Algorithms for Automatic Sensor Placement to Acquire Complete and Accurate Information

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Thesis

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Doctor of Philosophy
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ABSTRACT

There is a need to develop strategies to automatically position mobile sensors to acquire data that contains accurate and complete information. This data is required to build models that are needed for navigation, manipulation, inspection, object recognition and verification. Systematic, random and exhaustive sensing techniques are inefficient. Automatic placement strategies can economize on the use of resources and maximize the quality and content of the information that is acquired.

In this thesis methods are developed to generate the viewpoints (position and orientation) for a two dimensional laser range scanner and other sensors with a high angular field of view (> 180^0), a low angular resolution, and a limited range of detection can acquire accurate and complete information. A candidate viewpoint must satisfy a number of detectability constraints: the target feature must be visible and enclosed within the sensor's angular and range field of view. Two other constraints that affect the density and accuracy of the data must also be satisfied. The methods use descriptions of the sensor, the feature and analytic expressions of the constraints to generate a region in cartesian space that encloses the set of viewpoints that will satisfy each of the constraints. The intersection of the candidate viewpoints for all the constraints is the set of positions from where the sensor can acquire an accurate and complete image.

To simplify the problem a two step process is adopted. First, a position from where the sensor can detect the feature (satisfying all constraints) is selected, and then the sensor is oriented so that the feature is in the sensor's field of view. When a sensor is not be able to acquire accurate and complete information in a single image, multiple viewpoints are generated. To generate these viewpoints the feature is recursively subdivided until for each sub-feature there is at least one viewpoint from where the sensor can acquire accurate and complete information. The information contained in the images that are acquired from these viewpoints is sufficient to construct an accurate and complete description of the entre feature.

The methods developed in this dissertation are able to generate viewpoints that are inside the cavity of a concave feature. The methods also apply to sensors with wide angular field of view, limited detection range, and low angular resolution.

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To Lory and the kids ....

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Chapter 1

Introduction

> " Most past and present work in machine perception has involved extensive static analysis of passively sampled data. However, it should be axiomatic that perception is not passive, but active. Perceptual activity is exploratory, probing, searching; percepts do not simply fall on to sensors as rain falls onto ground. We don't just see, we look. "
>
> *Ruzena Bajcsy*[5]

There is a need to develop strategies to automatically position a laser range scanner to acquire an image that contains accurate and complete information. These images are needed to build models which can be used for navigation [27][39][56], object recognition [22][29], and automatic inspection and manipulation.

Sensing strategies which systematically, randomly or exhaustively scan an environment are inefficient. Systematic and random sensing strategies are not guaranteed to collect all the required information. Furthermore, all three sensing strategies may collect data that contains redundant, inaccurate or sparse information. Acquiring and processing this information consumes resources that may otherwise be limited or unavailable.

Planning the placement of a sensor to obtain new information is a strategy that may economize on the consumption of resources. However, it is difficult to determine the viewpoints (position and orientations) from where the sensor can acquire data that contains accurate and complete information. The quality and content of the information contained in an image is limited in part by the viewpoint, the characteristics of the sensor, and the geometry of the object that is being sensed. Therefore, although there are many possible sensor positions, only from a few locations is the sensor able to acquire accurate and complete information.

Automatic sensor placement is an area of research that has produced methods to determine where to position a sensor to acquire accurate and complete information of a desired object [44][15][53]. These methods rely on a description of the sensor, a description of the object geometry, and measures that characterize the quality and completeness of the image. By generating viewpoints from where a sensor can acquire accurate and complete information, these methods can economize on the use of computational resources while ensuring that all the necessary information is acquired.

The state of the art in sensor placement focuses on sensors with a narrow angular field of view, unlimited range of detection (infinite depth of field) or high angular resolution. Moreover, sensor placement methods have concentrated on obtaining a complete view (information) within a single image [44][15][53]. The conditions that are used in these methods to classify a candidate sensor viewpoint are not suitable for laser range scanners and other sensors that have a wide angular field of view, limited range of detection or low angular resolution. Furthermore, due to technical limitations, these methods are unable to a generate or evaluate a viewpoint that is inside the cavity of a con-convex object.

