Voronoi-based Coverage Control with Connectivity Maintenance for Robotic Sensor Networks

Wenhao Luo and Katia Sycara

***Abstract*— In this paper, we consider the problem of Voronoi-based coverage control for multi-robot sensor networks with connectivity constraints, where a team of mobile robots spread out over the workspace in order to optimize the overall coverage performance while preserving connectivity. In particular, we proposed a connectivity-aware coverage control approach to simultaneously optimize the coverage performance and ensure robotic network connectivity. Unlike most of existing works on connectivity control that have no guarantee on the primary task, our algorithm could maintain minimum connectivity that introduce minimal revision to the primary coverage controller due to invoked connectivity constraints. This ensures the robotic network stay connected at all time but also in an optimal way to provide highest freedom for achieving primary coverage task. Moreover, we prove the convergence of the proposed controller that guarantees continuously improved coverage performance in presence of connectivity constraints. Simulation results are given to demonstrate the effectiveness of our approach.**

I. Introduction

Multi-robot systems have been widely studied for extending its capability of doing complex tasks through cooperative behaviors in a number of applications, such as search and rescue [1], cooperative sensor coverage [2], [3], and environmental exploration [4]. The ability of collaboration in multi-robot systems often relies on the local information sharing and interaction among networked robot members through connected communication graph. As robots are often assumed to interact in a proximity-limited manner due to limited communication range [5]–[8], it is necessary to consider connectivity maintenance that ensures robots stay connected by constraining inter-robot distance while executing their original tasks.

In this paper, we are interested in the *Voronoi-based Multi-Robot Coverage* that have been widely applied to applications such as cooperative sensor coverage and environmental monitoring [2], [3], [9]–[11]. In such coverage task, a group of robots are deployed in an environment from given starting configurations and then seek for the final optimal placements such that the overall sensing/coverage performance over the environment from those particular locations is maximized, which is also known as the *Locational Optimization* problem [12]. Although the *Multi-Robot Coverage* problem [2] and its variants [3], [9], [11], [13] have been extensively studied with the optimal solutions of Centroidal Voronoi tessellation (CVT) [14], the results are often based on assumptions that the multi-robot network is always connected for sharing global or local information, which may not be applicable in real-world situations. Due to the limited communication range, the robotic sensor networks are very likely to get disconnected from the dispersing coverage movements and hence it is critical to address the connectivity constraints to ensure the desired task execution and performance.

The multi-robot coverage control with connectivity constraints is particularly challenging for existing work since (a) the connectivity maintenance with dispersing coverage motions introduces increased complexity for global connectivity control algorithms [6], [15], [16] due to the possible discontinuity from dynamic topology changes, as pointed out in [7], and (b) there are no theoretical guarantee on the optimality of imposed connectivity constraints, e.g. [5], [17]–[19] nor the guaranteed convergence in presence of the constraints, e.g. [6], [20]–[22]. Such issues could lead to overly conservative robots motion and thus inferior coverage performance, for example, dead locks that might prevent reaching towards the optimal coverage placements, and divergence of coverage behavior by the perturbation from connectivity control outputs. Hence, it is desired to derive an approach to maintain minimum satisfying connectivity so as to provide highest freedom for robots' original coverage controllers, while ensuring the convergence of the multi-robot coverage task.

The objective of this paper is thus to develop provably optimal connectivity controller for multi-robot Voronoi-based coverage. To formulate the connectivity constrained coverage control problem, we employ control barrier functions [17], [23] that characterize and enforce connectivity constraints over multi-robot controllers in an optimal way. However, the existing control barrier functions on connectivity requires either predefined fixed connectivity topology [5] or enumerating all possible combination of connectivity topology [17]. Such rigid constraints is not scalable nor feasible in as number of robots and behaviors increases. To that end, we propose to develop an optimal connectivity maintenance algorithm for coverage control with the following contributions: 1) a novel quantifiable relationship between original coverage controllers and the candidate connectivity constraints to invoke dynamic quantified minimum connectivity constraints that are *least violated* by the original unrevised coverage controllers, and 2) a connectivity-aware coverage controller with proof of convergence that minimally revises the robots' original coverage controllers in presence of activated connectivity constraints with guaranteed continuous

This work was funded by the DARPA Cooperative Agreement No.: HR00111820051, AFOSR awards FA9550-18-1-0097 and FA9550-15-1-0442.

The authors are with the Robotics Institute, Carnegie Mellon University, Pittsburgh, PA 15213, USA. Email: {wenhao, katia}@cs.cmu.edu.

improvement on the coverage performance.

II. Related Work

The general problem of connectivity maintenance has been widely studied in the past decade due to its importance in enabling local information sharing and collaboration for multi-robot systems in performing complex tasks. Given an initially connected multi-robot spatial communication graph, the goal of continual connectivity control is to couple the task-related controllers of robots with connectivity controller such that the communication graph over time remains connected. There have been two major classes of connectivity control methods: 1) *local methods* that seeks to preserve the initial connectivity graph topology over time [8], [24], [25], and 2) *global methods* that aims to preserve the global algebraic connectivity of the communication graph by deriving controllers to keep the second smallest eigenvalue of the graph Lapacian positive at all time [6], [7], [15], [16], [20]. While the global connectivity control provide better flexibility over local methods as it allows for changing network topology, neither of them is able to provide any formal guarantee over the primary task performance aside from connectivity maintenance. Meanwhile, both of the methodologies demands for the revision of original robot controllers more or less at all time, even if the robots' original behaviors won't lead to network disconnection. A recent work [26] proposed a bounded connectivity control framework to address both the algebraic connectivity maintenance and the achievement of the primary control objective with an input-to-state stability-like theoretical analysis. The overall control system was shown to approach a critical point of the potential energy function of the primary control objective under perturbation from global algebraic connectivity control, thus to ensure connectedness of the graph and the stability of the primary control objective. However, there is still no formal discussions on the optimality of the algebraic connectivity control perturbation over primary control objectives. This poses additional challenges when extending to dispersing behaviors from the multi-root coverage controllers that demands for minimum connectivity constraints to maximize the dispersing coverage performance.

