_size, h)
  probs = sigmoid(logits)
  labels = one_hot(dt, num_classes=h)
  loss = BinaryCrossEntropy(logits, labels)
  return loss.mean()

def actor_loss(states, goals):
  actions = policy.sample(states, goal=goals)  # (batch_size, action_dim)
  logits = classifier(states, actions, goals)  # (batch_size, batch_size, h)
  prob_ratio = exp(logits) # p(g|s,a,h) / p(g) = C(s,a,g)[h] / (1 - C(s,a,g)[h])
  Q = sum(discount ** range(h) * prob_ratio, axis=-1)  # (batch_size, batch_size)
  return -1.0 * Q.mean()
```

One way to look at our method is that we are learning a distributional critic to represent the likelihood of reaching the goal at each future timestep, as opposed to learning a single scalar unnormalized density over future goals [13, 43]. Adding this temporal dimension to the contrastive RL algorithm enables the critic network to break down a complex future density distribution into hopefully simpler per-timestep probabilities. This framework also allows one to *(i)* enforce structural consistency for probabilities across timesteps (closely related to n-step Bellman backup), *(ii)* make the critic more interpretable, and *(iii)* reason over future probabilities as distances.

5.1.1 Getting Distances from Distributional NCE

The Bayes optimal MC distance classifier can be obtained from normalizing the Bayes optimal distributional NCE classifier across the horizon:

$$P^{\pi_g}(H = h \mid s_t, a_t, g) = \frac{p^{\pi_g}(s_{t+h} = g \mid s_t, a_t)P(h)}{p^{\pi_g}(s_+ = g \mid s_t, a_t)} = \frac{w^\pi(s_t, a_t, g)[h]P(h)}{\sum_{h'} w^\pi(s_t, a_t, g)[h']P(h')}, \tag{5.4}$$

$$\text{where } w^\pi(s_t, a_t, g)[h] = \frac{C^\pi(s_t, a_t, g)[h]}{1 - C^\pi(s_t, a_t, g)[h]}.$$

5.1.2 Convergence Proof

The Q-function we obtain from aggregating the bins of the distributional NCE classifier with geometric weights (Eq. 5.3) is the same as the contrastive NCE method [14]. To prove convergence of the Distributional NCE algorithm, we make the same assumptions as the Contrastive NCE [14] work:

1. *Bayes-optimality of the Critic*: We assume that the distributional critic is Bayes-optimal for the current policy.
2. *Training Data Filtering*: We only consider (s_t, a_t, s_{t+h}) tuples for the policy improvement step, whose probability of the trajectory $\tau_{t:t+h} = (s_t, a_t, s_{t+1}, a_{t+1}, ..., s_{t+h})$ when sampled from $\pi(.|., s_g)$ under the commanded goal s_g is close to the probability of the same trajectory when sampled from $\pi(.|., s_{t+h})$, under the relabelled goal s_{t+h}.

Proposition 4. *When the above-mentioned assumptions hold, the Distributional NCE update corresponds to approximate policy improvement in tabular settings.*

Proof. We first point out that the Bayes optimal Distribuitional NCE critic can be used to obtain the Bayes optimal Contrastive NCE [14] critic, by geometrically averaging the classifier bins according to Eq. 5.10. Using this result, Proposition 4 is validated by the proof for the Contrastive NCE update corresponding to approximate policy improvement in tabular settings (Sec 4.5 and Appendix B in [14]). □

5.2 Self-Supervised Temporal Consistency Objective

The problem of learning goal-directed behavior exhibits a certain structure: if you can predict the probability of reaching a goal in 10 days starting today, then you can predict the probability of reaching that same goal in 9 days starting tomorrow. This idea highlights a simple identity that the distributional probabilities must satisfy to remain temporally consistent:

$$p^{\pi_g}(g \mid s_t, a_t, H) = \mathbb{E}_{(s_{t+1}, a_{t+1}) \sim (s_t, a_t)} \left[p^{\pi_g}(g \mid s_{t+1}, a_{t+1}, H-1) \right].$$

We use this result to derive a temporal consistency identity for distributional NCE, which is satisfied by the Bayes' optimal classifier (the solution to Eq. 5.2):

$$\frac{C^\pi(s_t, a_t, g)[H]}{1 - C^\pi(s_t, a_t, g)[H]} = \mathbb{E}_{(s_{t+1}, a_{t+1}) \sim (s_t, a_t)} \left[\frac{C^\pi(s_{t+1}, a_{t+1}, g)[H-1]}{1 - C^\pi(s_{t+1}, a_{t+1}, g)[H-1]} \right]. \tag{5.5}$$

We now turn this identity into a penalty for the distributional NCE classifier as follows:

$$\begin{aligned} L_{TC} = \mathbb{E}_{(s_t, a_t, g, s_{t+1}, a_{t+1})} \Big[&\lfloor C(s_{t+1}, a_{t+1}, g)[H-1] \rfloor \log C(s_t, a_t, g)[H] \\ &+ \lfloor (1 - C(s_{t+1}, a_{t+1}, g)[H-1]) \rfloor \log \left(1 - C(s_t, a_t, g)[H]\right) \Big], \end{aligned} \tag{5.6}$$

where $\lfloor . \rfloor$ denotes the stop-gradient operator. Because the identity in Eq. 5.5 is satisfied by the Bayes' optimal classifier, adding the corresponding penalty (Eq. 5.6) to the distributional NCE loss (Eq. 5.1) does not change the solution. In on-policy settings, we can generalize 1-step consistency to $k-$step consistency (more details in Sec. 5.3.2):

$$\begin{aligned} L^k_{TC} = \mathbb{E}_{(s_t, a_t, g, s_{t+k}, a_{t+k})} \Big[&\lfloor C(s_{t+k}, a_{t+k}, g)[H-k] \rfloor \log C(s_t, a_t, g)[H] \\ &+ \lfloor (1 - C(s_{t+k}, a_{t+k}, g)[H-k]) \rfloor \log \left(1 - C(s_t, a_t, g)[H]\right) \Big]. \end{aligned} \tag{5.7}$$

We will empirically study this k-step consistency in our experiments. We hypothesize that the temporal consistency objective, like temporal difference learning, enables information to flow back in time, accelerating the Monte-Carlo classifier training. We will test this hypothesis in our experiments in Chapter 7 (Fig. 7.2).

5.3 A Practical Algorithm

In this section, we introduce the modifications to the Distributional NCE framework from Sec. 5.1 to turn it into a practical algorithm. We start by introducing a catch-all bin in 5.3.1 to avoid truncation errors and optimize for the true (Markovian) RL objective. Subsequently, we provide the derivation for 1-step and Multi-step temporal consistency regularization in 5.3.2, highlighting their connections to temporal difference (TD) learning approaches.

5.3.1 Introducing a catch-all bin

In Sec. 5.1, we introduced the Distributional NCE algorithm (Alg. 1) that estimates the likelihood of reaching the goal at specific future timesteps (up to proportionality, Eq. 5.2). We then showed that these estimates can be aggregated using geometrically-decaying weights to optimize for the (Markovian) RL objective with $r(s, a, g) = \mathbb{1}(s = g)$ in Eq. 5.3. However, implementing this naively would require a large number of bins to prevent temporal truncation errors and could lead to ill-defined Q-values.

