# THE MEASURE OF DYNAMIC COUPLING OF SPACE ROBOT SYSTEMS

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## Abstract

*In this paper we discuss the dynamic coupling of a space robot system with a free-floating base which could be a spacecraft, space station, or satellite. We formulate the dynamic coupling factor of robot end-effector motion with respect to base motion, and vice versa. Based on the coupling factor, we define a measure to characterize the degree of the dynamic coupling. The measure can be considered as a performance index in planning robot motion, or in evaluating robot trajectory for minimizing base motion, or in optimizing the robot configuration design and selecting the robot base location. We give an example to illustrate the computational procedure and simulation results of the defined measure. We further develop a force coupling factor and the corresponding measure, by which we clarify that minimizing the base force transmission is not equivalent, but opposing, to minimizing the base motion coupling.*

## 1 Introduction

Robotic technology offers various potential benefits for future space exploration [7]. The control of space robots, however, is a challenge, especially when the robot mass and moment of inertia are not negligible in comparison to the base (spacecraft, space station, or satellite). Without considering the dynamic interaction of the space robot and the base, motion of space robots can alter the base trajectory. On the other hand, the robot end-effector may miss the desired target due to the base motion. This dynamic coupling causes an additional attitude control required and thus fuel consumption when the thrust jets are used for the base attitude control while the robot is working. If no attitude control is applied on the base, the robot motion must be carefully planned so as to minimize the base reaction.

Longman, Lindberg and Zadd [4] discussed the base reaction force and moment computation and compensation in the attitude control of the base. The discussion, however, is for the space robot system with an attitude controlled base only, i.e., the system is free in translation but not in rotation. Chung, Desa and deSilva [1] proposed a trajectory planning method to minimize the base reaction force of the robot manipulator by using a kinematic redundant robot. This work, however, is limited to a system where the base is fixed, and may not be reasonable when the mass and moment of inertia of the remotely-controlled robot and an additional facility, as well as the payloads may be as large as 1/3 of the spacecraft itself. Dubowsky and Torres [2] introduced a concept called disturbance mapping, which relates the robot joint motion to the base attitude disturbance. They then applied this concept to robot motion planning in minimizing the base attitude disturbance. This work is significant in understanding the relationship between the robot joint motion and the resultant base rotation, and is useful in minimizing fuel consumption for base attitude control. However, the robot task in space is normally specified in Cartesian space (i.e., Cartesian inertia space, or simply inertia space) of the end-effector. One would like to know how the robot end-effector motion produces movement, including translation and rotation, at the base.

On the other hand, the dynamic coupling may not be a disadvantage. Using the nonholonomic path planning method to control robot motion, the desired attitude of the base can be maintained as proposed by Fernandes, Gurvits and Li [3] and Nakamura and Mukherjee [5], and the approach will be best applied to a system with a strong dynamic coupling. Therefore, it is more important to fully understand the coupling of the system, rather than to minimize the coupling effect at the beginning. To this end, a measure to characterize this dynamic coupling, i.e., mutual dependency of robot motion and base motion, is still needed.

In this paper, we first develop a concept of the dynamic coupling factor that represents the coupling between the robot end-effector motion in inertia space and base motion in the same space. Eigenvalue analysis of the coupling factor results in the direction and relative magnitude of the maximum base (or end-effector) motion produced by end-effector (or base) motion. Based on the coupling factor concept, we define a measure to describe the degree of the dynamic coupling. The measure is a function of the robot configuration, the geometric and inertia parameters of the robot and spacecraft, and the robot base location with respect to the spacecraft. Thus the measure can be considered as an optimum criterion in planning robot motion and designing robot structure. Later, we give an example to illustrate the computational procedure and simulation results.

In the same manner, we define the force coupling concept that is the inverse of the motion coupling, that is, in the direction of maximum force transmission, the minimum motion is transmitted. Revealing this relationship is interesting, because some research has been directed to minimizing the base force, while other research has focused on minimizing the base motion disturbance, but now we see these two efforts are actually contradictory.

## 2 Dynamic Coupling Factor

A space robot attached to a spacecraft on the orbit is considered to be a free-flying system in the non-gravitational environment. The system is modeled as a set of $n + 1$ rigid bodies

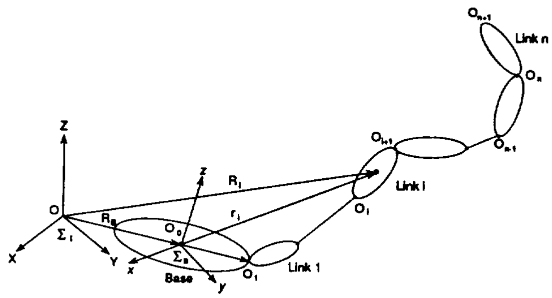


Figure 1: A model of a space robot system with a free-flying base.

connected by $n$ joints, which are numbered from 1 to $n$, and the joint position vector is $\mathbf{q} = (q_1, q_2, \cdots, q_n)^T$. Each body is numbered from 0 to $n$, and the base can be named as **B** in particular. The mass and inertia of $i$th body are denoted by $m_i$ and $\mathbf{I}_i$, respectively.