In the following chapters methods are developed to determine the viewpoints for a laser range scanner to be able to acquire accurate and complete information of a given object. In circumstances when no single viewpoint exists from where the sensor can acquire the image, methods are developed that generate multiple viewpoints from where the sensor can collect images that collectively contain accurate and complete information of the object.

1.1 The Problem

The problem is to find a set of viewpoints from where a laser range sensor is to acquire images that collectively contain accurate, dense and complete information of a given object. It is assumed that a geometric description of a target object, a description of the sensor and a description of the constraints that characterize a complete, dense and accurate image are given. Detailed definitions of objects, their features, sensors and image quality and accuracy constraints are given in section 1.4.

1.2 Approach

A geometric approach is used to generate all viewpoints from where a sensor can acquire an accurate and complete description of the object. For this purpose, an object is expressed as a collection of one or more *features* that have recognizable geometric properties.

A viewpoint is considered a *candidate viewpoint* if from that location the following five conditions are satisfied:

(i) The entire feature is visible.

(ii) The feature is completely enclosed by the sensor's angular field of view.

(iii) The feature is within the sensor's range of detection.

(iv) The maximum separation between two consecutively sampled points (which controls the density of the image) must not exceed a given threshold.

(v) The accuracy of the data (controlled by the angle of incidence of the beam) must be above a given threshold.

These constraints evaluate the accuracy and completeness of an image. The first three constraints, which form one class of constraints that refer to the *visibility*, *angular field of view* and *range field of view*, classify the set of viewpoints from where the feature is visible and can be detected by the sensor. The last two constraints, which form a second class of constraints that refer to the *maximum separation between two consecutive points* and *angle of incidence of the beam*, are intended to control the accuracy and density of the range image. These two classes are respectively labeled the *visibility and detectability constraints* and the *image quality constraints*.

Finding the set of feasible viewpoints (position and orientation) is accomplished in two steps. First, positions are generated for each of which there is at least one orientation along which the sensor is be able to detect the entire feature. Second, the sensor is oriented so that from a selected location, the feature is entirely in its field of view. Separating sensor position from sensor orientation simplifies the solution by reducing the search space and by simplifying the constraints that characterize the set of feasible solutions.

The set of positions that satisfy each constraint are generated from a description of the feature, the sensor and the constraints that characterize a feasible sensor position. The viewpoints are combined, using set intersection, to determine those positions that satisfy all of the constraints.

If the set of feasible viewpoints is empty, there is no single location that satisfies all of the constraints. Therefore, there is no single location from where the sensor can acquire accurate and complete information of the feature. In this case, by recursively subdividing the feature, multiple viewpoints are determined from which the sensor can image the feature. For each of the generated viewpoints the sensor can acquire a portion of the entire information; collectively, the sensor can accurately and completely describe the entire feature.

1.2.1 Advantages of the approach

A geometric approach to sensor placement has a number of advantages over numerical techniques. Contrary to numerical techniques that can only generate a single candidate viewpoint, a geometric approach is able to generate all feasible viewpoints. This is an advantage especially in situations in which the set of feasible solutions are grouped into disjoint collections as well as in situations where the set of feasible viewpoints is null (indicating that there was no solution). Unlike iterative numerical techniques, the geometric approach to generate the set of feasible viewpoints is procedural and requires a constant number of operations which only depend on the geometry of the feature.

The viewpoints that satisfy each constraint can be presented in a visual form. This enables interactive trade-off analysis which can be used to best position the sensor, design the parameters of the problem and design the characteristics of the sensor to enable a robust and optimal solution. Moreover, a geometrically based trade-off analysis enables the development of heuristics for quick positioning of a sensor.

Finally, since the space of feasible viewpoints is an exhaustive description of all possible solutions, this space can be combined with an exhaustive description of the viewpoints that satisfy any number of other constraints to determine the viewpoints that satisfy these other constraints from where the sensor can acquire accurate and complete information. For example, a feasible viewpoint is only valid if it can be reached by the sensor. Mobile robot path planners are able to determine the set of viewpoints that can be reached by a sensor. Combining the space that describes the configurations that the sensor can reach with the set of feasible viewpoints determines the viewpoints that can be reached from where the sensor can acquire accurate and complete information.

It should be noted that the regions produced by the geometric generation technique and algebraic representations of the constraints used by numerical methods are duals of one another. The boundaries of the regions that enclose the set of feasible solutions can be described as algebraic constraints. Similarly, the space of all possible solutions can be generated by spanning the environment and verifying if all of the constraints are satisfied.