The noise contrastive estimation (NCE) framework (Sec. 2.1, [18]) used in Distributional NCE and prior works [13, 14] can estimate any arbitrary positive goal distribution upto a proportionality, as long as one can draw samples from it. In the Distributional NCE implementation with h classifier bins, we repurpose the last bin to predict if the goal was sampled for $t \geq h$ rather than $t == h$ event, referring to it as the "catch-all" bin in the rest of the document. More precisely, the objective for the catch-all bin is as follows:

$$\max_C \mathbb{E}_{p(H \geq h)p(s_t, a_t)} \left[\mathbb{E}_{g \sim p^\pi(s_{t+H} | s_t, a_t)}[\log C(s_t, a_t, g)[h]] + \mathbb{E}_{p(g)}[\log(1 - C(s_t, a_t, g)[h])]\right], \tag{5.8}$$

where $p(H \geq h) = (1-\gamma)\gamma^{H-h} = \text{GEOM}(\gamma)[H-h]$ is a Geometric distribution shifted by h units. Then, the Bayes' optimal catch-all classifier for a policy π satisfies:

$$\frac{C^\pi(s_t, a_t, g)[h]}{1 - C^\pi(s_t, a_t, g)[h]} = \mathbb{E}_{p(H \geq h)} \left[\frac{p^{\pi_g}(s_{t+H} = g \mid s_t, a_t)}{p(g)}\right]. \tag{5.9}$$

This classifier can be used to maximize the (Markovian) RL objective with $r = \mathbb{1}(s = g)$

as follows:

$$\begin{aligned}
&\sum_{\Delta=1}^{h-1}\left((1-\gamma)\gamma^{\Delta-1}\frac{C^\pi(s_t,a_t,g)[\Delta]}{1-C^\pi(s_t,a_t,g)[\Delta]}\right)+\gamma^{h-1}\frac{C^\pi(s_t,a_t,g)[h]}{1-C^\pi(s_t,a_t,g)[h]}\\
&=\sum_{\Delta=1}^{h-1}\left((1-\gamma)\gamma^{\Delta-1}\frac{p^{\pi_g}(s_{t+\Delta}=g|s_t,a_t)}{p(g)}\right)+\gamma^{h-1}\mathbb{E}_{p(H\geq h)}\left[\frac{p^{\pi_g}(s_{t+H}=g\mid s_t,a_t)}{p(g)}\right]\\
&=\sum_{\Delta=1}^{h-1}\left((1-\gamma)\gamma^{\Delta-1}\frac{p^{\pi_g}(s_{t+\Delta}=g|s_t,a_t)}{p(g)}\right)+\gamma^{h-1}\sum_{\Delta=h}^{\infty}\left((1-\gamma)\gamma^{\Delta-h}\frac{p^{\pi_g}(s_{t+\Delta}=g|s_t,a_t)}{p(g)}\right)\\
&=(1-\gamma)\sum_{\Delta=1}^{\infty}\left(\gamma^{\Delta-1}\frac{p^{\pi_g}(s_{t+\Delta}=g|s_t,a_t)}{p(g)}\right)\\
&=\frac{p^{\pi_g}(s_+=g|s_t,a_t)}{p(g)} \qquad (5.10)
\end{aligned}$$

The expression on the RHS is the same as the objective in Contrastive NCE [14], which corresponds to maximizing the likelihood of the goal state under the discounted state occupancy measure.

In Distributional NCE, each classifier bin is crucial for estimating the corresponding component in the discounted future state density. An interesting future direction can be to employ redundancy in classifier bins, i.e., use multiple catch-all bins and exploit the relation between them as additional temporal consistency. Such a temporal ensembling procedure can be very similar to consistency training approaches [53] from semi-supervised learning literature.

5.3.2 Generalizing 1-step to Multi-step Temporal Consistency

In Sec. 5.2, we detailed the temporal consistency identity in Eq. 5.5 and proposed a 1-step temporal consistency regularization objective in Eq. 5.6 to enforce it. We also briefly introduced a k-step extension of this objective in Eq. 5.7. In this section, we formally derive the k-step consistency objective.

Deriving the multi-step consistency regularization. Let $p^{\pi(\tau)}(s_t,a_t,g,H)$ be the distribution over H-length state-action trajectories generated by the goal-conditioned policy $\pi_g=\pi(.|.,g)$ with s_t as the start state and a_t as the first action.

Then, the future state probabilities under π_g satisfy the following identity:

$$\begin{aligned} p^{\pi_g}(g \mid s_t, a_t, H) &= \mathbb{E}_{(s_{t+1}, a_{t+1}, s_{t+2}, a_{t+2}, \ldots, s_{t+H-1}, a_{t+H-1}) \sim p^{\pi(\tau)}(s_t, a_t, g, H-1)} \left[p(g \mid s_{t+H-1}, a_{t+H-1}) \right] \\ &= \mathbb{E}_{s_{t+1} \sim p(.|s_t, a_t), a_{t+1} \sim \pi(.|s_{t+1}, g)} \Big[\mathbb{E}_{(s_{t+2}, a_{t+2}, \ldots, s_{t+H-1}, a_{t+H-1}) \sim p^{\pi(\tau)}(s_{t+1}, a_{t+1}, g, H-2)} \\ &\qquad\qquad [p(g \mid s_{t+H-1}, a_{t+H-1})] \Big] \\ &= \mathbb{E}_{s_{t+1} \sim p(.|s_t, a_t), a_{t+1} \sim \pi_g(.|s_{t+1})} \left[p^{\pi_g}(g \mid s_{t+1}, a_{t+1}, H-1) \right]. \end{aligned} \tag{5.11}$$

This identity can be enforced over the distributional classifier using the 1-step temporal consistency regularization objective in Eq. 5.6. However, this property also holds for $k > 1$ steps:

$$p^{\pi_g}(g \mid s_t, a_t, H) = \mathbb{E}_{(\ldots, s_{t+k}, a_{t+k}) \sim p^{\pi(\tau)}(s_t, a_t, g, k)} \left[p^{\pi_g}(g \mid s_{t+k}, a_{t+k}, H-k) \right]. \tag{5.12}$$

We use the identity in Eq. 5.12 to derive a temporal consistency identity for distributional NCE. This identity is satisfied by the Bayes' optimal classifier (the solution to Eq. 5.2)[1]:

$$\frac{C^\pi(s_t, a_t, g)[H]}{1 - C^\pi(s_t, a_t, g)[H]} = \mathbb{E}_{(\ldots, s_{t+k}, a_{t+k}) \sim p^{\pi(\tau)}(s_t, a_t, g, k)} \left[\frac{C^\pi(s_{t+k}, a_{t+k}, g)[H-k]}{1 - C^\pi(s_{t+k}, a_{t+k}, g)[H-k]} \right], \tag{5.13}$$

and then turn this identity into an auxiliary, consistency objective:

$$\begin{aligned} L_{TC}^k = \mathbb{E}_{(s_t, a_t, g, s_{t+k}, a_{t+k})} \Big[& \lfloor C(s_{t+k}, a_{t+k}, g)[H-k] \rfloor \log C(s_t, a_t, g)[H] \\ & + \lfloor (1 - C(s_{t+k}, a_{t+k}, g)[H-k]) \rfloor \log \left(1 - C(s_t, a_t, g)[H]\right) \Big]. \end{aligned} \tag{5.14}$$

The consistency objective above is valid for any goal-conditioned policy $\pi(.|., g)$, as long as (s_{t+k}, a_{t+k}) is the k^{th} intermediate step on the Markov chain generated by the policy that connects s_t and g. In our practical implementation, we sample $(s_t, a_t, s_{t+k}, a_{t+k}, s_{t+H})$, $k < H$, from the replay buffer, and relabel the goal $g = s_{t+H}$ to train the critic via direct contrastive loss (Eq. 5.1) and k-step temporal consistency regularization (Eq. 5.14). As a result, the critic estimates the future state density of

[1]We use $C^\pi(s, a, g)$ to denote the Bayes' optimal classifier for a policy $\pi(a|s, g)$.