To achieve more flexible connectivity control with flexible behaviors, i.e. simultaneously exploring different regions, recent work [21], [22], [27], [28] have explored the idea of redeploying a certain number of robots to act as communication relays, while aiming to allow the rest of the robots to perform their original tasks. In particular, the communication relays can be derived by following certain structured behaviors such as lattice-based formations [27], [28], or by separate optimization process that explicitly assigns some of the robots as connectors [21], [22]. In order to find a more flexible communication relay structure with quantified pairwise connectivity, [29] proposed to employ minimum spanning tree topology and uses pairwise distance as heuristic to provide better freedom of robot motion, i.e. robots closer to each other are less restrictive. However, these heuristic methods have no theoretical guarantee that the selected connectivity constraints are minimum to the original task-related robot controllers.

For less restrictive multi-robot control with constraint satisfaction, control barrier functions have been employed to encode a variety of inter-robot constraints and the resulting constrained control outputs lead to forward invariance of the satisfying set, i.e. robots remain collision free and connected under predefined fixed communication topology [5], [17], [18]. Although the resultant control outputs are optimized to stay as close to the original controllers with constraints, the predefined fixed communication topology has no guarantee regarding its optimality to the robot behaviors. In our work, we are optimizing both the activated connectivity constraints as well as the constrained coverage controllers with proven optimality and convergence guarantees, so that the control revision with the invoked connectivity constraints is minimally invasive to the original coverage controllers, allowing for continuous coverage improvement.

III. Problem Formulation

Consider a team of n robots moving in a bounded environment $Q \subset \mathbb{R}^2$ and assume the environment can be discretized into a set of point $q \in Q$, with the position of each robot $i \in \{1, 2, \dots, n\}$ denoted by $x_i \in Q$ with single integrator dynamics $\dot{x}_i = u_i \in \mathbb{R}^2$. We assume the environment is free of obstacles and can be partitioned into n Voronoi cells, as done in most multi-robot sensor coverage algorithm [2], [9], [11].

$$V_i = \{q \in Q \,|\, \|q - x_i\| \leq \|q - x_j\|, \forall j \neq i\} \tag{1}$$

where $\|\cdot\|$ is the l^2-norm. Each Voronoi cell V_i corresponds to its generator robot x_i who is the closest robot to any points $q \in V_i$, and hence each robot x_i will be responsible for sensing all assigned points $q \in V_i$.

A. Voronoi-based Coverge Control

The evaluation of sensing/coverage performance for each robot i depends on the value of each sensed point $q \in V_i$ as well as the sensing capability over those points. In particular, there exists a given density function $\phi(\cdot) : Q \to \mathbb{R}_+$ that maps each point $q \in Q$ to a scalar value specified by $\phi(q) \in \mathbb{R}_+$. On the other hand, the sensing capability usually degrades as the distance between the robot and the point to sense increases. To that end, the sensing cost function of multi-robot sensor coverage can be formally defined as follows [2], [9].

$$\mathcal{H}(x_1, \dots, x_n) = \sum_{i=1}^{n} \int_{q \in V_i} \|q - x_i\|^2 \phi(q) dq \tag{2}$$

Hence the lower $\mathcal{H}(x_1, \dots, x_n)$ the better. Then by taking the gradient of (2), we have the well-known local optimal solutions for minimizing $\mathcal{H}(\cdot)$ for all $i \in \{1, \dots, n\}$ as follows.

$$x_i^* = \arg\min \mathcal{H}(x_1, \dots, x_n) = \frac{\int_{V_i} q\phi(q)dq}{\int_{V_i} \phi(q)dq} = C_{V_i} \tag{3}$$

where $C_{V_i} \in \mathbb{R}^2$ is also referred to as the centroid of each Voronoi cell V_i. Although this critical point of $\mathcal{H}$ is a local minimum, due to the intractable solution (NP-hard) to the global optimum $\mathcal{H}$ the local optimal solution x_i^* is often considered optimal (see [9], [11]). The decentralized gradient-based move-to-centroid controller [2] has been proven to navigate the robots to the local optimal locations.

$$\dot{x}_i = \hat{u}_i = k_p(C_{V_i} - x_i) \tag{4}$$

where k_p is a user-defined control gain. Although such conventional decentralized controller $\hat{u}_i$ ensures the convergence to the local optimal configurations of the robots, it always relies on the underlying assumption that the robots can communicate to each other with unlimited communication range to exchange information and collaboratively construct the Voronoi partition in (1) (e.g. [3], [9]–[11], [30]). However, it is often the case that the robots' communication range is limited and thus the motion of robots should be restrained from the optimal controller $\hat{u}_i$ to accommodate the connectivity constraint.

