Title: “Pilot Results of a Digital Manipulative for Elementary Mathematics”

Authors: J. Cross, B. Brewer, E. Hamner, L. Zito, S. Speer, M. Tasota

Final Version: Forthcoming

Citation:

J. Cross, B. Brewer, E. Hamner, L. Zito, S. Speer, and M. Tasota, “Pilot Results of a Digital Manipulative for Elementary Mathematics,” Presented at the 2020 Annual Meeting of the American Educational Research Association (AERA), 2020.

Pilot Results of a Digital Manipulative for Elementary Mathematics

Jennifer Cross[1] (Tufts University), **Bambi Brewer** (BirdBrain Technologies, LLC), **Emily Hamner** (Carnegie Mellon University), **Lauren Zito** (Carnegie Mellon University), **Samantha Speer** (Carnegie Mellon University), **Michael Tasota** (Carnegie Mellon University)

Abstract

The use of tangible manipulatives to improve math achievement, while popular, requires careful design and use decisions to provide benefit. We present a novel instructional technology, Owlet, that combines math manipulatives with robotics. Owlet guides student manipulative use, structuring transitions between concrete tangible representations and generalized digital representations. In doing so, we aim to easily integrate into existing classrooms and improve math instruction for elementary-aged students. To investigate the potential of Owlet, we pilot tested in three classrooms, integrating into their existing curricula. Through these pilots, we found evidence that Owlet supported our goals for learning and classroom compatibility.

Objectives

Mathematical comprehension and skills are the gateway to future participation of students in opportunities and careers ranging from biology and medicine to business and finance to engineering and design (NRC, 2013). Mathematics education is therefore critical to the future prosperity and growth of both students as individuals and as a part of the communities they live in. Active learning and manipulatives are extremely popular tools for improving student understanding and achievement in math (Burns 1996). However, evidence also suggests that educators must introduce manipulatives carefully to prevent students from becoming over focused on irrelevant details and to support knowledge transfer to abstract and generalizable concepts (McNeil & Uttal, 2009; Brown et al, 2009). Most elementary schools adopt traditional math textbooks and curriculum in order to address student math learning (Dossey, McCrone, & Halverson, 2016). These detailed materials are essential for teachers lacking confidence and resources to develop their own math lessons, but these curricula frequently contain limited support and guidance for teachers applying manipulatives and hands-on activities to actively engage students. In order to fill this gap, we present our design and pilot test results of a hands-on tool supported by app-based scaffolds and guidance that is flexible enough to be integrated into many popular curricula through teacher professional development.

This early-stage, exploratory study aims to investigate the affordances that robotic technologies can provide in elementary math through early-stage design, development and testing of prototype robotics-imbued math manipulatives. There have been a variety of investigations into virtual versions of the physical manipulatives typically used in elementary math lessons including Durmus & Karakirik (2006) and Sarama & Clements (2016) among others. While comparisons between physical and virtual manipulatives in elementary math are common, there are limited examples which bridge these physical and virtual tools (Rodić & Granić, 2018). For that reason,

[1] Corresponding Author: Jennifer Cross, Tufts Center for Engineering Education and Outreach, 200 Boston Ave, G810, Medford, MA 02155 jennifer.cross@tufts.edu

we created a digital manipulative to bridge the gap between physical and virtual tools combining the affordances of each. Digital manipulatives combine tangible manipulatives with computational power and sensing (Resnick et al,.1998). With this study, we aim to address the following research questions:

1. **Math Instruction (primary question)**: What affordances of robotics technologies can support engagement and learning within existing elementary-level math curricula?
2. **Classroom Integration (auxiliary question)**: What attributes must robotics technologies possess for elementary math classroom compatibility?

Theoretical Framework

Our past work supported the belief that integrating robotics into core, non-technology classes can provide more students with opportunities to build technological fluency (Cross et. al, 2017). However, we found that in order to support a student's self perception as someone that can excel in mathematics, science, and technology, we must create opportunities for *elementary* school-aged students. As a result, in Hamner et al., 2017, we describe a process of developing a robotics program for elementary school students designed to support the development of systems thinking and technological fluency. This program integrated into science content and provided students with open-ended construction kits for creating interactive robotic devices. As math is allocated more time and resources than science within elementary education, we shifted the project focus to a math context (NAEP, 2009). With this shift we modified the hardware, software, and instructional methodology to integrate with the learning objectives in math. This math focused program is referred to as *Owlet*.