We define two coordinate frames, the inertia frame $\sum_I$ on the orbit, and the base frame $\sum_B$ attached to the base body with its origin at the centroid of the base. As shown in Figure 1, let $\mathbf{R}_i$ and $\mathbf{r}_i$ be the position vectors pointing toward the centroid of $i$th body with reference to $\sum_I$ and $\sum_B$ respectively; then

$$\mathbf{R}_i = \mathbf{r}_i + \mathbf{R}_B \tag{1}$$

where $\mathbf{R}_B$ is the position vector pointing toward the centroid of the base with reference to $\sum_I$.

Let $\mathbf{V}_i$ and $\mathbf{\Omega}_i$ be linear and angular velocities of $i$th body with respect to $\sum_I$; let $\mathbf{v}_i$ and $\omega_i$ be that with respect to $\sum_B$. Then we have

$$\begin{aligned} \mathbf{V}_i &= \mathbf{v}_i + \mathbf{V}_B + \Omega_B \times \mathbf{r}_i \\ \mathbf{\Omega}_i &= \omega_i + \Omega_B \end{aligned} \tag{2}$$

where $\mathbf{V}_B$ and $\mathbf{\Omega}_B$ are linear and angular velocities of the centroid of the base with respect to $\sum_I$, and operator "×" represents the outer product of $R^3$ vector. The velocities in the base frame $\mathbf{v}_i$ and $\omega_i$ can be represented by

$$\begin{bmatrix} \mathbf{v}_i \\ \omega_i \end{bmatrix} = \mathbf{J}_i(\mathbf{q})\dot{\mathbf{q}} \tag{3}$$

where $\mathbf{J}_i(\mathbf{q})$ is the Jacobian of the $i$th body,

$$\mathbf{J}_i = \begin{bmatrix} \mathbf{J}_{Li}(\mathbf{q}) \\ \mathbf{J}_{Ai}(\mathbf{q}) \end{bmatrix} \tag{4}$$

$$\mathbf{v}_i = \mathbf{J}_{Li}(\mathbf{q})\dot{\mathbf{q}} \tag{5}$$

$$\omega_i = \mathbf{J}_{Ai}(\mathbf{q})\dot{\mathbf{q}} \tag{6}$$

In what follows, we derive the relationship between motion rate in the inertia frame and that in the base frame. The linear and angular momenta $\mathbf{M}_l$ and $\mathbf{M}_a$ are defined as

$$\mathbf{M}_l = \sum_{i=0}^{n} m_i \mathbf{V}_i \tag{7}$$

$$\mathbf{M}_a = \sum_{i=0}^{n} \mathbf{I}_i^B \Omega_i + m_i \mathbf{R}_i \times \mathbf{V}_i \tag{8}$$

where $\mathbf{I}_i^B$ is the inertia tensor in $\Sigma_B$. The centroid of the entire system can be determined by

$$m_c = \sum_{i=0}^{n} m_i \tag{9}$$

$$\mathbf{r}_c = \sum_{i=1}^{n} m_i \mathbf{r}_i / m_c \tag{10}$$

$$\mathbf{J}_c = \sum_{i=1}^{n} m_i \mathbf{J}_{Li} / m_c \tag{11}$$

Considering Equations (9,10,11), Equations (7,8) yield

$$\begin{bmatrix} \mathbf{M}_l \\ \mathbf{M}_a \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{VV} & \mathbf{H}_{V\Omega} \\ \mathbf{H}_{V\Omega}^T & \mathbf{H}_{\Omega\Omega} \end{bmatrix} \begin{bmatrix} \mathbf{V}_B \\ \mathbf{\Omega}_B \end{bmatrix} + \begin{bmatrix} \mathbf{H}_{Vq} \\ \mathbf{H}_{\Omega q} \end{bmatrix} \dot{\mathbf{q}} + \begin{bmatrix} \mathbf{O}_3 \\ \mathbf{R}_B \times \mathbf{M}_l \end{bmatrix} \tag{12}$$