To simplify the discussion, we assume the robots have the same known limited communication range $R_c \in \mathbb{R}_+$. Each robot can connect and communicate directly with other robots within its spatial proximity. The communication graph of the robotic team is defined as $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where each node $v \in \mathcal{V}$ represents a robot. If the spatial distance between robot $v_i \in \mathcal{V}$ and robot $v_j \in \mathcal{V}$ is less or equal to the communication radius R_c (i.e. $\|x_i - x_j\| \leq R_c$), then we assume the two can communicate and edge $(v_i, v_j) \in \mathcal{E}$ is undirected (i.e. $(v_i, v_j) \in \mathcal{E} \Leftrightarrow (v_j, v_i) \in \mathcal{E}$). It is also assumed the initial graph $\mathcal{G}$ at $t = 0$ is connected. To ensure successful multi-robot coordination and information exchange, it is required that the time-varying communication/connectivity graph $\mathcal{G}(t)$ stays connected at all time $\forall t > 0$.

In presence of such connectivity constraint as well as the physical constraints of the robots such as velocity limits, each robot i may have to modify their original move-to-centroid controller $\hat{u}_i$ in (4) to accommodate the constraints. To that end, the objective is to 1) coordinately invoke optimal set of active constraints to follow (particularly the connectivity constraints imposed between pair-wise robots), such that the modification to the primary controller $\hat{u}_i$ due to invoked connectivity constraints is minimum for the robotic team, and 2) compute the modified controllers for robots coverage with convergence guarantee. In the remaining of this section, we will discuss the formulation of the mentioned constraints in the form of Barrier Certificates on the controllers and will present the formalized optimization problem.

B. Connectivity Constraints using Barrier Certificates

First, we consider the pair-wise connectivity constraints among the robotic team with joint states $\mathbf{x} = \{x_1, \ldots, x_n\} \in \mathbb{R}^{2n}$. If the connectivity constraint is enforced between any pair-wise robots i and j, it is to ensure inter-robot distance not larger than communication range R_c, and we have the following condition.

$$\begin{aligned} h_{i,j}^c(\mathbf{x}) &= R_c^2 - \|x_i - x_j\|^2 \\ \mathcal{H}_{i,j}^c &= \{\mathbf{x} \in \mathbb{R}^{2n} : h_{i,j}^c(\mathbf{x}) \geq 0\} \end{aligned} \tag{5}$$

where the set of $\mathcal{H}_{i,j}^c$ indicates the feasible set on $\mathbf{x}$ from which robot i and j will never lose connectivity. Consider any connectivity graph $\mathcal{G}^c = (\mathcal{V}, \mathcal{E}^c) \subset \mathcal{G}$ to enforce, the corresponding constrained set can be composed as follows.

$$\mathcal{H}^c(\mathcal{G}^c) = \bigcap_{\{v_i, v_j \in \mathcal{V} : (v_i, v_j) \in \mathcal{E}^c\}} \mathcal{H}_{i,j}^c \tag{6}$$

In [17], the following pair-wise connectivity barrier certificates $\mathcal{B}^c(\mathbf{x}, \mathcal{G}^c)$ have been proposed to map the constrained connectivity set (6) of $\mathbf{x}$ to the admissible joint control space $\mathbf{u} \in \mathbb{R}^{2n}$. The result is summarized as follows.

$$\mathcal{B}^c(\mathbf{x}, \mathcal{G}^c) = \{\mathbf{u} \in \mathbb{R}^{2n} : \dot{h}_{i,j}^c(\mathbf{x}) + \gamma h_{i,j}^c(\mathbf{x}) \geq 0, \forall (v_i, v_j) \in \mathcal{E}^c\} \tag{7}$$

where γ is a user-defined parameter to confine the available sets. It has been proven in [17], [23] that the forward invariance of the satisfying set $\mathcal{H}^c$ is ensured as long as the joint control input $\mathbf{u}$ stays in set $\mathcal{B}^c(\mathbf{x}, \mathcal{G}^c)$. In other words, the robots will always stay connected if they are initially connected and the control input lies in the set $\mathcal{B}^c(\mathbf{x}, \mathcal{G}^c)$. Note that at any time point t with known current robot states $\mathbf{x}(t)$, the constrained control space in (7) corresponds to a class of linear constraints over pair-wise control inputs u_i and u_j for $\forall i > j$ under a given current connectivity graph $\mathcal{G}^c = (\mathcal{V}, \mathcal{E}^c) \subset \mathcal{G}$ to enforce. As there are many candidate $\mathcal{G}^c = (\mathcal{V}, \mathcal{E}^c) \subset \mathcal{G}$ that define various constrained control sets (7), it is desired to obtain an optimal $\mathcal{G}^c$ such that the primary controllers in (4) are least constrained from the invoked $\mathcal{B}^c(\mathbf{x}, \mathcal{G}^c)$ by $\mathcal{G}^c$. The overall optimization problem is formally defined in the following subsection.

