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DEXTERITY OF UNDERACTUATED MANIPULATORS

Marcel Bergerman **Yangsheng Xu**

The Robotics Institute
Carnegie Mellon University
Pittsburgh PA 15213

1 Abstract

The dexterity of a manipulator with passive joints, also known as an underactuated manipulator, differs from the dexterity of a fully actuated one, even if their mechanical structures are identical. Therefore, for fault tolerance and energy saving purposes, it is important to study the dexterity of underactuated manipulators. The purpose of this work is to quantify the dexterity of underactuated robot manipulators equipped with active and passive joints. We assume that the passive joints are locked at an arbitrary known position, and we compare the dexterity of the underactuated manipulator with that of a fully actuated redundant manipulator with identical structure. Based on this comparison we propose an optimization index to find the angles at which the passive joints should be locked to maximize the dexterity of the underactuated manipulator. We discuss three important dexterity measures: workspace volume, reachability, and manipulability. Examples obtained with a 3-link planar manipulator with an arbitrarily located passive joint are presented.
Keywords: underactuated manipulators, fault tolerance, dexterity measure

2 Introduction

We propose, in this work, to quantify the kinematic dexterity of serial link, rigid robot manipulators with passive joints. Most often, a passive joint results from a failure on a joint's actuator, control system, or other components. A passive joint may also be an inherent part of the manipulator, such as when a hyper-redundant manipulator is designed with more links than actuators for the sake of energy saving, cost reduction, or weight reduction. To study the kinematic dexterity of such mechanisms, we assume that the passive joints are locked, while the remaining (active) ones can move freely. If a passive joint arises from a joint failure, we assume that it is locked at the position where it failed by a high-level fault-detection system. If it arises from the manipulator's design, we assume that it is kept locked by a high-level decision system. We assume that the positions at which the passive joints are locked are known, either because their position sensors still work reliably, or because other sensors (e.g., an overhead camera) are available.

Study of the kinematic dexterity of underactuated manipulators is important from both a fault-tolerance and an energy saving/cost reduction points of view. Nonetheless, it is a topic to which not much attention was devoted in the past. Early work by Pradeep et al. [7] analyzed the capabilities of three different commercial manipulators with a failed joint. The analysis is qualitative in nature and is not valid for a general manipulator. More recently, Roberts [8] and Roberts and Maciejewski [9] discussed the decrease in a manipulator's kinematic manipulability index and worst-case dexterity due to the failure and locking of one or more of its joints. In these works the authors were concerned with finding pre-failure optimal configurations of the manipulator that guarantee that the post-failure manipulability is maximized.

We address the dexterity of underactuated manipulators in two unexplored fronts: first, we quantify the kinematic dexterity of an underactuated manipulator with an arbitrary number of locked passive joints, and compare it to the dexterity of a fully actuated redundant manipulator with identical structure. Second, we propose an optimization index that allows one to determine the positions at which the passive joints should be locked for the sake of maximizing the underactuated manipulator's dexterity. This proposition makes sense since several researchers have shown in the past how to control the positions of a manipulator's passive joints to any desired set-point [1], [2], [3], [4], [5]. Because the optimization is done with respect to a few variables only (the locking positions of the passive joints), analytical methods or an exhaustive search yield quickly the global optimal solution. Among the several dexterity measures proposed in the literature, we consider two important ones, and propose a third one: workspace volume, reachability, and the manipulability index [10].

To illustrate the proposed method we study in detail a 3-link planar redundant manipulator, and show how the various measures of dexterity vary with different locations of the passive joints.

3 Dexterity analysis and optimization

Let $D(q)$ represent the dexterity measure of an n-link redundant fully actuated manipulator locally evaluated at the configuration q. Suppose now that joints $p_1, \ldots, p_{n_p}$ (with $n_p < n$), are passive joints locked at an arbitrary position within their joint limits. For convenience we represent this set of joints by I_p, and the vectors of active and passive joints by q_a and q_p, respectively. We represent by $D_{I_p}(q)$ the dexterity of the resulting underactuated manipulator at the configuration q. We define the *relative dexterity loss* is defined as:

$$\tilde{D}_{I_p}(q) = 1 - \frac{D_{I_p}(q)}{D(q)} \tag{1}$$

The relative dexterity loss measures the decrease in dexterity of the fully actuated manipulator when one or more of its joints fail or are intentionally designed as passive joints. It allows one to ensure that the underactuated manipulator's dexterity is sufficient for completion of the manipulator's assigned task.