Each block submatrix is determined by

$$\mathbf{H}_{VV} = m_c \mathbf{U}_3 \qquad \in \Re^{3\times 3} \tag{13}$$

$$\mathbf{H}_{V\Omega} = -m_c[\mathbf{r}_c \times] \qquad \in \Re^{3\times 3} \tag{14}$$

$$\mathbf{H}_{Vq} = m_c \mathbf{J}_c \qquad \in \Re^{3\times n} \tag{15}$$

$$\mathbf{H}_{\Omega\Omega} = \sum_{i=1}^{n} [\mathbf{I}_i^B + m_i \mathbf{D}(\mathbf{r}_i)] + \mathbf{I}_B \qquad \in \Re^{3\times 3} \tag{16}$$

$$\mathbf{H}_{\Omega q} = \sum_{i=1}^{n} [\mathbf{I}_i^B \mathbf{J}_{Ai} + m_i [\mathbf{r}_i \times] \mathbf{J}_{Li}] \qquad \in \Re^{3\times n} \tag{17}$$

where $\mathbf{U}_3$ is a $3 \times 3$ identity matrix, and $\mathbf{O}_3$ is a $3 \times 3$ zero matrix. The matrix functions $[\mathbf{r}\times]$ and $\mathbf{D}(\mathbf{r})$ for a vector $\mathbf{r} = [r_x, r_y, r_z]^T$ are defined by

$$[\mathbf{r}\times] = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix} \tag{18}$$

$$\mathbf{D}(\mathbf{r}) = [\mathbf{r}\times]^T[\mathbf{r}\times] = \begin{bmatrix} r_y^2 + r_z^2 & -r_x r_y & -r_x r_z \\ -r_x r_y & r_x^2 + r_z^2 & -r_y r_z \\ -r_x r_z & -r_y r_z & r_x^2 + r_y^2 \end{bmatrix} \tag{19}$$

For the system with no attitude controlled base, and no gravitational force and external forces, the linear and angular momenta of the system are conserved. We first assume that the system is stationary in the initial state, i.e., the total linear and angular momenta are zero. In this case, from Equation (12), the relationship between the joint velocity and the velocity in the base frame is

$$\dot{\mathbf{q}} = -\begin{bmatrix} \mathbf{H}_{Vq} \\ \mathbf{H}_{\Omega q} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{H}_{VV} & \mathbf{H}_{V\Omega} \\ \mathbf{H}_{V\Omega}^T & \mathbf{H}_{\Omega\Omega} \end{bmatrix} \begin{bmatrix} \mathbf{V}_B \\ \mathbf{\Omega}_B \end{bmatrix} \tag{20}$$

if the matrix $\begin{bmatrix} \mathbf{H}_{Vq} \\ \mathbf{H}_{\Omega q} \end{bmatrix}^{-1}$ exists. (The issues when this matrix does not exist, are discussed later on). The end-effector velocity in the base frame is related to the joint velocity by

$$\begin{bmatrix} \mathbf{v}_E \\ \omega_E \end{bmatrix} = \begin{bmatrix} \mathbf{J}_L \\ \mathbf{J}_A \end{bmatrix} \dot{\mathbf{q}} \tag{21}$$

while the end-effector velocity in the inertia frame can be determined by

$$\begin{aligned} \mathbf{V}_E &= \mathbf{v}_E + \mathbf{V}_B + \boldsymbol{\Omega}_B \times \mathbf{r}_E \\ \boldsymbol{\Omega}_E &= \omega_E + \boldsymbol{\Omega}_B \end{aligned} \quad (22)$$

Combining these two equations, we can relate the end-effector velocity in the inertia frame to the base velocity in the same frame by

$$\begin{bmatrix} \mathbf{V}_E \\ \boldsymbol{\Omega}_E \end{bmatrix} = \mathbf{S} \begin{bmatrix} \mathbf{V}_B \\ \boldsymbol{\Omega}_B \end{bmatrix} \quad (23)$$

where

$$\mathbf{S} = \begin{bmatrix} \mathbf{U}_3 & -[\mathbf{r}_E \times] \\ \mathbf{O}_3 & \mathbf{U}_3 \end{bmatrix} - \begin{bmatrix} \mathbf{J}_L \\ \mathbf{J}_A \end{bmatrix} \begin{bmatrix} \mathbf{H}_{Vq} \\ \mathbf{H}_{\Omega q} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{H}_{VV} & \mathbf{H}_{V\Omega} \\ \mathbf{H}_{V\Omega}^T & \mathbf{H}_{\Omega\Omega} \end{bmatrix} \quad (24)$$

and its inverse relation is

$$\begin{bmatrix} \mathbf{V}_B \\ \boldsymbol{\Omega}_B \end{bmatrix} = \mathbf{P} \begin{bmatrix} \mathbf{V}_E \\ \boldsymbol{\Omega}_E \end{bmatrix} \quad (25)$$

where $\mathbf{P}$ is the inverse of $\mathbf{S}$.

The matrices $\mathbf{S}$ and $\mathbf{P}$ characterize how robot motion alters base motion, or vice versa, i.e., it represents the dynamic interaction between the robot and the base. Therefore, we call the matrix $\mathbf{P}$ the *dynamic coupling factor* of end-effector motion with respect to base motion, or *end-to-base coupling* for short, and call the matrix $\mathbf{S}$ the dynamic coupling factor of base motion with respect to end-effector motion, or simply *base-to-end coupling*.