C. Objective Function

Given the original primary coverage control input $\hat{u}_i$ from (4) for any robot i, the robotic team needs to determine whether and how to best modify each $\hat{u}_i$ in a minimally invasive manner so as to achieve the improved coverage performance *at best* while ensuring connectivity. With the defined forms of constraints in (7), we formally define the overall optimization problem as follows at each time step t.

$$\mathbf{u}^* = \underset{\mathcal{G}^c, \mathbf{u}}{\arg\min} \sum_{i=1}^{n} \|u_i - \hat{u}_i\|^2 \tag{8}$$

$$\text{s.t.} \quad \mathcal{G}^c = (\mathcal{V}, \mathcal{E}^c) \subseteq \mathcal{G} \quad \text{is connected} \tag{9}$$

$$\mathbf{u} \in \mathcal{B}^c(\mathbf{x}, \mathcal{G}^c), \quad \|u_i\| \leq \alpha_i, \forall i = 1, \ldots, n \tag{10}$$

$$\dot{\mathcal{H}}(\mathbf{x}) \leq 0 \tag{11}$$

The above Quadratic Programming (QP) optimization problem is to find the optimal active connectivity spanning subgraph $\mathcal{G}^c$ from current connected multi-robot connectivity graph $\mathcal{G}$ and the alternative joint control inputs $\mathbf{u}^* \in \mathbb{R}^{2n}$ bounded by maximum velocity α_i, such that the connectivity and velocity constraints described in (9) and (10) are satisfied while ensuring minimally invasive to the primary controller as shown in (8) with guaranteed coverage performance improvement (11). Most of the works on the global connectivity maintenance [6]–[8], [16], [17] are intractable to apply to our problem in (8) due to i) the lack of optimality from the invoked connectivity constraints, and ii) no guarantee on the convergence nor the optimality of the synthesized connectivity controller w.r.t. the coverage performance as demanded by (11).

In this paper, we propose to decouple the problem of coverage control with connectivity maintenance (8) into two dependent sub-problems, namely 1) to select provable optimal connected subgraph $\mathcal{G}^c = \mathcal{G}^{c*} \subseteq \mathcal{G}$ that invokes minimally invasive connectivity constraints over the original coverage controller (4), and then 2) to solve the optimization problem (8) with the obtained optimal connectivity subgraph $\mathcal{G}^c = \mathcal{G}^{c*}$. Such a solution could provide best flexibility for the controllers under the provably minimum connectivity constraints so as to achieve improved coverage performance at best.

IV. Coverage Control while Maintaining Minimum Connectivity

A. Minimum Connectivity Constraint Spanning Tree (MCCST)

First we consider the sub-problem of selecting optimal connectivity subgraph $\mathcal{G}^c = \mathcal{G}^{c*}(\mathcal{V}, \mathcal{E}^*) \subseteq \mathcal{G}$ in (8) that introduces minimum connectivity constraints in (9). As each edge $(v_i, v_j) \in \mathcal{E}^c$ in a candidate subgraph $\mathcal{G}^c$ enforces one pair-wise linear constraint over primary control inputs $\hat{u}_i$ and $\hat{u}_j$ for robot i and j as shown in (7), the graph $\mathcal{G}^c$ whose edges define the minimum connectivity constraints must exist among the set of all spanning trees $\mathcal{T}^c$ from current connectivity graph $\mathcal{G}$ that have all the vertices $\mathcal{V}$ covered with $n-1$ edges, which is the least number of edges for keeping all vertices (robots) in $\mathcal{V}$ connected.

Thus, the problem boils down to finding the optimal spanning tree $\mathcal{T}^{c*} = \mathcal{G}^{c*} \subseteq \mathcal{G}$ whose edges invoke the minimum connectivity constraints in the form of (7) over the robots' controllers. To quantify the strength of connectivity constraint by an edge $(v_i, v_j) \in \mathcal{E}$ in the original graph $\mathcal{G}$, we introduce the following weight assignment derived from (7).

$$w_{i,j} = \dot{h}^c_{i,j}(\mathbf{x}, \hat{u}_i, \hat{u}_j) + \gamma h^c_{i,j}(\mathbf{x}), \forall (v_i, v_j) \in \mathcal{E} \tag{12}$$

where $w_{i,j}$ indicates the violation of the pair-wise connectivity constraint between the two robots, with the higher value of $w_{i,j}$ the less violated the connectivity constraint. To that end, each candidate spanning tree $\mathcal{T}^c \subseteq \mathcal{G}$ is redefined as a weighted spanning tree $\mathcal{T}^c_w = (\mathcal{V}, \mathcal{E}^T, \mathcal{W}^T)$. Hence the optimal connectivity graph $\mathcal{G}^{c*}$ with constraints in (9) can be obtained as follows.

$$\mathcal{G}^{c*} = \arg\max_{\mathcal{T}^c_w} \sum_{(v_i,v_j)\in\mathcal{E}^T} w_{i,j} = \arg\min_{\mathcal{T}^c_w} \sum_{(v_i,v_j)\in\mathcal{E}^T} -w_{i,j} \tag{13}$$

The optimal solution to (13) corresponds to the Minimal Spanning Tree (MST) among the set of $\mathcal{T}^c_w \subseteq \mathcal{G}$ by definition, namely, the MST whose invoked connectivity constraints from its edges are least violated by and minimally invasive to the current original joint coverage controllers $\hat{\mathbf{u}} = \{\hat{u}_i, \dots, \hat{u}_n\}$, implying the highest freedom for revising the controllers to achieve improved coverage performance in presence of connectivity constraint at current time step. We call such weighted MST as the Minimum Connectivity Constraint Spanning Tree (MCCST). The computation of MCCST in (13) can be addressed by MST approaches such as [31] in centralized or decentralized manner.

B. Connectivity-aware Coverage Controller with MCCST

Given the derived MCCST $\mathcal{G}^{c*}$ from (13) that specifies the optimal connectivity constraints in (9) and (10), the resulting constrained controllers will ensure the multi-robot network to stay connected at all time due to the forward invariance property of the barrier certificates $\mathcal{B}^c(\mathbf{x}, \mathcal{G}^{c*})$. On the other hand, as the primary goal is to minimize the coverage cost defined in (2), the resulting controllers should also satisfy the coverage improvement constraint described in (11).