Ideally, one would wish that the relative dexterity loss be as small as possible, as this would imply that the presence of passive joints reduces little the manipulator's dexterity. In practice, however, this does not usually happen. One might be interested, then, on determining the positions at which the passive joints should be locked for the sake of minimizing the relative dexterity loss. This problem can be cast as an optimization problem, with the optimization index given by the value of the relative dexterity loss:

$$q_p^* = \arg\min[J_D(q)] = \arg\min[\tilde{D}_{I_p}(q)] \tag{2}$$

Global minimization of the optimization index $J_D(q)$ with respect to the positions of the passive joints yields the global minimum value of the underactuated manipulator's relative dexterity loss. At times, the relative dexterity loss may depend on the positions of the active as well as of the passive joints. Consequently, minimization of J_D with respect to q_p only is an ill-defined problem. One possible solution is to average the optimization function with respect to the positions of the active joints, and then minimize the resulting averaged optimization function, which is a function of q_p only:

$$\bar{J}_D(q_p) = \frac{\int J_D(q)\,dq_a}{\int dq_a} \tag{3}$$

4 Workspace analysis

Being able to reach a large number of points in Cartesian space is usually a design criterion and a desirable characteristic of a robot manipulator. When the manipulator has both active and passive joints, it is important to guarantee that the number of reachable points (which compose the reachable workspace) is as large as possible. Denote by V the volume or area of a fully actuated manipulator's workspace, and by V_{I_p} that of the corresponding underactuated manipulator. While V is a fixed quantity, V_{I_p} depends on the positions where the passive joints are locked. The *relative workspace loss* is defined as:

$$\tilde{V}_{I_p}(q_p) = 1 - \frac{V_{I_p}(q_p)}{V} \tag{4}$$

Clearly, $V \neq 0$ and the relative workspace loss varies between 0 and 1. If it is equal to zero, the presence of passive joints does not incur reduction of the manipulator's workspace; the closer it gets to 1, the smaller the volume of the underactuated manipulator's workspace. The relative workspace loss index was mentioned in [7], however, in that work the authors did not present a quantitative analysis of it.

To compute the angle at which joints q_p should be locked for relative workspace loss minimization we minimize the following optimization function:

$$J_{V_{I_p}}(q_p) = 1 - \frac{V_{I_p}(q_p)}{V} \tag{5}$$

Example 1 Consider a 3-link planar rotary manipulator with link lengths $l_1 = l_2 = l_3 = 0.3$ m, as shown in Figure 1. We assume without loss of generality that all joint limits are equal to $[-\pi, \pi]$; different joint limits can be easily considered by simply limiting the search for the global minimum of J_V to the angles between the new joint limits. The manipulator's original workspace has an area equal to:

$$V = \pi(l_1 + l_2 + l_3)^2 \mathrm{m}^2 = 0.81\pi\ \mathrm{m}^2$$

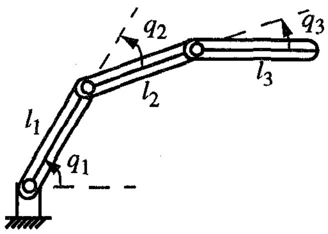


Figure 1: Three-link planar rotary manipulator.

If the manipulator is equipped with one passive joint at joint p, which is kept locked during, e.g., manipulation tasks, the manipulator is reduced in practice to a 2-link underactuated mechanism with a reduced workspace. The underactuated manipulator workspace's is an annulus with inner and outer radii $R_i(q_p)$ and $R_o(q_p)$, respectively. Its area is equal to:

$$V_p(q_p) = \pi[R_o^2(q_p) - R_i^2(q_p)]$$

Table 1 presents the expressions for R_i, R_o, and V_p for $p = 1, 2, 3$. The quantity l_{12} is the distance between the first and third joints and l_{23} is the distance between the second joint and the end-effector:

$$l_{12} = \sqrt{l_1^2 + l_2^2 + 2l_1 l_2 \cos(q_2)}, \quad l_{23} = \sqrt{l_2^2 + l_3^2 + 2l_2 l_3 \cos(q_3)}$$

Table 1: Boundary radii and workspace area for all possible locations of one passive joint on a 3-link planar manipulator.

passive joint	$R_i(q_p)$	$R_o(q_p)$	$V_p(q_p)$
1	$\lvert l_2 - l_3 \rvert$	$l_2 + l_3$	$4\pi l_2 l_3$
2	$\lvert l_{12} - l_3 \rvert$	$l_{12} + l_3$	$4\pi l_{12} l_3$
3	$\lvert l_1 - l_{23} \rvert$	$l_1 + l_{23}$	$4\pi l_1 l_{23}$