the average hindsight-relabeled policy over the replay buffer rather than the current policy, just like prior MC contrastive RL algorithms [14]. In our implementation, the future goal distance H is a random variable sampled from a Geometric distribution $H \sim \text{GEOM}(\gamma)$, and the intermediate state distance k is sampled from a truncated distribution to enforce that $k < H$. We call this method "Distributional NCE with Multi-step temporal consistency regularization." Like temporal difference methods [13], the temporal consistency regularization enables information and uncertainty over future states to flow back in time, thereby accelerating the Monte-Carlo classifier training.

Handling the edge-case: Consistency update for the catch-all bin. When applying the k-step temporal consistency loss, the catch-all bin gets mapped to $k+1$ bins from the future state, unlike regular classifier bins that have a $1:1$ mapping with a corresponding future classifier bin. This is an artifact of using a finite number of bins to represent the infinite-horizon discounted probabilities. More precisely, the equivalent temporal consistency identity (Eq. 5.13) for the catch-all bin, which is satisfied by the Bayes' optimal classifier (the solution to Eq. 5.2):

$$\frac{C^\pi(s_t,a_t,g)[h]}{1-C^\pi(s_t,a_t,g)[h]} =$$
$$\mathbb{E}_{(s_{t+k},a_{t+k})}\left[\sum_{i=1}^{h-1}\left((1-\gamma)\gamma^{i-1}\frac{C^\pi(s_{t+k},a_{t+k},g)[i-k]}{1-C^\pi(s_{t+k},a_{t+k},g)[i-k]}\right)+\gamma^{h-1}\frac{C^\pi(s_{t+k},a_{t+k},g)[h-k]}{1-C^\pi(s_{t+k},a_{t+k},g)[h-k]}\right],$$

where h is the total number of classifier bins and also the index of the catch-all bin. This identity can then be turned into a penalty as follows:

$$\begin{aligned}L_{TC}^k = \mathbb{E}_{(s_t,a_t,g,s_{t+k},a_{t+k})}\Big[&\lfloor w(s_{t+k},a_{t+k},g)\rfloor \log C(s_t,a_t,g)[H] \\ &+ \lfloor(1-w(s_{t+k},a_{t+k},g))\rfloor \log\left(1-C(s_t,a_t,g)[H]\right)\Big],\end{aligned}$$

where $w(s,a,g)=\sum_{i=1}^{h-1}\left((1-\gamma)\gamma^{i-1}\frac{C(s,a,g)[i-k]}{1-C(s,a,g)[i-k]}\right)+\gamma^{h-1}\frac{C(s,a,g)[h-k]}{1-C(s,a,g)[h-k]}$.

Chapter 6

Experimental Details

6.1 Task Descriptions

We conduct our experiments on four standard simulated robot manipulation tasks [41, 54] with increasing complexity: fetch_reach, fetch_push, sawyer_push, and sawyer_bin. All our tasks are framed as reward-free goal-reaching problems where the performance of the agent is tracked by the fraction of times it successfully reaches the goal.

fetch_reach: This task involves controlling a simulated fetch robotic arm to move the gripper to a specified 3D goal position. This is the simplest of all four tasks, where greedily moving the gripper toward the target position solves the task.

fetch_push: In this task, the same simulated fetch robotic arm needs to push a block placed on the table to a specified position. This is a harder task since the agent needs to reason about the dynamics of precisely pushing a block to a specified location. The agent needs to be careful as it can enter unrecoverable states, such as the block falling off the table if pushed incorrectly. Note that the gripper fingers are disabled, in order to force the agent to push the block to the goal rather than pick-and-place it at the goal.

sawyer_push: This task is similar to fetch_push but involves controlling a simulated sawyer robotic arm. A key difference is that this is a longer horizon task with $3\times$ as many steps as fetch_push in each episode before termination.

sawyer_bin: In this task, the same simulated sawyer robotic arm needs to

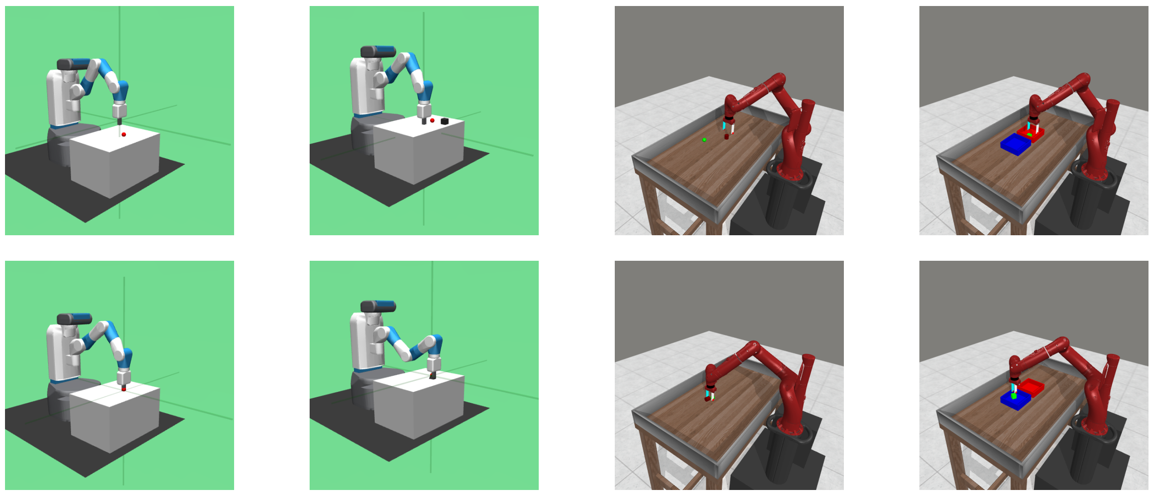

(a) fetch_reach (b) fetch_push (c) sawyer_push (d) sawyer_bin

Figure 6.1: Illustration of the goal-reaching tasks used in our experiments. The top row is a sample state at the time of initialization, and the bottom row is the corresponding goal state.

pick a block from a randomized position in one bin and put it in a goal location in another bin. This is a hard exploration problem[1] since the agent must learn the skills associated with (i) picking and dropping an object and (ii) moving the gripper to a desired location, and learn to coordinate these skills in the pick-move-drop sequence to solve the task. Failing to do even one of these skills/sub-tasks correctly will result in an unsuccessful outcome.

We also conduct our experiments on the following image-based variants of the above-mentioned tasks: fetch_reach_image, fetch_push_image, and sawyer_push_image. In these tasks, the low-dimensional observation space is replaced with a 64×64 image. We chose these tasks to demonstrate that the Distributional NCE algorithm is able to estimate the probability of reaching the goal over future timesteps directly from image observations. To get a better idea of the tasks, we visualized a random start state and the corresponding goal state for each of these tasks in Fig. 6.1. Moreover, the dimensionality of the observation and action space is described in Table 6.1.

[1]Note that the exploration in the **sawyer_bin** task is with regard to searching for a goal-reaching policy that is capable of solving the task. This work does not consider the problem of unsupervised exploration via goal selection [22, 39], since the goal is assumed to be provided by the environment in our framework.