The end-to-base coupling is important for avoiding a large disturbance movement of the base due to end-effector motion, while the base-to-end coupling is meaningful in investigating how robot end-effector motion is affected due to the base motion deviation, or attitude control errors.

The concept of the dynamic coupling factor is closely related to the generalized Jacobian matrix [6]. Using the same notation mentioned in our earlier paper [8], it is easy to obtain the following relationship

$$\mathbf{S} = \mathbf{N}\mathbf{K} \quad (26)$$

and

$$\mathbf{P} = \mathbf{K}^{-1}\mathbf{N}^{-1} \quad (27)$$

where

$$\mathbf{K} = -\begin{bmatrix} \mathbf{H}_{Vq} \\ \mathbf{H}_{\Omega q} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{H}_{VV} & \mathbf{H}_{V\Omega} \\ \mathbf{H}_{V\Omega}^T & \mathbf{H}_{\Omega\Omega} \end{bmatrix} \quad (28)$$

and $\mathbf{N}$ is the generalized Jacobian matrix. This relationship implies that, when a manipulator is in a singular configuration, i.e., Jacobian matrix is degraded, the coupling factor matrix is also degraded.

When the initial velocity of the system is not zero, the concept of the coupling factor is still valid to relate the two sets of motion, but there will be an additional term in (23) and (25) representing the constant momenta. It must also be noted that when the number of robot joints is less than the DOF of the base velocity that we are considering, Equation (20) does not exist, i.e., the independent number of the equations given by (12) is greater than the number of joints. However, in this case, not all equations given by (22) are independent, and the number of independent equations will be equal to the number of joints in general. By eliminating the dependent equations in (22), and combining the result with (20), the required equations become available for solving the problem. When the number of joints is greater than the DOF of the base velocity, i.e., the robot is kinematically redundant, an unique expression of joint velocity can not be determined by the base velocity and Equation (20) does not exist.

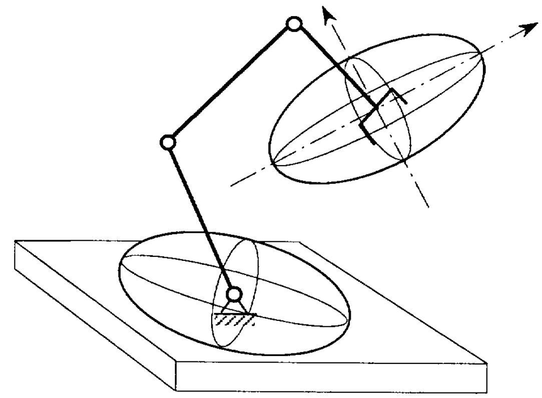

Figure 2: Geometric interpretation of the dynamic coupling measure.

## 3 Measure of Dynamic Coupling

If we denote the end-effector velocity in the inertia frame by $\dot{\mathbf{X}}_E$ and the base velocity in the same frame by $\dot{\mathbf{X}}_B$, the ratio of the velocity magnitude is determined by

$$\frac{\|\dot{\mathbf{X}}_B\|}{\|\dot{\mathbf{X}}_E\|} = \frac{<\mathbf{P}\dot{\mathbf{X}}_E, \mathbf{P}\dot{\mathbf{X}}_E>}{<\dot{\mathbf{X}}_E, \dot{\mathbf{X}}_E>} = \frac{<\dot{\mathbf{X}}_E^T(\mathbf{P}^T\mathbf{P})\dot{\mathbf{X}}_E>}{<\dot{\mathbf{X}}_E, \dot{\mathbf{X}}_E>} \quad (29)$$

This relationship describes how much base motion is produced by a given end-effector motion. If the end-effector motion is velocity, this relationship results in an eigenvalue problem of the matrix $\mathbf{P}^T\mathbf{P}$. That is, if the end-effector velocity $\|\dot{\mathbf{X}}_E\|$ is unit, the base velocity is bounded within an ellipsoid expanded by the eigenvectors of the matrix $\mathbf{P}^T\mathbf{P}$, and the volume of the ellipsoid is determined by $\sqrt{\det(\mathbf{P}^T\mathbf{P})} = \sigma_1\sigma_2...\sigma_m$, where $\sigma_i$ is the $i$th eigenvalue of the matrix $\mathbf{P}^T\mathbf{P}$.

Therefore, it is rational to define

$$w = \det(\mathbf{A}) = \det(\mathbf{P}^T\mathbf{P}) \quad (30)$$

as the measure of the dynamic coupling factor of the space robot system. The measure physically characterizes the degree of coupling of end-effector motion with respect to base motion. In the same way, we define

$$u = \det(\mathbf{B}) = \det(\mathbf{S}^T\mathbf{S}) \quad (31)$$

We call $w$ the measure of end-to-base coupling, and $u$ the measure of base-to-end coupling. The well-known geometric interpretation of the eigenvalue problem can be shown in Figure 2 to illustrate the physical meaning of the measure defined. In the direction of the eigenvector corresponding to the maximum eigenvalue of the matrix $\mathbf{A}$ (or $\mathbf{B}$), the maximum motion at the base (or end-effector) will be generated by an unit end-effector motion (or unit base motion).