1.3 Examples

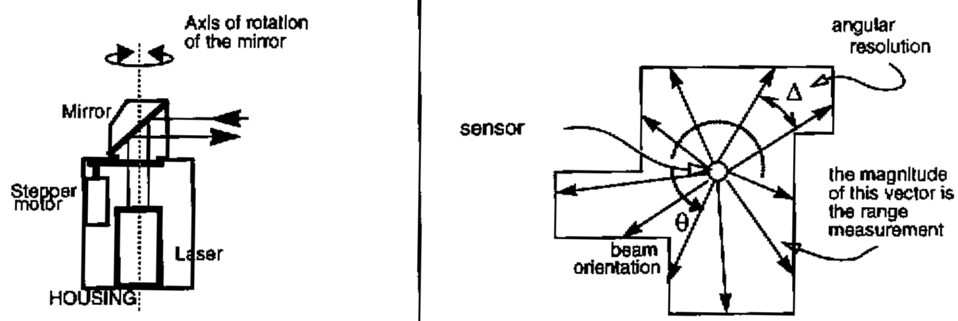
Two examples are illustrated in this section. The first example illustrates the methodology used to generate a viewpoint from where a sensor can scan a polygonal feature. The second example illustrates how multiple viewpoints are generated by subdividing the feature. The first part of this section defines the components in the problem. In both examples, the task is to determine the viewpoints from where a laser range scanner can acquire an accurate and complete description of the feature. A valid viewpoint is one that satisfies each of the five visibility, detectability and image quality constraints.

1.3.1 The sensor

A laser range scanner is the sensor that is used to scan the feature. A laser range scanner is a range sensing device often used to build depth images of an environment. It consists of a laser range sensor and a scanning mirror. The range sensor measures the distance to a single point by projecting a narrow laser beam and timing the flight of the reflection of the beam back onto the sensor. By orienting the beam with the scanning mirror, the sensor can scan hundreds of points in the environment. Since the range is sampled at regular beam orientation intervals, the image that is generated is a discrete representation of the environment.

Figure 1.1-a is an illustration of the Cyclone a two dimensional laser range scanner [50]. The sensor consists of a laser range sensor, a mirror and a housing. Figure 1.1-b is an illustration of the area that is scanned by the sensor.

Four parameters describe the sensing characteristic of a laser range scanner: *angular field of view*, *range field of view*, *angular resolution* and *range accuracy*. The angular field of view is given by the beam orientation angles that are spanned by the sensor. The range field of view determines at what



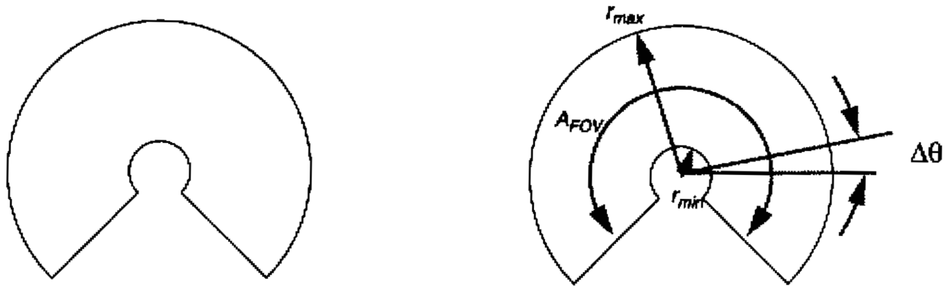
(a) The Cyclone Spot Laser Range Scanner[50]

(b) Top view of the 2D area scanned by the Cyclone

Figure 1.1 The configuration of two laser range scanners.

distance a feature can be detected by the sensor. The angular resolution is the angular distance between two consecutive beams. These three parameters describe what can be detected by the sensor.

The range accuracy is a measure of the difference between each measure and the true target distance. The angular field of view, range field of view and angular resolution are three parameters that depend on the design of the sensor. The range accuracy depends on the viewing angle of the sensor. A schematic that describes the parameters of the sensor is illustrated in figure Figure 1.2.



a) The sensor

b) the characteristics of the sensor

Figure 1.2 The parameters that describe a sensor.