Task	Observation Space (state and goal)	Action Space	Max Episode Length
fetch_reach	20	4	50
fetch_push	50	4	50
sawyer_push	14	4	150
sawyer_bin	14	4	150
fetch_reach_image	$64 \times 64 \times 6$	4	50
fetch_push_image	$64 \times 64 \times 6$	4	50
sawyer_push_image	$64 \times 64 \times 6$	4	150

Table 6.1: Environment details for the selected goal-reaching tasks.

6.2 Implementation Details

We used the official contrastive RL codebase[2] [14] in the JAX framework [5] to run the contrastive RL baselines: Contrastive NCE [14] and C-Learning [13]. Moreover, we implemented the distributional NCE algorithms by modifying this codebase as follows:

1. Change the last layer in the critic's architecture to output h bins (Alg. 2).
2. Change the Contrastive NCE objective to the Distributional NCE objective (Alg. 1).
3. Add the consistency loss (Eq. 5.7,5.14) to the classifier training module.

The actor is trained using the actor loss from soft actor-critic [20], while the critic is optimized for the contrastive classification objective in Eq. 5.1. For non-contrastive RL baselines, we use the sparse reward function for training the policy for a fair comparison with the reward-free contrastive RL framework. In all our experiments, we report the mean performance and the 95% confidence interval computed across 5 random seeds. We ran all our experiments on a single RTX 2080 Ti GPU with 11GB memory.

6.2.1 Distributional Critic Implementation

In this section, we go over the pseudo-code to implement a distributional critic network with h classifier bins in Alg. 2. The output of the distributional critic is a ternary tensor with the first two axes corresponding to the state-action and goal indices,

[2]https://github.com/google-research/google-research/tree/master/contrastive_rl

and the last axis h is the classifier bin index. The main diagonal along the first two axes corresponds to positive examples, i.e., state-action representations paired with their corresponding future states (reachable goals). Every other off-diagonal term corresponds to a negative example, i.e., a state-action representation paired with a randomly sampled goal.

Algorithm 2 DISTRIBUTIONAL CLASSIFIER: The contrastive classifier block, where the main diagonal corresponds to positive examples and off-diagonal entries correspond to negative examples. `h` is the number of bins in the classifier output, which may be less than the task horizon. Comments denote the shapes of tensors.

```
def classifier(states, actions, goals):
  sa_repr = sa_encoder(states, actions)  # (batch_size, h, repr_dim)
  g_repr = g_encoder(goals)  # (batch_size, h, repr_dim)
  logits = einsum('ikl, jkl->ijk')  # (batch_size, batch_size, h)
  # logits[i, j, k] is the probability of going from s[i] to s[j] in k steps.
  return logits
```

6.2.2 Network Architecture

We use the same architecture as the Contrastive NCE baseline while modifying the last layer in the critic. The policy is a standard 2-layer MLP with ReLU activations and 256 hidden units. The critic network comprises of a state-action encoder and a goal encoder (Alg. 2), which are each 2-layer MLP with ReLU activations and 256 hidden units, and a final dimension of $repr\_dim \times h$ ($repr\_dim = 64$ and $h = 21$ in all our experiments). For image-based tasks, we use the standard Atari CNN encoder [14, 34] to project the state and goal image observations into the latent space before passing them into the policy and critic networks.

6.2.3 Hyperparameters

We keep the default hyperparameters of Contrastive NCE [14] for all our experiments (Table 6.2). The proposed Distributional NCE algorithm only introduces one extra hyperparameter - the number of classifier bins, which is set to 21 in all the experiments.

Hyperparameter	Value
number of classifier bins (h)	21
batch_size	256
EMA target network (τ)	0.005
discount (γ)	0.99
hidden dims (policy and critic representations)	(256, 256)
critic representation dimension	64
learning rate	3e-4
policy and critic optimizers	Adam ($\beta_1 = 0.9, \beta_2 = 0.999$)
goal relabelling ratio for actor loss	0.5
maximum replay buffer size	1,000,000
minimum replay buffer size (initial data from random policy)	10,000

Table 6.2: A list of important hyperparameters used in our method and the baselines.

Chapter 7

Results and Analysis

In this section, we provide empirical evidence to answer the following questions:

1. Does the distributional NCE algorithm offer any benefits over the MC distance regression and distance classifier in deterministic goal-reaching environments, with function approximation and a stochastic policy?
2. Can distributional NCE accurately estimate the probability of reaching the goal at a specific future time step?
3. Are there any benefits to using the distributional architecture for classifier learning?
4. Does the temporal consistency term accelerate the distributional NCE training?

Environments. We selected seven standard goal-conditioned environments [41, 54] to test these hypotheses: fetch_reach, fetch_push, sawyer_push, sawyer_bin, fech_reach_image, fetch_push_image, and sawyer_push_image. The latter three environments have image-based observations. fetch_reach is the simplest task; The fetch_push and sawyer_push environments are more challenging, and require the robot to use its gripper to push an object to the specified goal position. Lastly, the pick-and-place in sawyer_bin presents a hard exploration challenge. See Sec. 6.1 for more details about the tasks.

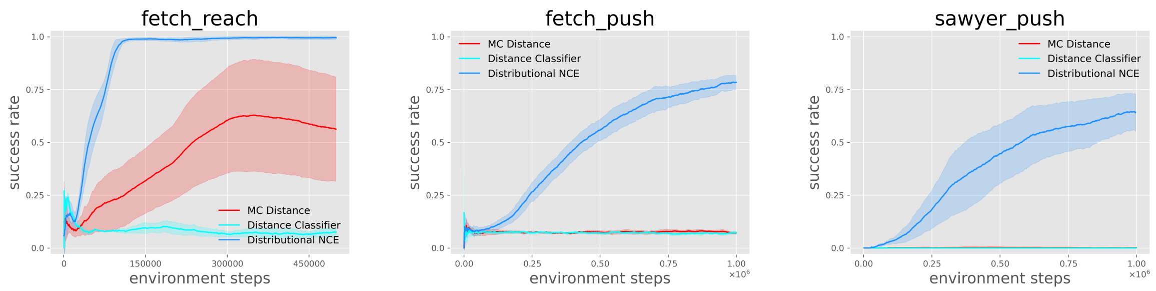


Figure 7.1: Distributional NCE is able to solve all the goal-reaching tasks with a good success rate, whereas the MC distance functions fail at almost all the tasks. This result supports our hypothesis that MC distances are not a good choice for the Q-function of goal-conditioned RL tasks.

7.1 Comparison with distance regression

In Chapter 4, we showed that using the MC distance metric can be very suboptimal for stochastic MDPs with a countable state space, where the optimal policy was known beforehand. Our first experiment is designed to test if distance functions learned via MC regression and distance classifier can be used in the place of a Q-function to greedily optimize a stochastic policy. We hypothesize that the stochasticity in action sampling from the policy, along with the associated risk of choosing the shortest path are ignored by MC distance functions, which will result in suboptimal behavior. We test out the MC distance regression and distance classifier algorithms on the three following tasks with increasing difficulty: fech_reach, fetch_push, and sawyer_push. We also included a comparison with distributional NCE to check if the proposed algorithm fills in the shortcomings of using MC distance functions. We use the same number of classifier bins for both the distance classifier and the distributional NCE.

Our results from Fig. 7.1 suggest that MC distance regression only succeeds at fetch_reach, the simplest of the selected tasks, which only requires greedily moving to a target goal position. Surprisingly, MC distance classifier fails at all the tasks. In every other setting, MC distance functions are not able to do considerably better than a randomly initialized policy. On the other hand, the distributional NCE algorithm is able to learn a policy that solves all the tasks.