The use of robotics to support student learning of math concepts has been investigated in a variety of studies (Benitti, 2012). Allen (2013), for example, discusses how robotics activities can be matched with common core math standards. Hussain et al. (2006) evaluate how robotics impacted student mathematics and problem-solving skills. Typically, these activities revolve around robotics as an application of math skills and do not seek to directly improve math learning or correct misconceptions.

Active learning, "any instructional method that engages students in the learning process … to do meaningful learning activities and think about what they are doing" (Prince, 2004), has been shown to improve student understanding and achievement in math (Freeman et al, 2014; Boaler & Greeno, 2000). Math manipulatives are widely accepted and expected tools for supporting student learning and understanding, however, mixed evidence about their benefits also suggest that their design and their application to instruction are crucial considerations (Boggan et al., 2010; McNeil & Jarvin, 2007).

Martin (2009) presents a theory for how to learn new mathematical concepts with physical manipulatives, or physically distributed learning, where use of the tangible manipulatives (actions) and general mental structures for math understanding (ideas) are coevolved through repeated sequences of actions and ideas. The students' ideas and actions are then developed through a process of failures and constraints applied by the actions to the ideas, and by the ideas to the actions. While actions with manipulatives are beneficial, they alone are not sufficient for math learning. The Owlet prototype builds on this theory by providing both a tangible representation to encourage actions, and the digital app to encourage ideas and connections to general math structures.

McNeil & Uttal (2009) summarize a series of articles on manipulatives and highlight some risks commonly found with manipulatives, and in particular note that, a physical manipulative on its own without guidance is unlikely to promote learning. A manipulative with extraneous details may prove distracting for students, leading them to focus on irrelevant and superficial information (Kaminsky et al 2009). This can in turn lead to challenges transferring their learning to more general contexts.

We hypothesize that a digital manipulative with both tangible and virtual representations can help teachers utilizing manipulatives to avoid these pitfalls by: 1) providing continual guidance alongside the manipulative and 2) presenting two simultaneous representations to support generalizability. By developing the Owlet program, a digital manipulative supported by intentional instructional activities and tools, we aim to promote student achievement and positive attitude shifts toward mathematics. This paper addresses our examination of this program and technologies as used in elementary math subjects.

Methods

We selected three elementary teachers (grades 2, 4, and 5) to implement the Owlet program in their math classes. These teachers had 7 to 15 years of teaching experience and taught the same curriculum in the same grade the previous academic year. Given the early stage of this design and development work, we selected and applied qualitative research methods. We conducted thematic analysis to answer our broad research questions. Data were collected via the following three methods.

1. **Teacher Interviews** (20-45 minutes): Teachers were asked to reflect on their experience using the intervention in their classroom.
2. **Student Interviews** (roughly 5 minutes, with 42 out of 53 students): Students were asked about math concepts and the technological tools.
3. **Classroom Observations** (between 120-180 minutes per grade): We focused on instruction, student use of the interface, student misconceptions, and student-demonstrated mathematical knowledge.

All class activities were observed by at least two researchers. All interviews were audio-recorded and transcribed. Teacher interviews and observation notes were combined using affinity diagramming (Holtzblatt et al, 2005) to inductively sort small pieces of evidence into larger themes. Student interviews were coded to provide descriptive statistics of their experiences.

Materials and Evidence

The prototype manipulatives were pre-constructed from a custom-developed microcontroller (Figure 1), standard RC servo motors, and tricolor LEDs. The prototype hardware was controlled by BirdBlox, a custom block-based programming language similar to Scratch.