The concept of the coupling measure is of significance in various practical problems. First, since the measure is a function of robot configuration, it can be considered as a performance index in planning robot motion for a given task, or in evaluating the generated trajectory for minimizing the base motion disturbance. Second, since the measure is determined by robot

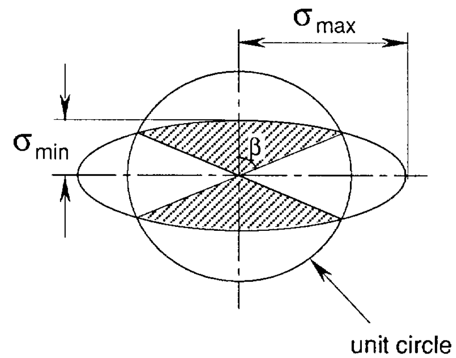


Figure 3: The directions in which the end-effector movement produces maximally the same amount of movement at the base.

geometric and inertia parameters, it can be used as an optimum criterion in designing the robot structure, and the relative mass/inertia distribution. Third, the measure is a function of robot base location with respect to the body of a spacecraft or space station. By giving a set of tasks, the concept proposed here allows us to select the best location for a robot to be installed or latched.

As discussed, the magnification ratio of end-effector motion to base motion is bounded by

$$\sigma_{max} \geq \frac{||\dot{\mathbf{X}}_B||}{||\dot{\mathbf{X}}_E||} \geq \sigma_{min} \tag{32}$$

In controlling a space robot, the resultant base motion from robot motion is usually undesirable. If we consider the movement under the following condition as *stable movement*, i.e., the end-effector movement produces maximally the same amount of the movement at the base,

$$\frac{||\dot{\mathbf{X}}_B||}{||\dot{\mathbf{X}}_E||} \leq 1 \tag{33}$$

then the direction of end-effector movement must be selected. Figure 3 shows two dimensional case where the shadow cone represents the allowable direction of the stable movement. The constraint is applicable for planning robot motion in generating a desired base motion. The angle $\beta$ in Figure 3 can be determined by

$$\beta = \tan^{-1}\sqrt{\frac{\sigma_{max}^2\sigma_{min}^2 + \sigma_{min}^2}{-\sigma_{max}^2\sigma_{min}^2 + \sigma_{min}^2}} \tag{34}$$

Sometimes it is convenient to decompose linear motion and angular motion. To this end, we partition the coupling factors into four submatrices for linear motion and angular motion, respectively.

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_{VV} & \mathbf{P}_{V\Omega} \\ \mathbf{P}_{\Omega V} & \mathbf{P}_{\Omega\Omega} \end{bmatrix} \tag{35}$$

and

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_{VV} & \mathbf{S}_{V\Omega} \\ \mathbf{S}_{\Omega V} & \mathbf{S}_{\Omega\Omega} \end{bmatrix} \tag{36}$$

$\mathbf{P}_{VV}$ stands for the linear velocity coupling factor of end-to-base, and $\mathbf{P}_{\omega\omega}$ represents the angular velocity coupling factor of end-to-base. $\mathbf{P}_{V\Omega}$ is the coupling factor of the end-effector angular velocity with respect to the base linear velocity, while $\mathbf{P}_{\Omega V}$ is that of the end-effector linear velocity with respect to the base angular velocity. The same definition can be given for the base-to-end coupling, $\mathbf{S}$. The decomposition of translation from rotation sometimes helps in easily understanding the coupling factor in the linear motion or angular motion individually, because the importance of the coupling in linear motion or angular motion varies with tasks. For example, translational motion of the spacecraft is significant for rendezvous from one orbit to another, because in this case substantial force is acting on a spacecraft due to significant acceleration, and the effect of robot motion on translation of the spacecraft must be minimized. While for docking process, a good attitude control of a spacecraft is more important.