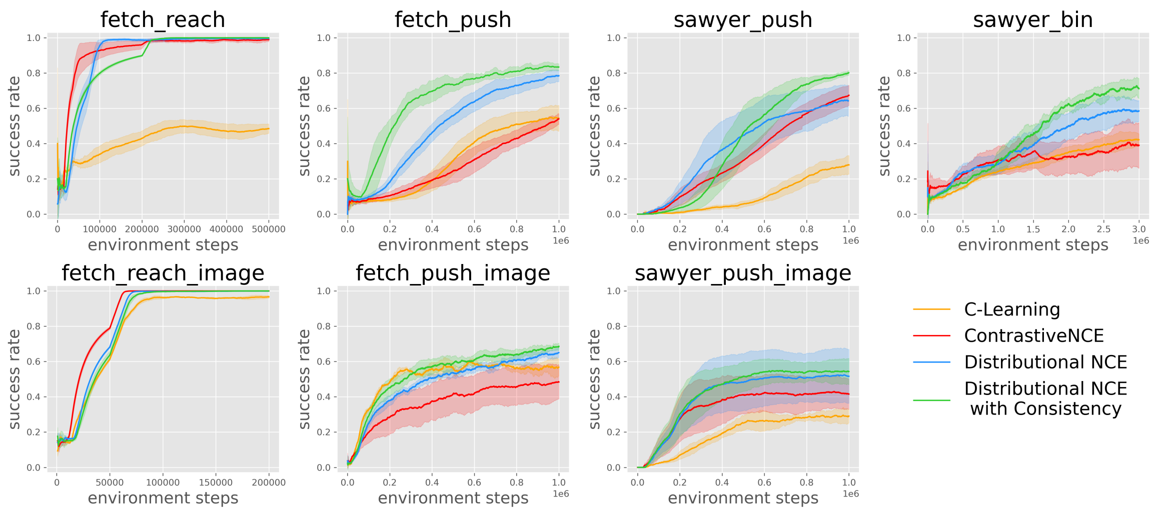


Figure 7.2: **Comparison with baselines.** Distributional NCE outperforms the Contrastive NCE [14] and C-Learning [13] in all but the easiest tasks (fetch_reach, fetch_reach_image). Applying temporal consistency on top of Distributional NCE accelerates learning and boosts asymptotic performance.

7.2 Comparing to prior goal-conditioned RL algorithms

We now compare the performance of distributional NCE against two high-performance goal-conditioned RL algorithms: Contrastive NCE and C-learning algorithms. Comparing against Contrastive NCE directly allows us to study whether our distributional architecture boosts performance, relative to a contrastive method (contrastive NCE) that predicts a single scalar value. Distributional NCE is an on-policy algorithm, so the comparison with C-learning (an off-policy algorithm) lets us study whether this design decision decreases performance.

The results, shown in Fig. 7.2, demonstrate that distributional NCE is roughly on par with the prior methods on the easiest tasks (fetch_reach and fetch_reach_image), but can perform notably better on some of the more challenging tasks; relative to the strongest baseline, distributional NCE realizes a +24% improvement on fetch_push and a +20% improvement on sawyer_bin.

As discussed in Sec. 5.2, the predictions from distributional NCE should obey a certain consistency property: the probability of getting to a goal after t time steps from the current state should be similar to the probability of getting to that same

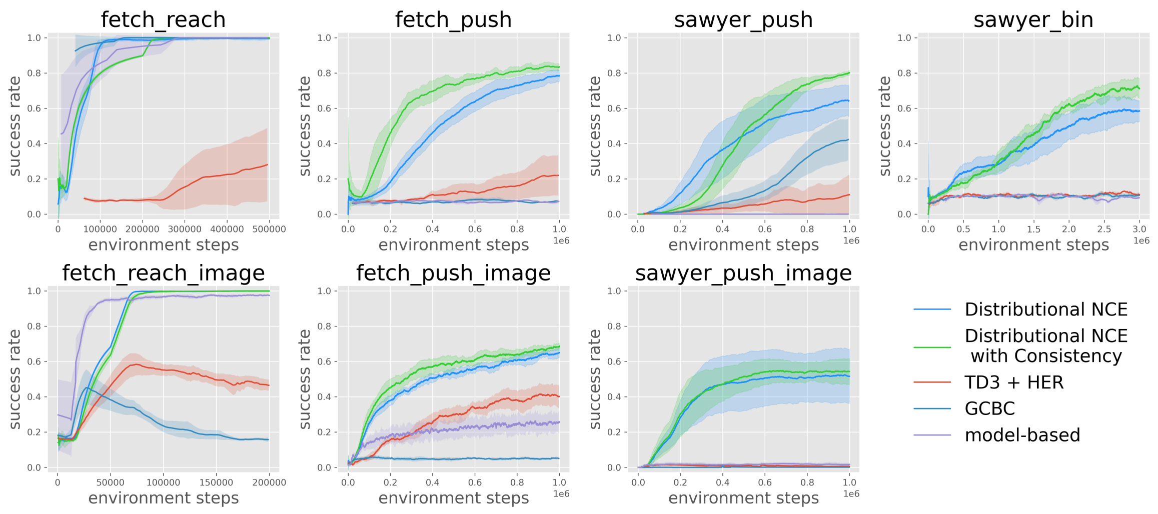


Figure 7.3: **Comparison with (non-contrastive) RL baselines.** Distributional NCE methods outperform the prior (non-contrastive) GCRL baselines, TD3+HER [2], Goal-conditioned behavior cloning (GCBC) [9] and model-based approach [8, 10, 24], across all the tasks.

goal after $t-1$ steps starting at the next state. We equip distributional NCE with the auxiliary objective proposed in Eq. 5.7 based on this property.

We show the results from this variant of distributional NCE ("distributional NCE with consistency") in green in Fig. 7.2. While we see no effect on the easiest tasks (fetch_reach, fetch_reach_image), the auxiliary term improves the sample efficiency of the fetch_push task (2.8×10^5 fewer samples to get to 60% success rate) and improves the asymptotic performance on the sawyer_push and sawyer_bin tasks by +16% and +13% respectively.

7.3 Comparing to prior (non-contrastive) GCRL algorithms

Unlike contrastive RL methods [13, 14], prior works have also used hindsight relabeling [2] and generative modeling over the discounted future state occupancy to solve goal-reaching tasks. In Fig. 7.3, we compare the proposed Distributional NCE methods with three exemplar (non-contrastive) GCRL algorithms:

1. *TD3+HER*: A common approach to solve goal-conditioned RL tasks is to use

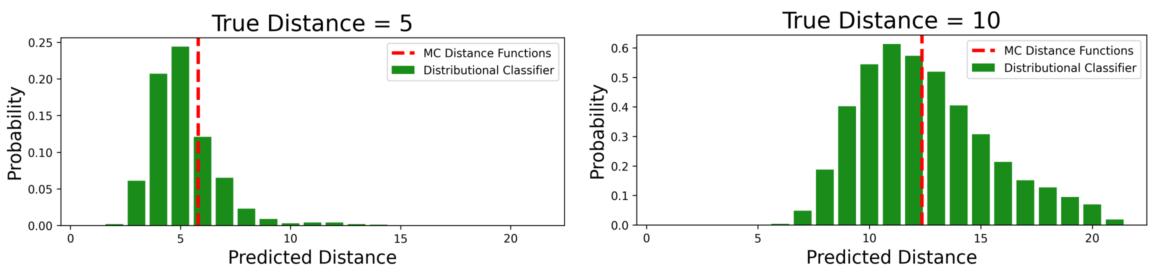


Figure 7.4: Visualizing the probabilistic distance predictions for future goals that are 5 *(Left)* and 10 *(Right)* steps away, on the fetch_push task. These results confirm that the distance predictions offered by distributional NCE correlate well with the true distance and are well-calibrated in uncertainty.