1.3.2 Example 1: Finding a single viewpoint

The task for the above sensor is to scan the “front” face of a feature. The feature is illustrated in Figure 1.3. The “front” of the feature is denoted by the solid line whereas the “back” is denoted by the hash line.

Figure 1.3 The feature.

The characteristics of the sensor and the parameters that define an acceptable image are summarized in Table 1. The maximum separation between two consecutive sampled points has been defined to guarantee that two points are sampled for each segment along the contour of the feature[1] The angle of incidence has been set arbitrarily at 59°.

Table 1: The sensor characteristics and other parameters that define the image constraint.

A_{FOV}	r_{min}	r_{max}	$\Delta\theta$	*minimum #hits/ segment*	*Angle of incidence*
270°	1.00	5.00	1°	2	59°

The viewpoints that satisfy each constraint are synthesized using the techniques that are developed in Chapters 3 and 4 (see Figure 1.4). Once the set of feasible viewpoints has been generated for each constraint, these are combined (set intersection) to produce the viewpoints that satisfy all of the constraints simultaneously.

One of the points is chosen as the sensor position. The sensor is then oriented so that the entire feature is in the field of view of the sensor. The algorithm to orient the sensor is developed in Chapter 5 Figure 1.4 is an illustration of the set of viewpoints that satisfy each of the constraints. Figure 1.5 is an illustration of the orientation of the sensor.

1. This number has been selected for this example to determine the minimum separation between two consecutively sampled points. At least two points are required to determine the orientation of a line segment.