Since the matrices $\mathbf{S}_{VV}$ and $\mathbf{S}_{\Omega\Omega}$ are always square, and $\mathbf{S}$ and $\mathbf{P}$ are mutually inverse, the following algorithm facilitates the calculation of $\mathbf{P}$ from $\mathbf{S}$.

$$\mathbf{P}_{\Omega\Omega} = (\mathbf{S}_{\Omega\Omega} - \mathbf{S}_{\Omega V}\mathbf{S}_{VV}^{-1}\mathbf{S}_{V\Omega})^{-1} \tag{37}$$

$$\mathbf{P}_{V\Omega} = -\mathbf{S}_{VV}\mathbf{S}_{V\Omega}\mathbf{P}_{\Omega\Omega} \tag{38}$$

$$\mathbf{P}_{\Omega V} = -\mathbf{P}_{\Omega\Omega}\mathbf{S}_{\Omega V}\mathbf{S}_{VV}^{-1} \tag{39}$$

$$\mathbf{P}_{VV} = \mathbf{S}_{VV}^{-1} - \mathbf{P}_{V\Omega}\mathbf{S}_{\Omega V}\mathbf{S}_{VV}^{-1} \tag{40}$$

In the same manner, we also can calculate $\mathbf{S}$ from $\mathbf{P}$.

It must be noted that, from Equations (23) and (25), the elements of the matrices $\mathbf{S}$ and $\mathbf{P}$ are of different dimensions due to linear and angular velocities involved. Therefore, the elements of the matrices $\mathbf{A}$ and $\mathbf{B}$ in Equations (30) and (31) are also of different dimensions, and this causes the lose of physical meaning when the measures $w$ and $u$ are used as a performance index in optimization. One way to obtain dimensionless elements of the matrix, say $\mathbf{P}$, is to divide or multiply the submatrices $\mathbf{P}_{V\Omega}$ and $\mathbf{P}_{\Omega V}$ by the total length of the robot $L$, i.e., the new matrix $\mathbf{P}'$

$$\mathbf{P}' = \begin{bmatrix} \mathbf{P}'_{VV} & \mathbf{P}'_{V\Omega} \\ \mathbf{P}'_{\Omega V} & \mathbf{P}'_{\Omega\Omega} \end{bmatrix} \tag{41}$$

and

$$\mathbf{P}'_{VV} = \mathbf{P}_{VV}, \quad \mathbf{P}'_{V\Omega} = \mathbf{P}_{V\Omega}/L,$$
$$\mathbf{P}'_{\Omega V} = \mathbf{P}_{\Omega V} \cdot L, \quad \mathbf{P}'_{\Omega\Omega} = \mathbf{P}_{\Omega\Omega}$$

where $L = \sum_{i=1}^{n}\sqrt{a_i^2 + d_i^2}$, $a_i$ and $d_i$ are D-H parameters of link $i$. We also can obtain $\mathbf{S}'$ from $\mathbf{S}$ in the same way. Then, based on $\mathbf{P}'$ and $\mathbf{S}'$, we can redefine $w$ and $u$.

# 4 An Example

Let us consider an example to better understand the concept we discussed previously. A model of a three-link, two dimensional space robot is located on the centroid of the base, as shown in Figure 4. This is not a kinematic redundant manipulator, if the three variables, $(x, y, \theta)$ at the end-effector are considered. The position vectors of the base, link 1, link 2, and link 3 in the inertia frame are denoted by $\mathbf{R}_0$, $\mathbf{R}_1$, $\mathbf{R}_2$ and $\mathbf{R}_3$, respectively, and the position vectors of link 1, link 2, link 3 in the base frame are $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$.

$$\mathbf{R}_0 = \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix} \qquad \mathbf{R}_1 = \begin{bmatrix} X_1 \\ Y_1 \end{bmatrix}$$

$$\mathbf{R}_2 = \begin{bmatrix} X_2 \\ Y_2 \end{bmatrix} \qquad \mathbf{R}_3 = \begin{bmatrix} X_3 \\ Y_3 \end{bmatrix} \tag{42}$$

$$\mathbf{r}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} l_1c_1 \\ l_1s_1 \end{bmatrix} \tag{43}$$

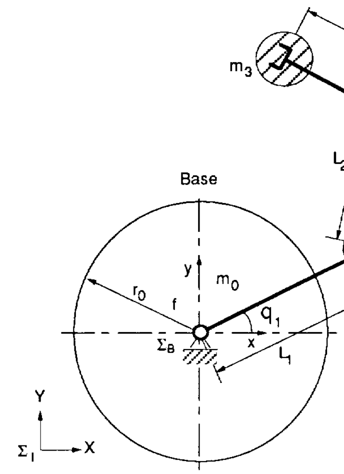


Figure 4: An example of a free-flying space robot system.

$$\mathbf{r}_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} l_1c_1 + l_2c_{12} \\ l_1s_1 + l_2s_{12} \end{bmatrix} \tag{44}$$

$$\mathbf{r}_3 = \begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} l_1c_1 + l_2c_{12} + l_3c_{123} \\ l_1s_1 + l_2s_{12} + l_3s_{123} \end{bmatrix} \tag{45}$$

where $c_1 = cos(q_1)$, $s_{12} = sin(q_1 + q_2)$, etc.