a high-performance off-policy RL algorithm with goal-relabeling via hindsight experience replay (HER) [2, 26, 42, 44]. As our first baseline, we choose TD3 [16], an off-policy actor-critic algorithm, and combine it with hindsight relabeling [27].

2. *Goal-Conditioned Behavior Cloning (GCBC)*: A simple baseline where a goal-conditioned policy is trained directly with behavior cloning on hindsight relabeled trajectories [9]. Unlike regular behavior cloning, where expert trajectories are used for supervision, GCBC uses suboptimal trajectories from the replay buffer. While this approach can result in sub-optimal policies in theory, prior work [9] has demonstrated impressive empirical results.

3. *Model-based approach*: This baseline is simply fitting a generative model on the future state occupancy distribution $p^{\pi(\cdot|\cdot)}(s_{t+}|s, a)$ [8, 10, 24]. A policy is then trained to maximize the likelihood under this density model. Unlike contrastive RL methods that estimate the probabilities directly, these methods learn a generative model and thus do not scale favorably with high-dimensional state-spaces such as image-based observations.

It can be seen in Fig. 7.3 that the proposed Distributional NCE methods outperform TD3+HER, GCBC and model-based approach on all the tasks. While the difference is marginal on the easier fetch_reach environments, the Distributional NCE method achieves greater than +50% higher median success rate on the harder pushing tasks.

7.4 Analyzing distributional NCE's predictions

To better understand the success of distributional NCE, we visualize its predictions. We do this by taking two observations from the fetch_reach task that take 5 steps to transit between under a well-trained policy. We show the predictions from distributional NCE in Fig. 7.4 *(left)*. Note that distributional NCE outputs a probability for each time step t. The highest probability is for reaching the goal after exactly 5 steps, but the method still predicts that there is a non-zero probability of reaching the goal after 4 steps or after 6 steps. We also compare to the predictions of the "MC Distance" baseline from Fig. 7.1. We see that this baseline makes an accurate estimate for when the goal will be reached.

We include another visualization of the distributional NCE predictions in Fig. 7.4 *(right)*, this time for two observations that occur 10 steps apart. Again, the predictions from distributional NCE appear accurate: the goal has the highest probability of being reached after 10 – 12 steps. These predictions highlight an important property of the distributional NCE predictions: they do not sum to one. Rather, it may be likely that the agent reaches the goal after 9 steps and remain at that goal, so the probability of being in that goal after 11 steps is also high.

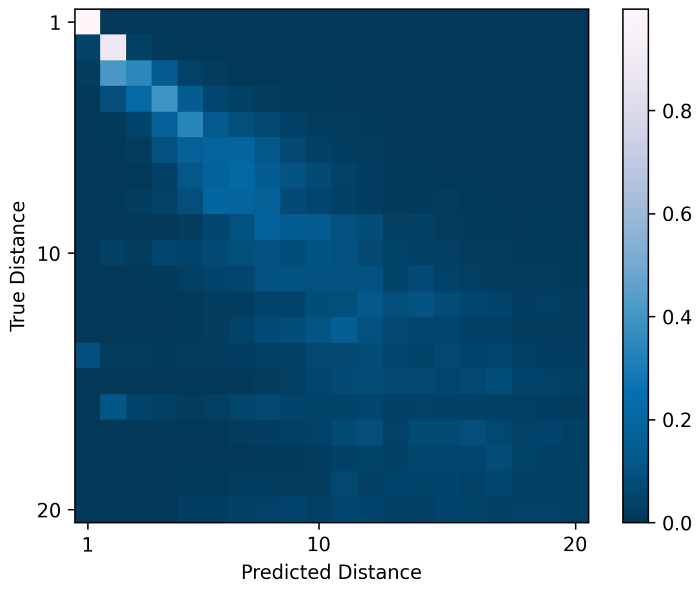


Figure 7.5: The predictions from distributional NCE can be converted into a distance classifier by normalization. See text for details.

Our final visualization draws a connection between distributional NCE and the "Distance Classifier" baseline from Fig. 7.1. We can recover the Bayes' optimal distance classifier from the distributional NCE predictions by normalizing the predicted probabilities (Eq. 5.4). We visualize a confusion matrix of these predictions in Fig. 7.5 by averaging over 1000 states sampled from a trained policy. We observe that there is a clear trend along the diagonal, indicating that the distributional NCE predictions (after normalization) can be used to estimate "distances." This visualization not only provides a further sanity check that distributional NCE makes reasonable predictions, but also highlights that (by normalization) distributional NCE retains all the capabilities of distance prediction.

7.5 How well does consistency regularization work in practice?

We empirically found that 1-step temporal consistency regularization does not improve the performance of Distributional NCE; occasionally it decreases performance. On the other hand, we found that the multi-step temporal consistency regularization significantly boosts the performance of Distributional NCE in Fig. 7.6.

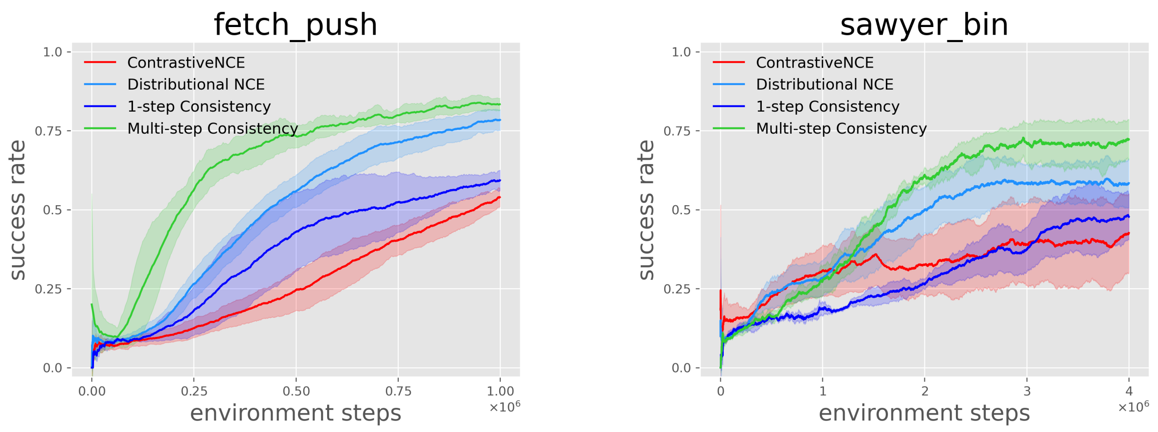


Figure 7.6: Multi-step temporal consistency regularization is significantly more effective than 1-step consistency regularization. In some cases, 1-step consistency regularization actually hurts the performance of the Distributional NCE algorithm, but Multi-step consistency almost always improves the performance.