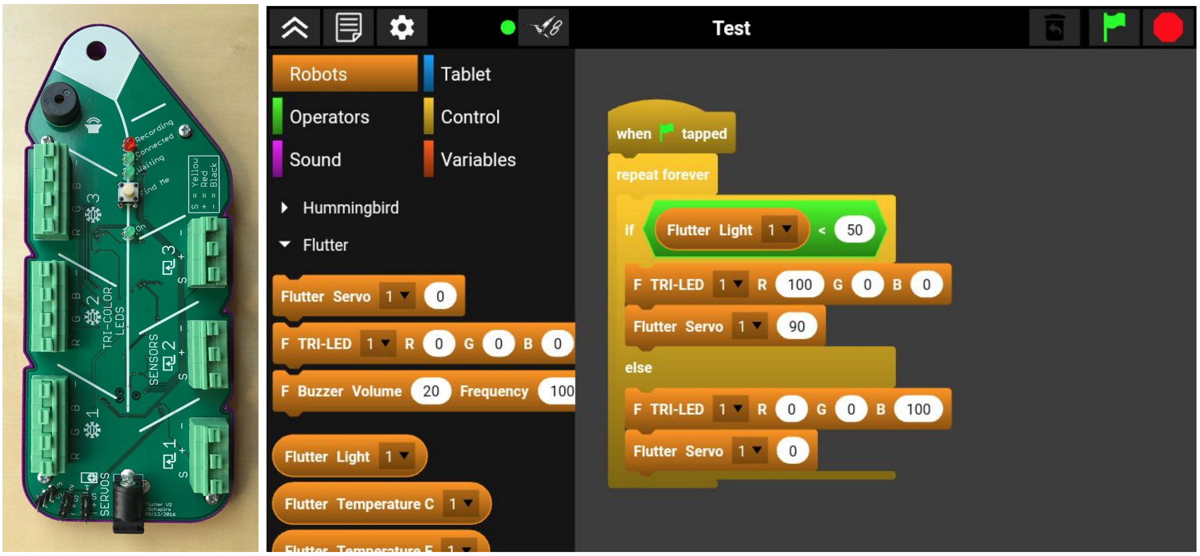

Figure 1. The prototype microcontroller (left) and a screenshot of the prototype BirdBlox programming language (right).

To identify curriculum areas to target, we asked each participating teacher to choose three to five units from their curriculum that they would be willing to teach with Owlet. Each unit covered two to three math lessons. For each pilot, we selected one unit that could be addressed in a novel way with the prototype hardware and BirdBlox. The lessons corresponding to each chosen topic were rewritten to cover the same content while incorporating Owlet, as described in Table 1.

Table 1: Pilot and Lesson Activities

Grade Level & Curriculum	Duration	Lesson Titles and Descriptions
2 EnVision Math (Pearson)	120 minutes	**Lesson 1 - Thinking Addition to 10 to Subtract:** Students created programs to light up two LEDs and wrote addition and subtraction equations describing the blocks in their programs, e.g., 8 total blocks - 3 yellow blocks = 5 orange blocks. **Lesson 2 - Thinking Addition to 18 to Subtract:** Students created programs that directly modeled subtraction equations. A custom servo motor manipulative depicted each subtraction equation by moving along a semicircular number line from 0 - 18.
4 Everyday Math (McGraw Hill)	140 minutes	**Lesson 1 - Equal Sharing and Equivalence:** A lesson on equal sharing was rewritten to model equal sharing by dividing a time period into equal parts. For example, students divided a 3 second time period by writing a program to turn an LED red for 1½ seconds and then blue for 1½ seconds.

		Lesson 2 - Fraction Circles and Equivalence: The second lesson modeled equivalent fractions using a custom servo motor manipulative. Students entered fractions into a program to control the position of a servo motor. Students labeled the servo motor positions to explore which fractions were equivalent (Figure 3).
5 Math Expressions (Houghton Mifflin Harcourt)	180 minutes	**Lessons 1-3 - Long Division:** Three lessons focused on the practice of long division techniques framed the division problems in terms of making a small robot move and light up (Figure 4). For example, students calculated the angles required to make the robot stop seven times between 0° and 180°.

Prior to the first pilot, all three teachers participated in a seven-hour professional development session. They were introduced to Owlet, as well as the curricular materials for all three pilots. Teachers received lesson plans, student workbooks, template programs in BirdBlox, and any manipulatives required for their lessons. Manipulatives included a servo powered semicircular number line which was used to illustrate addition and subtraction in second grade (Figure 2, right) as well as fractions in fourth grade (Figure 3), and a small robot which was programmed to move and light up based on division solutions in fifth grade (Figure 4). Other lessons used on-screen blocks as virtual manipulatives, controlling physical tri-color LEDs, to support addition and subtraction (Figure 2, left).

Figure 2. Students in the second grade pilot working on lesson 1 “Thinking Addition to 10 to Subtract” (left) and lesson 2 “Thinking Addition to 18 to Subtract” (right).