We further simplify the model by assuming $l_1 = l_2 = l_3 = l$ and $m_1 = m_2 = m_3 = m$ thereafter. The Jacobian matrices for each link are

$$\mathbf{J}_{L1} = \begin{bmatrix} -ls_1 & 0 & 0 \\ lc_1 & 0 & 0 \end{bmatrix} \tag{46}$$

$$\mathbf{J}_{L2} = \begin{bmatrix} -l(s_1 + s_{12}) & -ls_{12} & 0 \\ l(c_1 + c_{12}) & lc_{12} & 0 \end{bmatrix} \tag{47}$$

$$\mathbf{J}_{L3} = \begin{bmatrix} -l(s_1 + s_{12} + s_{123}) & -l(s_{12} + s_{123}) & -ls_{123} \\ l(c_1 + c_{12} + c_{123}) & l(c_{12} + s_{123}) & lc_{123} \end{bmatrix} \tag{48}$$

and

$$\mathbf{J}_{A1} = [1,0,0] \qquad \mathbf{J}_{A2} = [1,1,0] \qquad \mathbf{J}_{A3} = [1,1,1] \tag{49}$$

The centroid of the whole system can be determined by

$$m_c = m_0 + 3m \tag{50}$$

$$\mathbf{r}_c = \frac{m}{m_c}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3) \tag{51}$$

$$\mathbf{J}_c = \frac{m}{m_c}(\mathbf{J}_{L1} + \mathbf{J}_{L2} + \mathbf{J}_{L3}) \tag{52}$$

Following derivations in Section 2, we have

$$\mathbf{H}_{VV} = m_c\mathbf{U}_2 \ \in \Re^{2\times 2} \tag{53}$$

$$\mathbf{H}_{V\Omega} = -m_c[\mathbf{r}_c\times] = m \begin{bmatrix} y_1 + y_2 + y_3 \\ -x_1 - x_2 - x_3 \end{bmatrix} \in \Re^{2\times 1} \tag{54}$$

$$H_\Omega = \sum_{i=0}^{3} I_i + m\sum_{i=1}^{3}(x_i^2 + y_i^2) \ \in \Re^{1\times 1} \tag{55}$$

$$\mathbf{H}_{Vq} = m(\mathbf{J}_{L1} + \mathbf{J}_{L2} + \mathbf{J}_{L3}) \ \in \Re^{2\times 3} \tag{56}$$

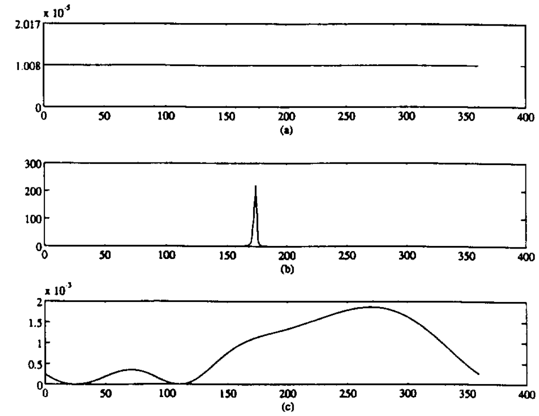


Figure 5: The base-to-end coupling measure with respect to the robot joint position (in degree): (a) joint 1, (b) joint 2, (c) joint 3.

$$\mathbf{H}_{\Omega q} = (\sum_{i=1}^{3} I_i, \sum_{i=2}^{3} I_i, I_3) + \sum_{i=1}^{3}(my_i, mx_i)\mathbf{J}_{Li} \ \in \Re^{1\times 3} \tag{57}$$

$$H_B = H_\Omega - m_cD(r_c) = H_\Omega - m_c(r_{cx}^2 + r_{cy}^2) \ \in \Re^{1\times 1} \tag{58}$$

$$\mathbf{H}_M = \mathbf{H}_{\Omega q} - m_c(-r_y, r_x)\mathbf{J}_c \ \in \Re^{1\times 3} \tag{59}$$

The manipulator parameters $m$ are considered here: $m = 50kg$, $I = 26kg.m^2$, and $l = 2.5m$. The base parameters are: $m_0 = 100kg$ and $I = 52kg.m^2$. We assume the initial configuration is $q_1 = 20^o$, $q_2 = 20^o$, and $q_3 = 20^o$. For simplicity, we do not use the dimensionless matrices $\mathbf{P}'$ and $\mathbf{S}'$. Figure 5 shows the end-to-base coupling measure, $w$, with respect to the joint position of the manipulator. When joint 1 moves while other two joints are stationary, the measure of coupling is a constant. The rationale behind this is that in this case the manipulator can be viewed as a single arm rotating around the center of a disk; the resulting effect to the disk is identical in any angular motion of the arm. Figure 5(b) and (c) show the variation of the coupling measure with respect to the positions of joint 2 and joint 3.