To that end, we propose an improvement barrier certificate $\mathcal{B}^d(\hat{\mathbf{u}})$ defined as follows to ensure the revised constrained controller will always seek to improve the coverage performance, namely, the constraint of $\dot{\mathcal{H}}(\mathbf{x}) \leq 0$ in (11) holds true at all time.

$$\mathcal{B}^d(\hat{\mathbf{u}}) = \{\mathbf{u} \in \mathbb{R}^{2n} : \mathbf{u} \cdot \hat{\mathbf{u}} \geq 0\} \tag{14}$$

Intuitively, the dot product of $\mathbf{u} \cdot \hat{\mathbf{u}} = 0$ defines a hyperplane $\hat{\mathbf{u}}^\perp \subset \mathbb{R}^{2n}$ spanned by all vectors perpendicular to the vector of $\hat{\mathbf{u}} = \{\hat{u}_i, \dots, \hat{u}_n\}$ and passing through the origin in the joint control input space (also intersect with the vector $\hat{\mathbf{u}}$ at the origin). Hence the barrier certificate constraint in (14) ensures that the satisfying controller $\mathbf{u}$ has the positive projection component along the original controller $\hat{\mathbf{u}}$ in (4), indicating the guaranteed decrease of the resulting sensing cost $\mathcal{H}(\cdot)$ in (2) and hence the improved coverage performance.

With the obtained specifications of optimal connectivity constraint and improvement constraint in (9)-(11) w.r.t. control inputs $\mathbf{u}$, we thereby re-write the optimization problem (8) at each time step as follows with the new forms of the constraints from (13) and (14).

$$\mathbf{u}^* = \arg\min_{\mathcal{G}^c, \mathbf{u}} \sum_{i=1}^{n} \|u_i - \hat{u}_i\|^2 \tag{15}$$

$$\text{s.t.} \quad \mathbf{u} \in \mathcal{B}^c(\mathbf{x}, \mathcal{G}^{c*}) \bigcap \mathcal{B}^d(\hat{\mathbf{u}}), \quad \|u_i\| \leq \alpha_i, \forall i = 1, \dots, n \tag{16}$$

This yields our proposed connectivity-aware coverage controller with minimum connectivity maintenance guarantee that is minimally invasive to the original coverage controller (4). Note that the constraints from the composition of the two barrier certificates $\mathcal{B}^c(\mathbf{x}, \mathcal{G}^{c*}) \bigcap \mathcal{B}^d(\hat{\mathbf{u}})$ simply implies a class of linear constraints over $\mathbf{u}$ and hence could be solved very efficiently in real-time using standard quadratic programming (QP) solver. It can also be further decentralized with distributed barrier certificates computation as done in [18], [23].

In particular, the convergence of our proposed connectivity-aware coverage controller (15) can be formalized in the following theorem.

Theorem 1. *Using the proposed connectivity-aware coverage controller (15), the robots will either converge to the centroids of their Voronoi cells,*

$$\lim_{t\to\infty} ||x_i(t) - C_{V_i}|| = 0 \quad \forall \quad i = 1, \ldots, n \tag{17}$$

or converge to (stop at) certain configurations where the sensing cost function cannot be further minimized due to constraints.

Proof: Consider the sensing cost function $\mathcal{H}(\cdot)$ in (2) as a Lyapunov-like function. By taking the time derivative of $\mathcal{H}$, we have

$$\dot{\mathcal{H}} = -2\sum_{i=1}^{n}\int_{q\in V_i}(q - x_i)\phi(q)\dot{x}_i dq \tag{18}$$

If the constraints in (15) are not active given the original controller $\hat{\mathbf{u}}$, namely, $\hat{\mathbf{u}}$ already satisfies all the constraints, then for each robot i we have $\dot{x}_i = u_i^* = \hat{u}_i$ and thus

$$\begin{aligned}\dot{\mathcal{H}} &= -2\sum_{i=1}^{n}\int_{q\in V_i}(q - x_i)\phi(q)k_p(C_{V_i} - x_i)dq \\ &= -2\sum_{i=1}^{n}k_p\int_{q\in V_i}\phi(q)dq||C_{V_i} - x_i||^2 \le 0\end{aligned} \tag{19}$$

With LaSalle's Invariance Principle [32] it is straightforward that the robots will converge to the invariante set where $\dot{\mathcal{H}} = 0$ and $x_i \to C_{V_i}$, $\forall i = 1, \ldots, n$.

On the other hand, if the original controller $\hat{\mathbf{u}}$ violates constraints in (16), it will be revised to $\dot{\mathbf{x}} = \mathbf{u}^* \neq \hat{\mathbf{u}} \in \mathbb{R}^{2n}$ which is projected from $\hat{\mathbf{u}}$ to the hyperplanes described by the constraints (16) in the joint control input space. In particular, the condition of $\dot{\mathcal{H}} < 0$ will always hold when the projected solution $\mathbf{u}^*$ does not lie on the hyperplane of $\mathcal{B}^d(\hat{\mathbf{u}})$. On the other hand, if $\mathbf{u}^*$ become the projection of $\hat{\mathbf{u}}$ to the hyperplane of $\mathcal{B}^d(\hat{\mathbf{u}})$, it is guaranteed that $\mathbf{u}^* \to 0$ and $\dot{\mathcal{H}} \to 0$ as the hyperplane of $\mathcal{B}^d(\hat{\mathbf{u}})$ and vector $\hat{\mathbf{u}}$ are orthogonal at the origin in the joint control input space by definition (14). To that end, when not able to converge to the centroids of Voronoi cells (17), the robots will asymptomatically converge to zero velocity at certain configurations where $\dot{\mathcal{H}} \to 0$, which concludes the proof.