Figure 3. Students from the fourth grade pilot labeling equivalent fractions (Lesson 2).

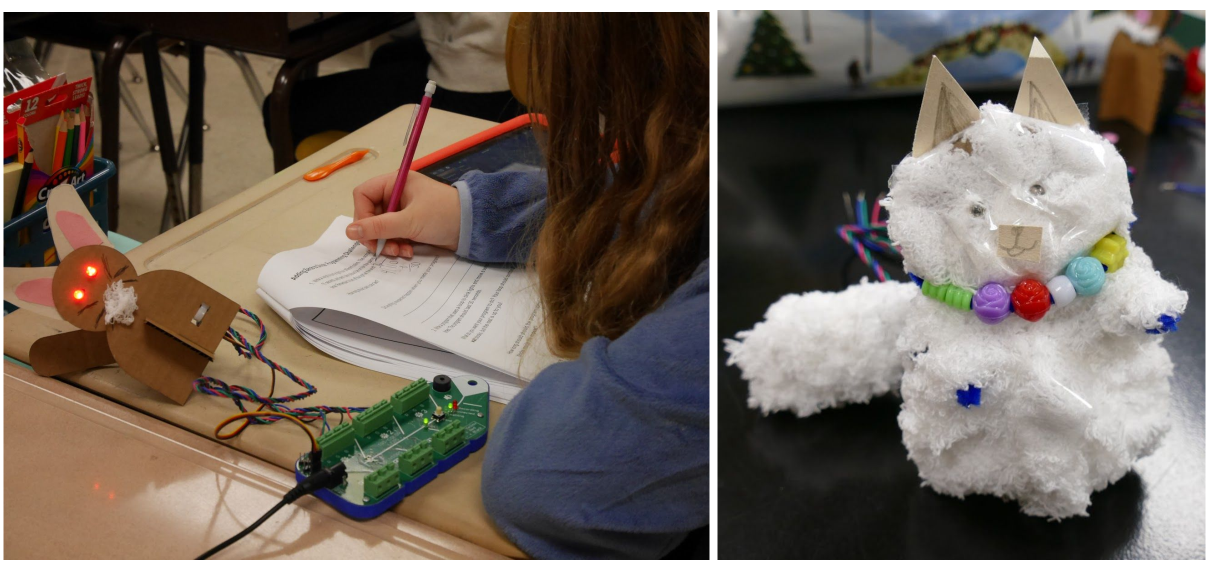

Figure 4. Student in the fifth grade pilot using division to complete challenges with a small robot (left). Students were able to personalize their robots with craft materials (right).

Results

Below we discuss the major results that arose during analysis of student interviews, teacher interviews, and classroom observations.

Student Interviews

We interviewed 42 students (ages 7 to 11). Forty-one students (97%) said that they liked the Owlet project. When asked what they liked about it, the two largest response categories were: custom manipulatives and creating colors with the tri-color LEDs (36%). When asked what they learned, 27 students' (64%) responses involved programming or technology, while 14 students' (33%) responses involved math. When asked what they learned about math, answers varied widely depending on the topics covered. Table 2 details the student category response distribution.

Table 2: Student Interview Responses to the Question "What did you learn about math?"

What did you learn about math?			
Grade	Response Code	Number of Students	Percentage of Students
2nd	Modeling subtraction with Flutter	6	46%
	Math strategies	4	31%
4th	Equivalent fractions	8	53%
	Equal Shares	2	13%
	Comparing fractions	2	13%
5th	Applying math	7	50%
	Importance of practice	3	21%

Supporting Math Instruction

The first themes synthesized from analysis of observations and teacher interviews address how teachers use Owlet to support math instruction. These themes are well aligned with the Math Instruction research question and seemed to be mediated by how Owlet integrated with teacher instruction.

Using Owlet to Support Math Engagement

Teachers described Owlet as supporting their students' engagement with math through open-ended, playful exploration; their success during tasks perceived as challenging; and students' ability to customize and take ownership of the project. One teacher explained:

> *"[...] the kids did not get tired, disengaged [...] none of that kind of stuff that typically happens in a regular class. [...] I was sort of surprised by [...] just how quickly things went with the engagement"*

Using Owlet to Support Math Learning

Teachers also used Owlet to support student learning of specific math knowledge and skills. One teacher reported that the Owlet-supported math lesson met more math standards than the lesson it replaced but that it was also more time consuming.