Figure 2 presents the value of the relative workspace loss for all possible locations of the passive joint, as the passive joint's angle varies from $-\pi$ to π. Figure 3 presents the original workspace V (a circle with radius 0.9 m) and the workspace V_2 when joint 2 is passive and is locked at a few selected angles. We conclude from these figures that whenever joints 2 or 3 are passive, they must be locked at 0 rd if the underactuated manipulator's workspace is to be maximized. One can also conclude from Figure 2 that the optimal value of the workspace loss is only about 12%, which indicates that this underactuated manipulator retains 88% of the dexterity of the corresponding fully actuated one even with a passive joint. Additionally, the closer to π rd joints 2 or 3 are locked, the smaller the workspace, which vanishes at that angle (see also the lower right graph in Figure 3, which shows that, when q_2 is locked at π rd, the workspace is a circumference with zero area). On the other hand, when joint 1 is the passive one, the workspace area is constant, independent of the locking angle. In this case one might choose to lock the first joint at the angle q_1 that maximizes another dexterity measure.

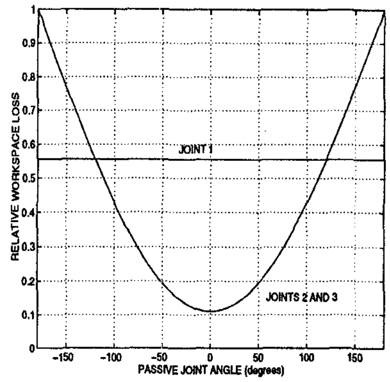


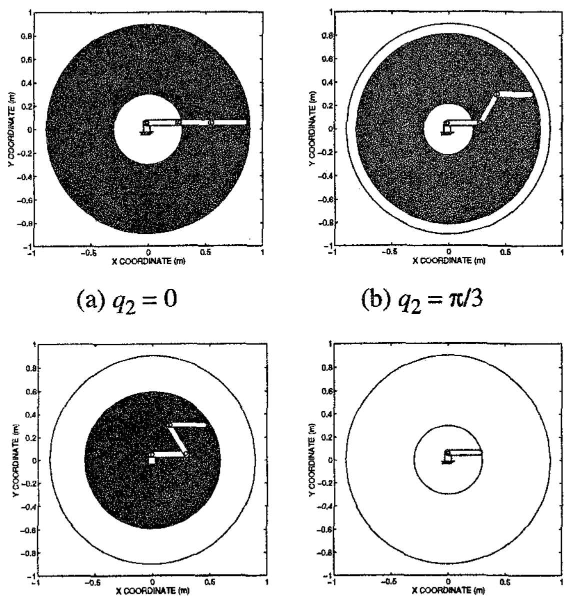
Figure 2: Relative workspace loss for all possible locations of a 3-link planar manipulator's passive joint.

5 Reachability analysis

Another important property of a robot manipulator is its capability to reach a number of pre-defined points in Cartesian space for manipulation tasks. This property is not guaranteed to exist when the manipulator contains a passive joint, even if the passive joint is locked at the angle which maximizes the manipulator's workspace volume. To see this, consider again Figure 3; if joint 2 is locked at $q_2 = 0$, then points close to the robot's base cannot be reached. On the other hand, locking joint 2 at $q_2 = 2\pi/3$ rd does allow the end-effector to reach those points. We propose in this work a new dexterity measure, which we call the *reachability* of the underactuated manipulator. The reachability is computed as the sum of the individual distances between all points that must be reached by the manipulator's end-effector and the respective closest point in the workspace boundary:

$$d_{I_p}(q_p) = \sum_{i=1}^{P} \lVert X_i - B_i(q_p) \rVert \tag{6}$$

In Equation (6), P is the number of Cartesian points to be reached, X_i are the coordinates of the i-th point, and $B_i(q_p)$ is the point in the workspace boundary closest to X_i. Since workspace boundaries are directly related to kinematic



(a) $q_2 = 0$ (b) $q_2 = \pi/3$

(c) $q_2 = 2\pi/3$ (d) $q_2 = \pi$

Figure 3: Absolute workspace of a 3-link planar manipulator with joint 2 passive. The outermost circle represents the boundary of the corresponding fully actuated manipulator's workspace.
(a) the workspace has maximum area when joint 2 is locked at $q_2 = 0$;
(b), (c) as the locking angle increases, the area of the workspace decreases;
(d) if joint 2 is locked at $q_2 = \pi$ the workspace reduces to a circumference with zero area.

singularities, the reachability measures how close or far from a singularity the underactuated manipulator will be able to reach the desired points.