7.6 Performance of Distributional NCE with different number of bins

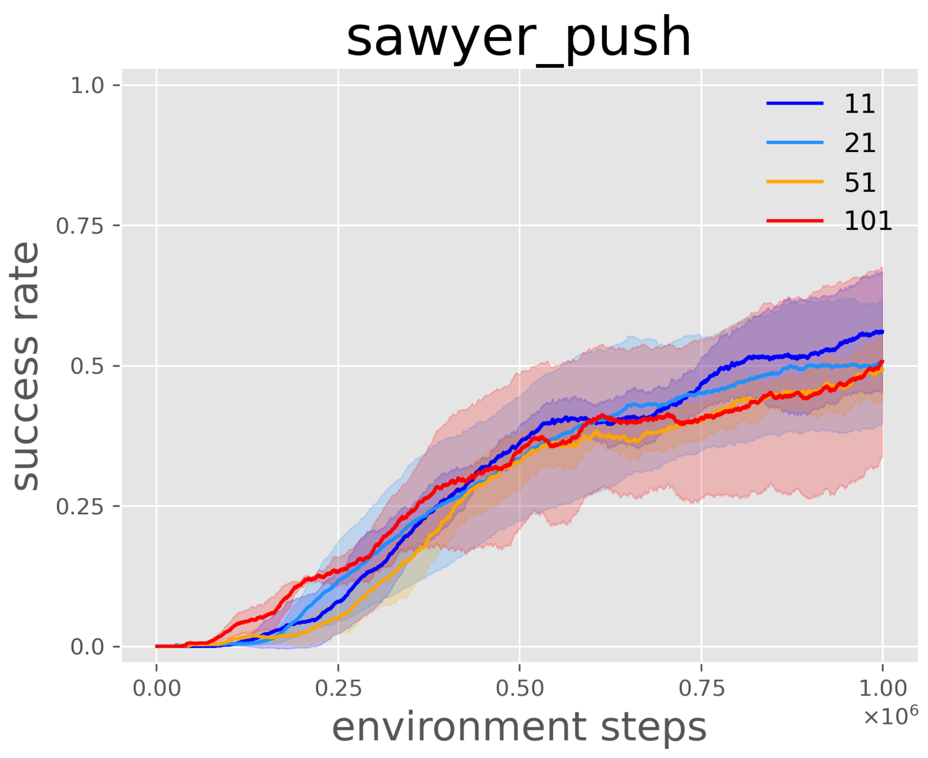


Figure 7.7: Varying the number of classifier bins has little effect on the performance of Distributional NCE for sawyer_push task.

In this section, we study if the choice of the number of classifier bins has an impact on the performance of the Distributional NCE algorithm. In theory, this hyperparameter should have no effect on the final policy since any Bayes-optimal distributional classifier can be mapped to the Bayes optimal contrastive NCE classifier (num_bins=1) as shown in Eq. 5.10. We verify this empirically on the sawyer_push task in Fig. 7.7, wherein we see very little difference in performance for four distinct choices for the number of classifier bins: 11, 21, 51, and 101. For the rest of the experiments in the report, we fix the number of classifier bins to 21.

7.7 Comparison with the last-layer ensemble baseline

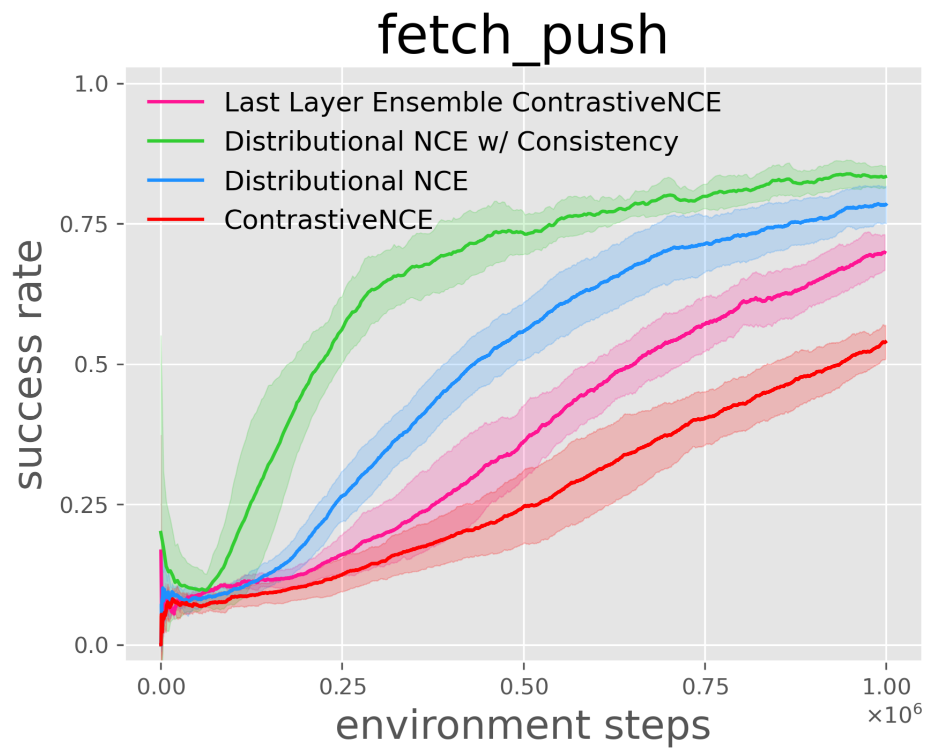


Figure 7.8: Distributional Critic trained with Contrastive NCE (last layer ensemble baseline) does not match Distributional NCE, highlighting the importance of the training algorithm over architectural choice.

The Distributional NCE algorithm uses a distributional critic with h classifier bins, while the Contrastive NCE [14] uses a regular critic with 1 output bin to directly denote the probability of reaching the goal in the future (up to proportionality). For the distributional critic, we simply modified the last linear layer in the regular critic network to have h outputs in all our experiments. In this section, we examine the importance of the distributional critic architecture by training a distributional critic with the Contrastive NCE algorithm. We do this by treating the distributional critic as an ensemble of critic networks, with all but the last-layer parameters shared.

We report the performance of the last-layer ensemble baseline in comparison to Contrastive NCE and Distributional NCE algorithms on the fetch_push task in

Fig. 7.8. We observe that the last-layer ensemble baseline outperforms the Contrastive NCE algorithm by +10% higher success. It can also be seen that the Distributional NCE algorithm outperforms this ensemble baseline by +7%. Further, Distributional NCE with consistency loss offers a +15% improvement over the ensemble baseline. This experiment confirms that the algorithm used to train the distributional critic has a huge impact on the overall performance: Distributional NCE with Consistency $>$ Distributional NCE $>$ Contrastive NCE.

7.8 Exploring the loss landscape of Distributional Classifiers

Prior works [3, 48] have identified that distributional RL methods enjoy stable optimization and better learning signal compared to their counterpart RL methods. In particular, Sun et al. [48] demonstrates that distributional value function approximations have a desirable smoothness property during optimization, which is characterized by small gradient norms. In this section, we try to examine if using the proposed distributional NCE algorithm enjoys some of these benefits. Note that prior works use the distributional critic to estimate the continuous return distribution with discretized bins [3, 7], which is different from our work that estimates the distributional probabilities of reaching the goal at discrete future timesteps.

In Fig. 7.9, we visualize the training loss and the gradient norm for the actor and critic networks over the course of training when optimized with the contrastive NCE and Distributional NCE algorithms. We note that the training loss for the critic network remains nearly unchanged, and the gradient norm is slightly smaller when switching from contrastive NCE to the Distributional NCE objective. On the other hand, we observe that actors trained with distributional critics receive gradients with smaller norms and achieve an overall lower loss. Note that plots in Fig. 7.9 are not a fair comparison since the Distributional NCE and Contrastive NCE agents were trained on different data, one that was collected by their respective actors interacting with the environment. However, we find the consistently low actor loss and smaller actor gradient norms with Distributional critics as compelling evidence to inspire future research works to study these optimization advantages more rigorously.