> *"I would say [Owlet] was definitely not the most efficient use of time [...] but I do think it took in more standards than [the conventional] lessons required [...] I think it gave them a broader base on more skills than just what was required of those two lessons."*

We found that concrete Owlet lessons enhanced student engagement with math content, leading us to conclude that careful consideration for Owlet integration was crucial to student learning. One teacher observed that abstractly writing subtraction equations to describe the number of blocks in a program controlling LEDs was less successful than using the traditional counters at connecting to the lesson concepts. She stated that the fraction dial supported a more direct and useful connection:

> *"that first lesson, I think they were just so wrapped up in the lights [...] that I think they weren't really focused on how many blocks they had [...] whereas that second lesson [...] you could see the dial moving to a number"*

Integrating Owlet into Instruction

We modified standard curriculum lessons that the teachers used in the previous year to include Owlet in two ways: as a manipulative that provides visual formative assessment and supports specific math concepts, and as a motivational tool that encouraged repetition as students practice applying arithmetic operations.

> *"I think that the engagement and the application of [...] long division, to have something that is so engaging be the end result, or the reward for doing the division, I think that made a big difference."*

Improving Classroom Integration

The other themes center on requirements for implementing robotics in a typical math classroom and are well aligned with the Classroom Integration research question. We identified two key areas of focus: providing adequate support for teachers, and structuring students' equipment sharing.

Supporting Teacher Success

Teachers found the professional development and supplied materials very useful and requested regular refresher lessons as a follow up to the professional development.

> *"I felt, like, super prepared coming out of that training that we had, as well as the notebook and all the binders and stuff that you guys had provided for me. I felt that [...] did an awesome job at explaining exactly what I needed to know."*

Using Owlet with Student Groups

In the second and fourth grade pilots, pairs worked with a single Owlet device and tablet. This form of collaboration posed challenges for student engagement, as both could not see and interact with the screen at the same time. However, students were well engaged when completing a task with two clear roles on a single device. Pairs of fifth grade students encountered fewer challenges collaborating on tasks while using individual devices.

Adapting Owlet for Classroom Compatibility

Classroom implementation must address classroom constraints, including time limits and physical constraints. For example, limited outlet availability in the classroom suggests the importance of battery-powered technologies. Setup was also more time consuming than regular math classes. The second grade teacher observed,

> "*that first day took so much longer than I expected it to take, just because they didn't really know how to connect, how to hook up*"

This suggests that the prototypes, which required significant modification for different activities, limited math classroom compatibility. We observed that while most students were able to operate the tablet and manipulate on-screen elements, it was not something that all students were familiar with.

Scholarly Significance

Much scholarly work is currently focused on the development of new methodologies for improving support for students and student achievement in mathematics. We contribute to this effort through the development of a new robotics-powered, digital manipulative for elementary math instruction. This paper describes pilot tests of Owlet in three grades and content areas.

We applied qualitative analysis to address research questions into how math learning and classroom integration can be achieved with digital manipulatives. All pilot teachers reported meeting required standards using Owlet lessons, and 97% of students reported that they enjoyed the lessons. We found that Owlet lessons with simple, concrete visualizations resulted in more self-reported student gains, while more abstract tasks with extraneous details, such as selecting LED colors, were engaging but not well connected to math learning. These results are aligned with the findings of Kaminsky et al (2009). Our observations also highlighted key classroom compatibility issues for informing future design, including setup and power requirements . These insights are important for digital manipulatives development; however, further study beyond this limited testing is necessary to evaluate claims. In future development and evaluation, we will refine the system to limit extraneous details and better support the interleaving of physical actions and cognitive ideas to develop skills through the processes proposed by Martin (2009).

Acknowledgements

This material is based upon work supported by the National Science Foundation under Grant No. (1648747). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

References

Allen, K. C. (2013). Robots bring math-powered ideas to life. *Mathematics Teaching in the Middle School*, *18*(6), 340-347.

Benitti, Fabiane Barreto Vavassori. "Exploring the educational potential of robotics in schools: A systematic review." *Computers & Education* 58.3 (2012): 978-988.

Boaler, J, Greeno, J.G. (2000). Identity, Agency, and Knowing in Mathematics Worlds, *Multiple Perspectives on Mathematics Teaching and Learning*. Westport, CT: Ablex Publishing.