Usually more than one locking angle will allow the end-effector to reach a set of pre-defined points in Cartesian space. To compute the optimal one, we propose to minimize the *relative reachability loss* defined as:

$$\tilde{d}_{I_p}(q_p) = 1 - \frac{d_{I_p}(q_p)}{d} = 1 - \frac{\sum_{i=1}^{P} \|X_i - B_i(q_p)\|}{\sum_{i=1}^{P} \|X_i - B_i\|} \tag{7}$$

The denominator in the right-hand-side of (7) is the constant reachability, d, of the corresponding fully actuated manipulator. Clearly, if $d = 0$, then all pre-defined Cartesian points are situated at the boundary of the workspace. Consequently, if the manipulator is equipped with passive joints, then either $d_{I_p} = 0$ or one or more of the Cartesian points will not be reachable by the end-effector. We must then consider only the case $d \neq 0$.

The relative reachability loss indicates, on a 0 to 1 scale, how closer to an workspace boundary a Cartesian point becomes when passive joints are present in the manipulator. It is important to mention that the minimization of $\tilde{d}_{I_p}(q_p)$ is only carried over the joint angles q_p for which all points X_i are actually reachable. If any of the X_i is not reachable for all possible q_p, then the underactuated manipulator cannot complete its task because of the presence of the passive joints, and is defined as non-fault tolerant [6]. Minimization of $\tilde{d}_{I_p}(q_p)$ effectively leads one to choose the optimal workspace whose boundaries are as far as possible from the desired Cartesian points.

Example 2 Consider the 3-link manipulator in Figure 1, with joint 1 passive, programmed to reach the set of points $X = \{(-0.3, 0.3), (-0.3, 0.6), (-0.6, 0.3), (-0.6, 0.6)\}$. Figure 4 shows the value of $\tilde{d}_1(q_1)$ in this case. This is not a continuous function because, when several Cartesian points have to be reached, one cannot expect that in general all of them will be located at the workspace boundary for some value of the passive joint position (and consequently yield $\tilde{d}_1 = 0$). It can be seen that the set of Cartesian points cannot be reached whenever joint 1 is locked outside the range [–105°, 165°]. Inside this range, the Cartesian points are optimally reached (i.e., they are farthest from the workspace boundary) when $q_1^* = 135^\circ$. The optimal reachability is only 6% smaller than the reachability of a corresponding fully actuated manipulator. Figure 5 presents the workspace that enables the manipulator to optimally reach all four desired Cartesian points, i.e., when $q_1 = 135^\circ$.

6 Manipulability analysis

The manipulability index, introduced by Yoshikawa [10], is a measure of how far from a kinematic singularity a manipulator is. It has been used for many years as a tool for posture optimization of redundant manipulators, as well as for singularity avoidance. The manipulability index is defined as:

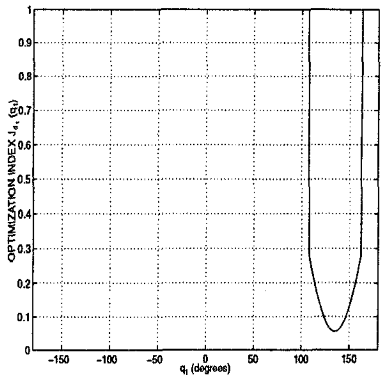


Figure 4: Optimization function $\tilde{d}_1(q_1)$ for $X = \{(-0.3,0.3), (-0.3,0.6), (-0.6,0.3), (-0.6,0.6)\}$.

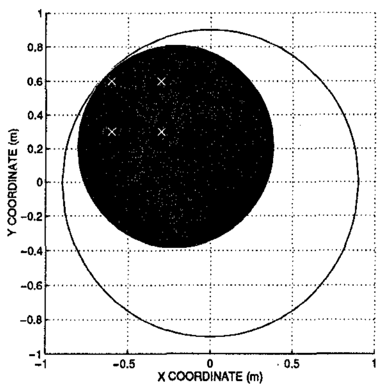


Figure 5: Optimal workspace for the set of points $X = \{(-0.3,0.3), (-0.3,0.6), (-0.6,0.3), (-0.6,0.6)\}$.

$$w(q) = \sqrt{\det[J(q)J(q)^T]} \tag{8}$$

where $J(q)$ is the position-dependent manipulator's Jacobian. Let j_i represent the i-th column of J. When the manipulator is equipped with a set of passive joints, located at joints $I_p = \{p_1, \ldots, p_{n_p}\}$, its new Jacobian is obtained by deleting from J all columns corresponding to the passive joints:

$$J_{I_p} = \left[j_1 \cdots j_{p_1-1}\, j_{p_1+1} \cdots j_{p_{n_p}-1}\, j_{p_{n_p}+1} \cdots j_n \right] \tag{9}$$