V. Results

To evaluate our proposed connectivity-aware coverage controller (15), we designed experiments in Matlab simulation with $n = 8$ robots moving in a rectangular environment Q with the uniform density function $\phi(q)$. Each robot has a limited communication range of $R_c = 0.3$m and the same control gain $k_p = 0.2$ for the original coverage controller (4).

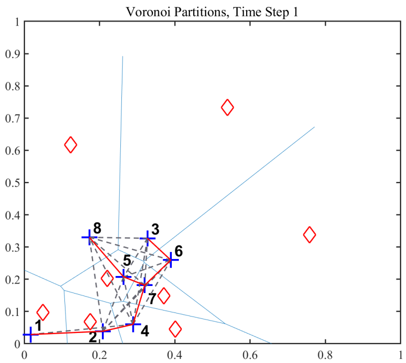
Figure 1a shows the initial robots' positions around the left-bottom corner and the current centroids of Voronoi cells. While many connectivity edges exists due to the cluttered robots distribution, our algorithm could derive a unique MCCST (red) out from the grey connected graph to invoke minimum connectivity constraints to enforce for our connectivity-aware coverage controller. Figure 1b shows an intermediate configurations of the robots at time step= 15. It is observed the robots (blue) already spread out while still maintaining connectivity. The converged configurations in Figure 1c indicates the robots under our controllers (15) managed to approach the optimal locations (i.e. centroids of Voronoi cells) at best but still stay connected at all time.

In comparison, we present converged results from another two controllers: i) move-to-centroid coverage controller (4) without considering connectivity constraints (Figure 1d), and ii) coverage controller with initial connectivity constraint which was built with our connectivity-aware controller but with connectivity constraints from the initial connectivity graph to simulate the results from local connectivity control methods such as [8], [25] (Figure 1e). Although the move-to-centroid controller could converge to the optimal configurations, the robot team has already been disconnected and in real-world case they would not have been able to communicate and collaboratively construct the dynamic Voronoi partition for rendering the move-to-centroid controllers. For the coverage controller with initial connectivity graph, the rigidness of the graph lead to redundant and overly restrictive connectivity constraints that keep the robot team from approaching to the optimal locations. We also demonstrate the quantitative results over time in Figure 2, showing our connectivity-aware coverage controller could lead to suboptimal solutions (Figure 2a) while being able to ensure minimum connectivity (Figure 2b) that yield minimal revision to the original move-to-centroid controller (Figure 2c).

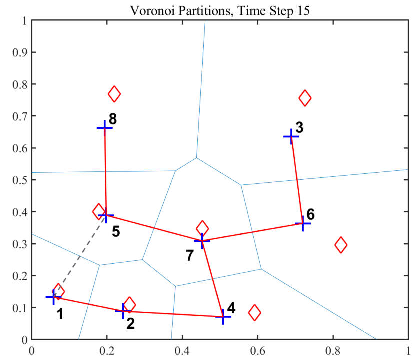
VI. Conclusion

In this paper, we consider the problem of Voronoi coverage control in multi-robot sensor coverage with connectivity maintenance. In particular, we proposed a novel connectivity-aware coverage controller to optimize coverage performance while preserving minimum connectivity, which leads to minimal revision to the original coverage controller. This allows for maximizing the coverage performance at best in presence of connectivity constraints from dynamic and possibly discontinuous communication topology. The convergence of the proposed controller is proved to ensure sub-optimality and continuous improvement of coverage performance at all time. Experimental results validate our method in comparison to other conventional methods. The proposed connectivity-aware coverage controller is flexible and general to other variants of Voronoi-based coverage problem as well to achieve sub-optimal performance while preserving minimum connectivity.

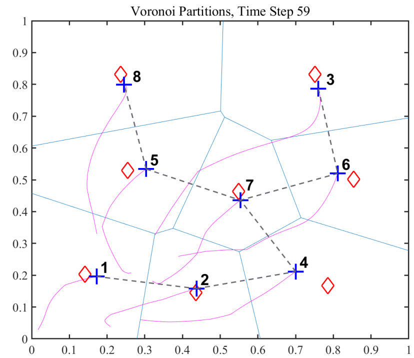
Future work includes extensions to fully decentralized connectivity-aware coverage control and prove its performance in other variants of Voronoi coverage problem. We will also implement the algorithms on physical robotic platforms to incorporate uncertainties in communication and control in real-world applications.