Boggan, M., Harper, S., & Whitmire, A. (2010). Using Manipulatives to Teach Elementary Mathematics. Journal of Instructional Pedagogies, 3.

Brown, M. C., McNeil, N. M., & Glenberg, A. M. (2009). Using concreteness in education: Real problems, potential solutions. Child Development Perspectives, 3(3), 160-164.

Burns, M. (1996). How to Make the Most of Math Manipulatives. Instructor, 105(7), 45-51.

Cross, J.L., E. Hamner, L. Zito, and I. Nourbakhsh. (2017) Student Outcomes from the Evaluation of a Transdisciplinary Middle School Robotics Program. In Proceedings of 2017 IEEE Frontiers in Education Conference (FIE), Indianapolis, IN.

Dossey, J.A., McCrone, S.S., Halverson, K.T. (2016, July). The Implemented Curriculum. In *Mathematics Education in the United States 2016*. Reston, VA: The National Council of Teachers of Mathematics, Inc.

Durmus, S., & Karakirik, E. (2006). Virtual Manipulatives in Mathematics Education: A Theoretical Framework. *Turkish Online Journal of Educational Technology-TOJET*, *5*(1), 117-123.

Freeman, S., Eddy, S.L., McDonough, M., Smith, M.K., Okoroafor, N., Jordt, H., Wenderoth, M.P. (2014, June). Active Learning Increases Student Performance in Science, Engineering, and Mathematics, *Proceedings of the National Academy of Sciences*, Vol. 111, Issue 23, pp. 8410-8415.

Hamner, E., L. Zito, J. L. Cross, M. Tasota, P. Dille, S. Fulton, M. Johnson, I. Nourbakhsh, and J. Schapiro. (2017). Development and Results from User Testing of a Novel Robotics Kit Supporting Systems Engineering for ElementaryAged Students. In Proceedings of 2017 IEEE Frontiers in Education Conference (FIE), Indianapolis, IN.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American educational research journal*, *42*(2), 371-406.

Holtzblatt, K., Burns, J., & Wood, S. (2005). Building an Affinity Diagram. In *Rapid Contextual Design: A How-to Guide to Key Techniques for User-Centered Design* (pp. 159-179). Morgan Kaufmann.

Hussain, S., Lindh, J., & Shukur, G. (2006). The effect of LEGO training on pupils' school performance in mathematics, problem solving ability and attitude: Swedish data. *Journal of Educational Technology & Society*, *9*(3), 182-194.

Kaminski, J. A., Sloutsky, V. M., & Heckler, A. (2009). Transfer of mathematical knowledge: The portability of generic instantiations. Child Development Perspectives, 3(3), 151-155.

Martin, T. (2009). A theory of physically distributed learning: How external environments and internal states interact in mathematics learning. Child Development Perspectives, 3(3), 140-144.

McNeil, N., & Jarvin, L. (2007). When theories don't add up: disentangling the manipulatives debate. Theory into practice, 46(4), 309-316.

McNeil, N. M., & Uttal, D. H. (2009). Rethinking the use of concrete materials in learning: Perspectives from development and education. Child development perspectives, 3(3), 137-139.

National Research Council (NRC). (2013). The Mathematical Sciences in 2025. Washington, DC: The National Academies Press.

Prince, M. (2004). Does active learning work? A review of the research. *Journal of engineering education*, *93*(3), 223-231.

Resnick, M., Martin, F., Berg, R., Borovoy, R., Colella, V., Kramer, K., & Silverman, B. (1998). Digital manipulatives: new toys to think with. In *Proceedings of the SIGCHI conference on Human factors in computing systems* (pp. 281-287).

Rodić, L. D., & Granić, A. (2018). Tangible User Interfaces for Enhancement of Young Children's Mathematical Problem Solving and Reasoning: A Preliminary Review of Relevant Literature. In *Central European Conference on Information and Intelligent Systems* (pp. 77-84). Faculty of Organization and Informatics Varazdin.

Sarama, J., & Clements, D. H. (2016). Physical and Virtual Manipulatives: What Is “Concrete”?. In *International perspectives on teaching and learning mathematics with virtual manipulatives* (pp. 71-93). Springer, Cham.

U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, and National Assessment of Educational Progress (NAEP). (2009). 2009 Mathematics Assessments.