Figure 6(a) and (b) shows the base-to-end coupling measure and end-to-base measure, with respect to the mass/inertia ratio $m_0/m = I_0/I = \rho$. It is interesting to note that when the mass/inertia ratio increases, i.e., the base becomes more massive, the end-to-base coupling reduces exponentially, while the base-to-end coupling increases almost linearly.

# 5 Force Coupling

The dynamic coupling factor defined previously represents basically the coupling of motion, i.e., the interaction between robot end-effector motion and base motion. Similarly, we also can derive the coupling factor for force transmission from the end-effector to the base, or vice versa.

From the Virtual Work concept, we can easily determine the force relationship

$$\mathbf{F}_E = \mathbf{P}^T\mathbf{F}_B \tag{60}$$

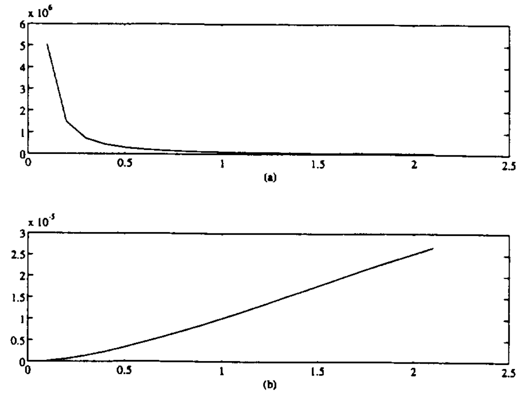


Figure 6: The end-to-base coupling measure (a) and the end-to-base coupling measure (b) vary with the mass and inertia ratio of the robot with respect to the base.

or

$$\mathbf{F}_B = \mathbf{P}^{-T}\mathbf{F}_E = \mathbf{S}^T\mathbf{F}_E \quad (61)$$

It is interesting to compare the above force coupling factor with the motion coupling factor,

$$\dot{\mathbf{X}}_B = \mathbf{P}\dot{\mathbf{X}}_E \quad (62)$$

or

$$\dot{\mathbf{X}}_E = \mathbf{P}^{-1}\dot{\mathbf{X}}_B = \mathbf{S}\dot{\mathbf{X}}_B \quad (63)$$

This duality implies that, at the direction where end-effector motion produces the minimal base motion, the force at the end-effector is transmitted maximally to the base. In other words, when we minimize the base motion disturbance, by carefully planning the robot motion, or by designing feasible controller and robot structure, we may maximize the force transmission which is also undesirable. Especially, it is noted that some researchers, such as [1], have been focusing on minimizing the base force, while some others, such as [2], are working on minimizing the base motion disturbance. Now these two approaches are actually contradictory.

In practice, sometimes the motion coupling is more important, while for other cases, the force coupling must be considered, depending on the tasks required, the payload to be manipulated, and working conditions of the space vehicles. For certain tasks, it may be desirable to minimize the motion coupling in certain directions, while minimizing the force coupling in some other directions.

Based on the concept of force and motion coupling factors, the dynamic equation of the system can be described in terms of the base variables. By differentiating the motion coupling factor,

$$\ddot{\mathbf{X}}_E = \dot{\mathbf{S}}\dot{\mathbf{X}}_B + \mathbf{S}\ddot{\mathbf{X}}_B \quad (64)$$

and recalling the dynamic equation represented by the end-effector variables [8],

$$\mathbf{F}_E = \mathbf{H}\ddot{\mathbf{X}}_E + \mathbf{B}\dot{\mathbf{X}}_E \quad (65)$$

we obtain

$$\mathbf{F}_B = \mathbf{H}_B\ddot{\mathbf{X}}_B + \mathbf{B}_B\dot{\mathbf{X}}_B \quad (66)$$

where

$$\mathbf{H}_B = \mathbf{S}^T\mathbf{H}\mathbf{S}, \quad \mathbf{B}_B = \mathbf{S}^T(\mathbf{H}\dot{\mathbf{S}} + \mathbf{B}\mathbf{S}) \quad (67)$$

Therefore, the system dynamics and the inertia matrix can be represented in a standard form in terms of the base variables, i.e., the force and motion at the base. When the base variables are measurable in some applications, this representation is useful.

## 6 Conclusion

We have presented formulation of the dynamic coupling factors relating robot end-effector motion to base motion, and vice versa. Based on the concept of the coupling factor, we define a measure to characterize the degree of the coupling. The use of the defined measure is of significance in planning robot motion or evaluating robot trajectory for minimizing base motion, or optimizing the robot configuration design and selecting the robot base location. We discuss a case study to show the computational procedure and simulation results. We further developed a concept of force coupling factor and its corresponding measure by which the capability of transferring force from the base to the robot end-effector, or vice versa, is described. The motion and force coupling measures allow us to understand the mutual dependency of the robot dynamics and the base, and will be a valuable tool in the analysis, design, and planning of a space robot system when the robot mass/inertia effect must be considered with respect to the base.

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