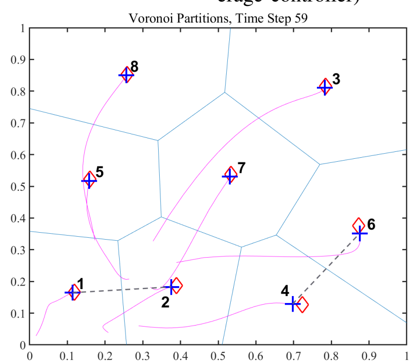
(a) Time Step = 1



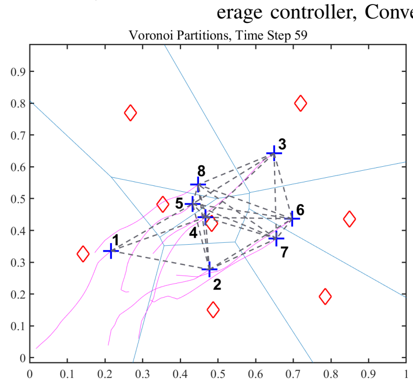
(b) Time Step = 15 (connectivity-aware coverage controller)



(c) Time Step = 59 (connectivity-aware coverage controller, Converged)

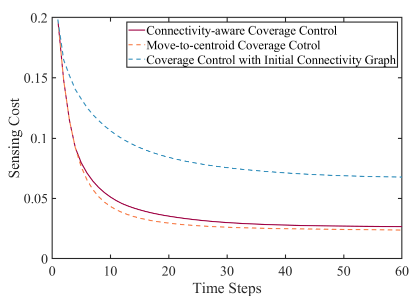


(d) Time Step = 59 (Move-to-centroid coverage controller, Converged)

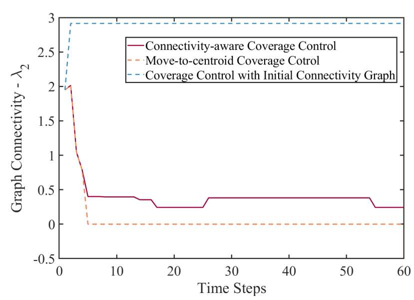


(e) Time Step = 59 (coverage controller with initial connectivity graph, Converged)

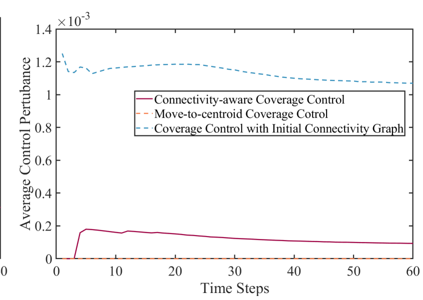
Fig. 1: Simulation example of 8 robots (blue plus sign) in coverage task with Voronoi boundary (blue lines) and centroids of Voronoi cell (red diamonds). Grey dashed lines in (a)-(e) denote current connectivity edges and red lines in (a),(b) denote current active MCCST invoking pair-wise connectivity constraints. Magenta curves in (c)-(e) denotes the whole trajectories of each robot. Compared to convergence result of (d) move-to-centroid controller (4) without considering connectivity, and (e) coverage control preserving initial connectivity graph, our connectivity-aware coverage control with MCCST drive the robots towards centroids of Voronoi at best while preserving minimum connectivity constraint.



(a) Sensing cost



(b) Algebraic graph connectivity



(c) Average control perturbation

Fig. 2: Performance comparison of simulation example in Figure 1 w.r.t. different metrics: (a) sensing cost computed from $\mathcal{H}$ in (2), (b) algebraic graph connectivity evaluated by second smallest eigenvalue of laplacian matrix of multi-robot connected network. Positive meaning connectivity ensured and the larger the stronger. (c) control perturbation computed by $\frac{1}{n}\sum_{i=1}^{n}\left\|u_i^* - \hat{u}_i\right\|^2$ (the smaller the better).

REFERENCES

[1] G. Kantor, S. Singh, R. Peterson, D. Rus, A. Das, V. Kumar, G. Pereira, and J. Spletzer, "Distributed search and rescue with robot and sensor teams," in *Field and Service Robotics*. Springer, 2003, pp. 529–538.

[2] J. Cortes, S. Martinez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks," *IEEE Transactions on robotics and Automation*, vol. 20, no. 2, pp. 243–255, 2004.

[3] W. Luo and K. Sycara, "Adaptive sampling and online learning in multi-robot sensor coverage with mixture of gaussian processes," in *2018 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2018, pp. 6359–6364.

[4] W. Burgard, M. Moors, C. Stachniss, and F. E. Schneider, "Coordinated multi-robot exploration," *IEEE Transactions on robotics*, vol. 21, no. 3, pp. 376–386, 2005.

[5] A. Li, L. Wang, P. Pierpaoli, and M. Egerstedt, "Formally correct composition of coordinated behaviors using control barrier certificates," in *Intelligent Robots and Systems (IROS), IEEE/RSJ International Conference on*. IEEE, 2018.

[6] L. Sabattini, N. Chopra, and C. Secchi, "Decentralized connectivity maintenance for cooperative control of mobile robotic systems," *The International Journal of Robotics Research*, vol. 32, no. 12, pp. 1411–1423, 2013.

[7] R. K. Williams, A. Gasparri, G. S. Sukhatme, and G. Ulivi, "Global connectivity control for spatially interacting multi-robot systems with unicycle kinematics," in *Robotics and Automation (ICRA), 2015 IEEE International Conference on*. IEEE, 2015, pp. 1255–1261.

[8] M. M. Zavlanos, A. Jadbabaie, and G. J. Pappas, "Flocking while preserving network connectivity," in *Decision and Control, 2007 46th IEEE Conference on*. IEEE, 2007, pp. 2919–2924.

[9] J. Cortés, “Coverage optimization and spatial load balancing by robotic sensor networks,” *IEEE Transactions on Automatic Control*, vol. 55, no. 3, pp. 749–754, 2010.