7.9 Performance on driving task

Most goal-conditioned RL benchmarks [29] exclude navigation tasks, which we find surprising. In this section, we consider a goal-reaching driving task with bicycle model dynamics. In particular, the agent's state is determined by a 5-D tuple (x, y, θ, v, ϕ), the x and y coordinates, orientation/yaw θ, velocity v, and steering angle ϕ. The goal is specified by a 2-D tuple indicating the target x and y coordinates. The agent's action space comprises of acceleration $\dot{v}$ and steering angle rate $\dot{\phi}$. Under the bicycle model dynamics, the state is updated as follows:

$$\begin{aligned}
x &= x + v\cos\theta * \Delta t \\
y &= y + v\sin\theta * \Delta t \\
\theta &= \theta + \frac{v\tan\phi}{L} * \Delta t \\
v &= v + \dot{v} * \Delta t \\
\phi &= \phi + \dot{\phi} * \Delta t,
\end{aligned}$$

where $\Delta t = 0.1$ sec is the time discretization, and $L = 1$ meter is the longitudinal length of the vehicle.

In this environment, we test the Distributional NCE algorithm, along with TD3, a performant off-policy algorithm, trained with sparse 0-1 rewards and dense rewards that measure progress toward the goal. From Table 7.1, we see that all three methods achieve a high success rate, but the TD3 algorithm trained with dense rewards reaches the goal twice as fast as the other. We hypothesize this is because reward-free or sparse 0-1 reward methods are trained to maximize the likelihood of reaching the goal, and are not explicitly rewarded for anything else, such as "minimum-time" behaviors. While the discount factor implicitly encodes the urgency to reach the goal, we observe that this enforcement alone is insufficient in practice.

One way to address this issue is to explicitly use a reward-maximization objective to optimize the policy and train the Distributional NCE critic as a standalone probabilistic distance function. This distance function offers a well-calibrated approximation of the distribution over dynamical distances under the policy, and can be used for stochastic shortest-path planning. Another future research direction can be to extend

Algorithm	Success	Episode Steps
Distributional NCE	0.85	37.5
TD3 (sparse rewards)	0.8	59.3
TD3 (dense rewards)	1.0	20.1
True Dynamics + MPC (near optimal)	1.0	18.5

Table 7.1: Limitations of reward-free setting: Maximizing the likelihood of reaching the goal is a poor incentive to reach the goal sooner. The results are averaged over 100 trajectories across three random seeds.

the proposed Distributional NCE framework to reward-maximization settings. One possible approach to doing this is by constructing an importance sampling estimator of the policy optimization objective (Eq. 3.1) using the learned density model over future states [24].

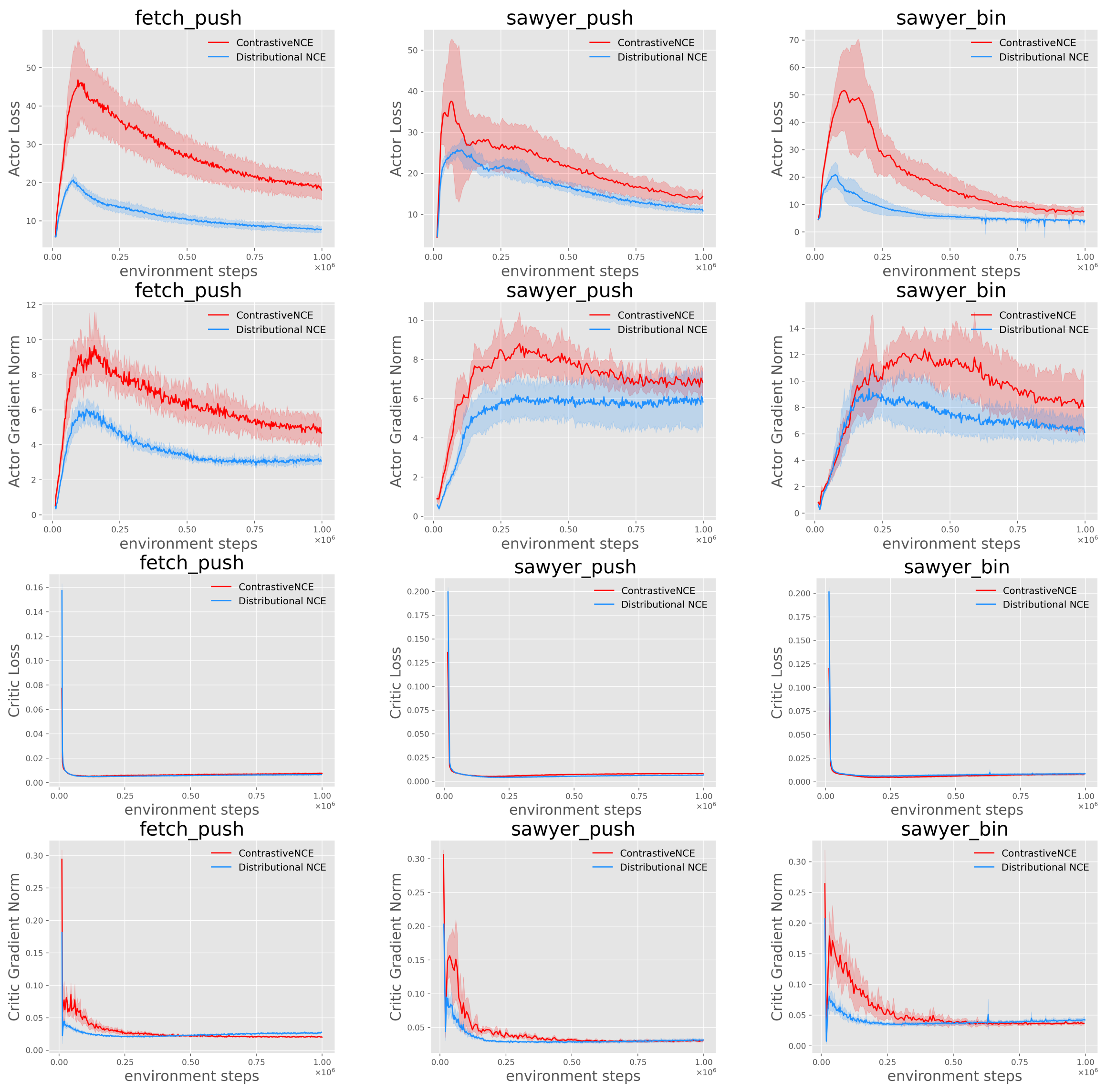


Figure 7.9: Visualization of the training losses and gradient norms for the actor and critic networks over the course of training. We do not see a huge difference in critic loss or gradients but observe that the actor loss is consistently lower and has a smaller gradient norm for Distributional NCE relative to Contrastive NCE.

Chapter 8

Conclusions

This work takes aim at the tension between two conflicting objectives for goal-reaching: maximizing the probability of reaching a goal, and minimizing the distance (number of steps) to reach a goal. Our analysis shows that distance-based objectives can cause poor performance on both didactic and benchmark tasks. Based on our analysis, we propose a new method that predicts the probability of arriving at the goal at many different time steps; this method outperforms prior goal-conditioned RL methods, most notably those based on regressing to distances. Our analysis also suggests a temporal-consistency regularizer, which can be added to boost performance. Together, we believe that these results may prove hopeful both to new researchers attempting to build a mental model for goal-conditioned RL, as well as veteran researchers aiming to develop ever more performant goal-conditioned RL algorithms.

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