The manipulability of the resulting underactuated manipulator is computed as:

$$w_{I_p}(q) = \sqrt{\det\left[J_{I_p}(q)J_{I_p}(q)^T\right]} \tag{10}$$

The *relative manipulability loss* is defined as the complement in [0,1] of the relative manipulability index [8]:

$$\tilde{w}_{I_p}(q) = 1 - \frac{w_{I_p}(q)}{w(q)} \tag{11}$$

Roberts showed that $0 \le \tilde{w}_{I_p}(q) \le 1$ [8]. The relative manipulability loss indicates how closer to a singularity a manipulator at configuration q becomes when some of its joints are passive. If the manipulability index is zero, we can consider instead the constrained manipulability index [9], which is equal to the product of the non-zero singular values of J. In this case the relative manipulability loss will indicate how much further the dexterity of the manipulator decreases when passive joints are present.

To compute the joint angles q_p which minimize $\tilde{w}_{I_p}$, and therefore minimize the manipulability loss, we propose to minimize the optimization function:

$$J_{w_{I_p}}(q) = 1 - \frac{w_{I_p}(q)}{w(q)} \tag{12}$$

Usually the Jacobian, and therefore the manipulability, depends on the values of all joint angles. Consequently, minimization of J_w with respect to q_p only is an ill-defined problem. When this is the case we choose to minimize the averaged optimization index as given by Equation (3).

Example 3 Consider again the 3-link manipulator in Figure 1. Its Jacobian is equal to:

$$J = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \end{bmatrix}$$

and its manipulability index is given by:

$$w = \sqrt{(l_2 l_3 s_3)^2 + (l_2 l_3 s_3 + l_1 l_3 s_{23})^2 + (l_1 l_2 s_2 + l_1 l_3 s_{23})^2}$$

where s and c are the standard abbreviations of the sine and cosine function, respectively.

Assume that one of the joints is passive; according to the location of this passive joint, the manipulability index is given by:

$$w_1 = |l_2 l_3 s_3|$$

$$w_2 = |l_2 l_3 s_3 + l_1 l_3 s_{23}|$$

$$w_3 = |l_1 l_2 s_2 + l_1 l_3 s_{23}|$$

The manipulability when either joint 2 or joint 3 are passive is a function of both q_2 and q_3. To find the minimum of, say, $J_{w_2}(q_2)$ we use Equation (3):

$$q_2^* = \arg\min\left[\bar{J}_{w_2}(q_2)\right] = \arg\min\left[\frac{\int_{-\pi}^{\pi} J_{w_2}(q_2, q_3)dq_3}{\int_{-\pi}^{\pi} dq_3}\right]$$

Figure 6 presents the value of $\bar{J}_{w_i}(q_i)$ for $i = 2, 3$ as a function of q_i. When joint 2 is passive the relative manipulability loss is minimized by locking joint 2 at $q_2^* = 0$. The manipulability loss is about 19%. The value of q_3 that minimizes J_{w_3} is the same, $q_3^* = 0$. The manipulability loss, however, is much smaller, about only 11%. These results indicate that, for the sake of maximizing the manipulability of a 3-link planar manipulator with one passive joint located at either joints 2 or 3, one must lock the passive joint at its maximum extended position.

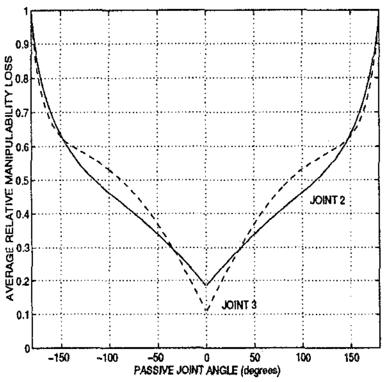


Figure 6: Average relative manipulability loss of a 3-link planar manipulator when joint 2 or joint 3 are passive.

Although we presented only examples of underactuated manipulators with one passive joint, the methods proposed in this work accommodate the presence of several passive joints. When optimizing the workspace or the reachability of underactuated manipulators, one has to search for the global minimum of the optimization functions through the n_p-dimensional space consisting of all passive joints' positions. When optimizing the manipulability, one first has to delete from the Jacobian matrix all columns corresponding to all passive joints, and then proceed to search for the global minimum on an n_p-dimensional space. Of course, as more passive joints are added to the manipulator, analytical solutions become more and more difficult to obtain because of the larger dimension of the search space. If that is the case one may resort to numerical optimization techniques to compute the optimal locking angle of the passive joints.

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