[10] M. Schwager, M. P. Vitus, S. Powers, D. Rus, and C. J. Tomlin, “Robust adaptive coverage control for robotic sensor networks,” *IEEE Transactions on Control of Network Systems*, vol. 4, no. 3, pp. 462–476, 2017.

[11] A. Pierson, L. C. Figueiredo, L. C. Pimenta, and M. Schwager, “Adapting to sensing and actuation variations in multi-robot coverage,” *The International Journal of Robotics Research*, vol. 36, no. 3, pp. 337–354, 2017.

[12] A. Okabe and A. Suzuki, “Locational optimization problems solved through voronoi diagrams,” *European Journal of Operational Research*, vol. 98, no. 3, pp. 445–456, 1997.

[13] M. Schwager, D. Rus, and J.-J. Slotine, “Decentralized, adaptive coverage control for networked robots,” *The International Journal of Robotics Research*, vol. 28, no. 3, pp. 357–375, 2009.

[14] Q. Du, M. Emelianenko, and L. Ju, “Convergence of the lloyd algorithm for computing centroidal voronoi tessellations,” *SIAM journal on numerical analysis*, vol. 44, no. 1, pp. 102–119, 2006.

[15] L. Sabattini, C. Secchi, N. Chopra, and A. Gasparri, “Distributed control of multirobot systems with global connectivity maintenance,” *IEEE Transactions on Robotics*, vol. 29, no. 5, pp. 1326–1332, 2013.

[16] P. Yang, R. A. Freeman, G. J. Gordon, K. M. Lynch, S. S. Srinivasa, and R. Sukthankar, “Decentralized estimation and control of graph connectivity for mobile sensor networks,” *Automatica*, vol. 46, no. 2, pp. 390–396, 2010.

[17] L. Wang, A. D. Ames, and M. Egerstedt, “Multi-objective compositions for collision-free connectivity maintenance in teams of mobile robots,” in *Decision and Control (CDC), 2016 IEEE 55th Conference on*. IEEE, 2016, pp. 2659–2664.

[18] ——, “Safe certificate-based maneuvers for teams of quadrotors using differential flatness,” *arXiv preprint arXiv:1702.01075*, 2017.

[19] M. M. Zavlanos, M. B. Egerstedt, and G. J. Pappas, “Graph-theoretic connectivity control of mobile robot networks,” *Proceedings of the IEEE*, vol. 99, no. 9, pp. 1525–1540, 2011.

[20] P. R. Giordano, A. Franchi, C. Secchi, and H. H. Bülthoff, “Bilateral teleoperation of groups of uavs with decentralized connectivity maintenance.” in *Robotics: Science and Systems*. Citeseer, 2011.

[21] J. Banfi, N. Basilico, and S. Carpin, “Optimal redeployment of multirobot teams for communication maintenance,” in *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2018, pp. 3757–3764.

[22] N. Majcherczyk, A. Jayabalan, G. Beltrame, and C. Pinciroli, “Decentralized connectivity-preserving deployment of large-scale robot swarms,” *arXiv preprint arXiv:1806.00150*, 2018.

[23] U. Borrmann, L. Wang, A. D. Ames, and M. B. Egerstedt, “Control barrier certificates for safe swarm behavior.” Georgia Institute of Technology, 2015.

[24] D. V. Dimarogonas and K. H. Johansson, “Decentralized connectivity maintenance in mobile networks with bounded inputs,” in *Robotics and Automation, 2008. ICRA 2008. IEEE International Conference on*. IEEE, 2008, pp. 1507–1512.

[25] M. Ji and M. Egerstedt, “Distributed coordination control of multiagent systems while preserving connectedness,” *IEEE Transactions on Robotics*, vol. 23, no. 4, pp. 693–703, 2007.

[26] A. Gasparri, L. Sabattini, and G. Ulivi, “Bounded control law for global connectivity maintenance in cooperative multirobot systems,” *IEEE Transactions on Robotics*, vol. 33, no. 3, pp. 700–717, 2017.

[27] M. Chandarana, W. Luo, M. Lewis, K. Sycara, and S. Scherer, “Decentralized method for sub-swarm deployment and rejoining,” in *2018 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*. IEEE, 2018, pp. 1209–1214.

[28] D. Krupke, M. Ernestus, M. Hemmer, and S. P. Fekete, “Distributed cohesive control for robot swarms: Maintaining good connectivity in the presence of exterior forces,” in *Intelligent Robots and Systems (IROS), 2015 IEEE/RSJ International Conference on*. IEEE, 2015, pp. 413–420.

[29] R. Aragues, C. Sagues, and Y. Mezouar, “Triggered minimum spanning tree for distributed coverage with connectivity maintenance,” in *Control Conference (ECC), 2014 European*. IEEE, 2014, pp. 1881–1887.

[30] S. Kemna, J. G. Rogers, C. Nieto-Granda, S. Young, and G. S. Sukhatme, “Multi-robot coordination through dynamic voronoi partitioning for informative adaptive sampling in communication-constrained environments,” in *ICRA*. IEEE, 2017, pp. 2124–2130.

[31] G. Pandurangan, P. Robinson, and M. Scquizzato, “A time-and message-optimal distributed algorithm for minimum spanning trees,” in *Proceedings of the 49th Annual ACM SIGACT Symposium on Theory of Computing*. ACM, 2017, pp. 743–756.

[32] F. Bullo, J. Cortes, and S. Martinez, *Distributed control of robotic networks: a mathematical approach to motion coordination algorithms*. Princeton University Press, 2009, vol. 27.