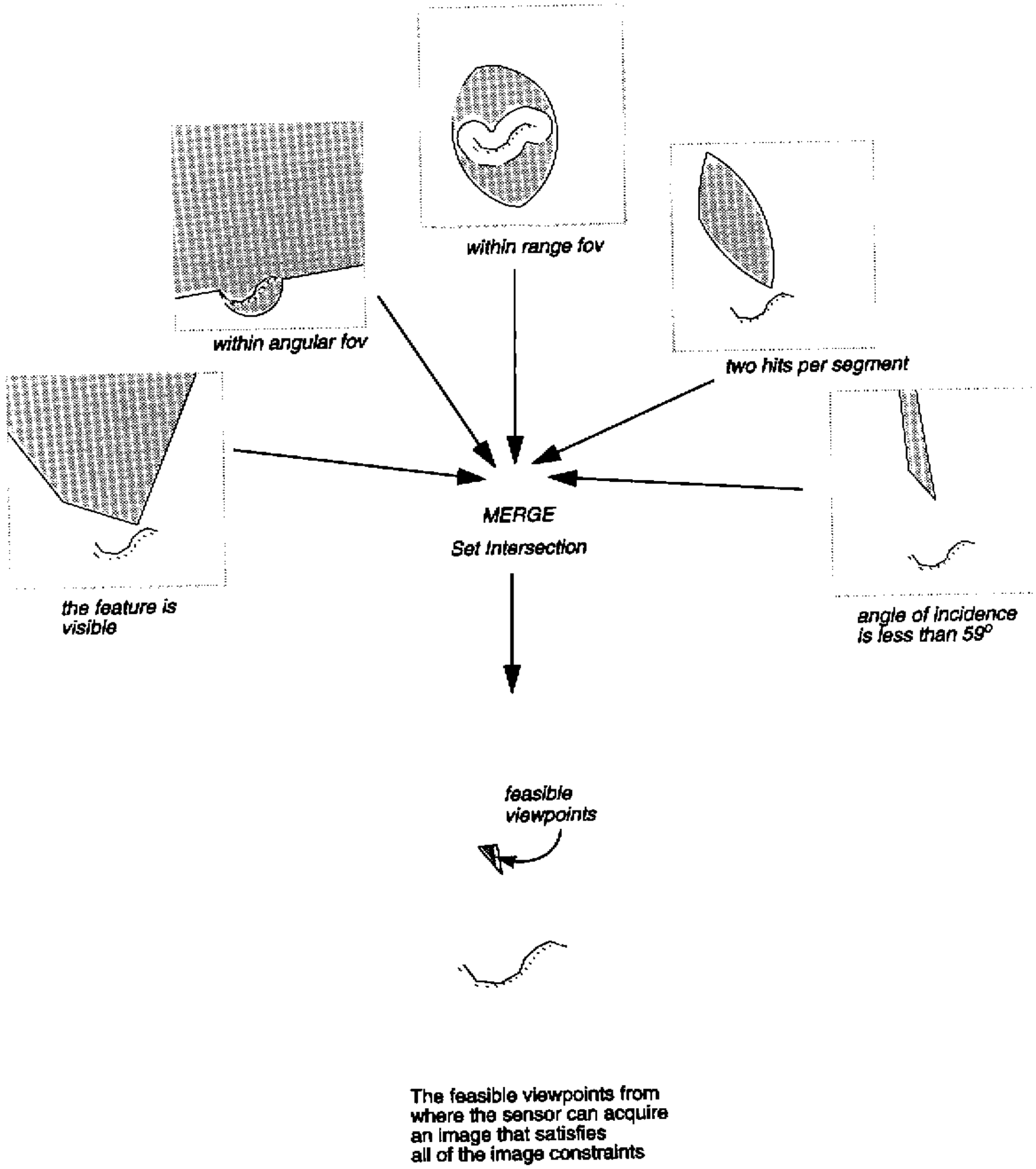


Figure 1.4 The set intersection of all of the points that satisfy each constraint is the set of feasible viewpoints.

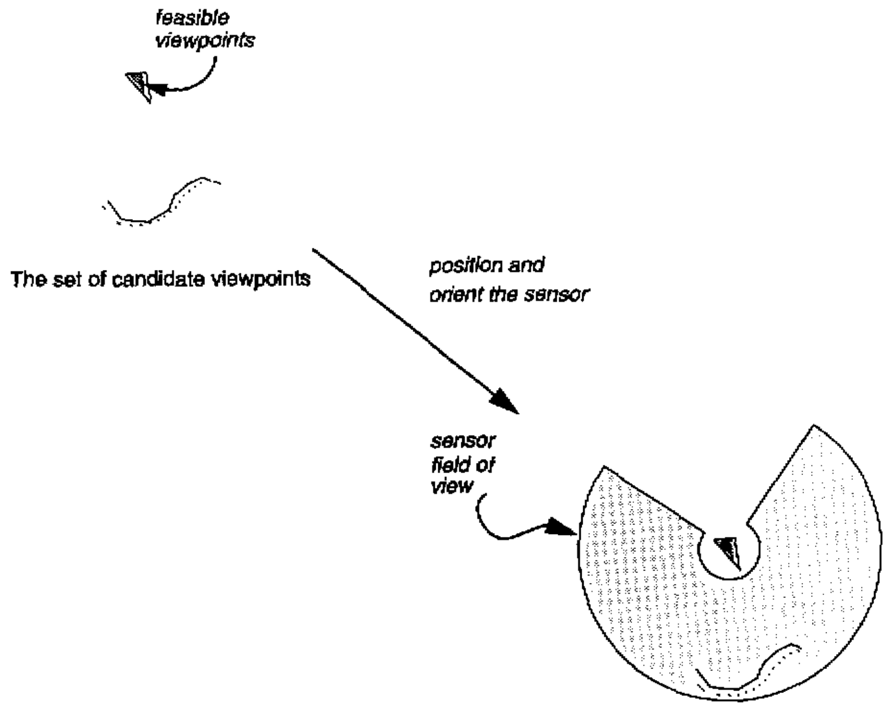


Figure 1.5 A point from the space of feasible viewpoint is selected to position the sensor. The sensor is then oriented so that the feature is in the sensor's field of view.

1.3.3 Example 2: Finding multiple viewpoints

In this example, the same sensor is used to scan the feature shown in Figure 1.6. As illustrated by the sequence in Figure 1.7, there is no viewpoint that will satisfy all of the constraints for the feature. For this particular example, it can be seen that there is no viewpoint from where the orientation of the beam striking every point along the contour of the feature is arbitrarily set to be less than 59^0.

Figure 1.6 The feature for example 2.

The viewpoints from where the sensor can acquire sufficient information to accurately and completely describe the feature are generated by recursively diving the feature until there is at least one viewpoint from where the sensor can acquire accurate and complete information for each sub-feature. The feature is subdivided at a point of inflection which divides the feature into concave and convex features. The viewpoints from where the sensor can acquire two images that contain complete and accurate information for the entire feature is illustrated in Figure 1.8.

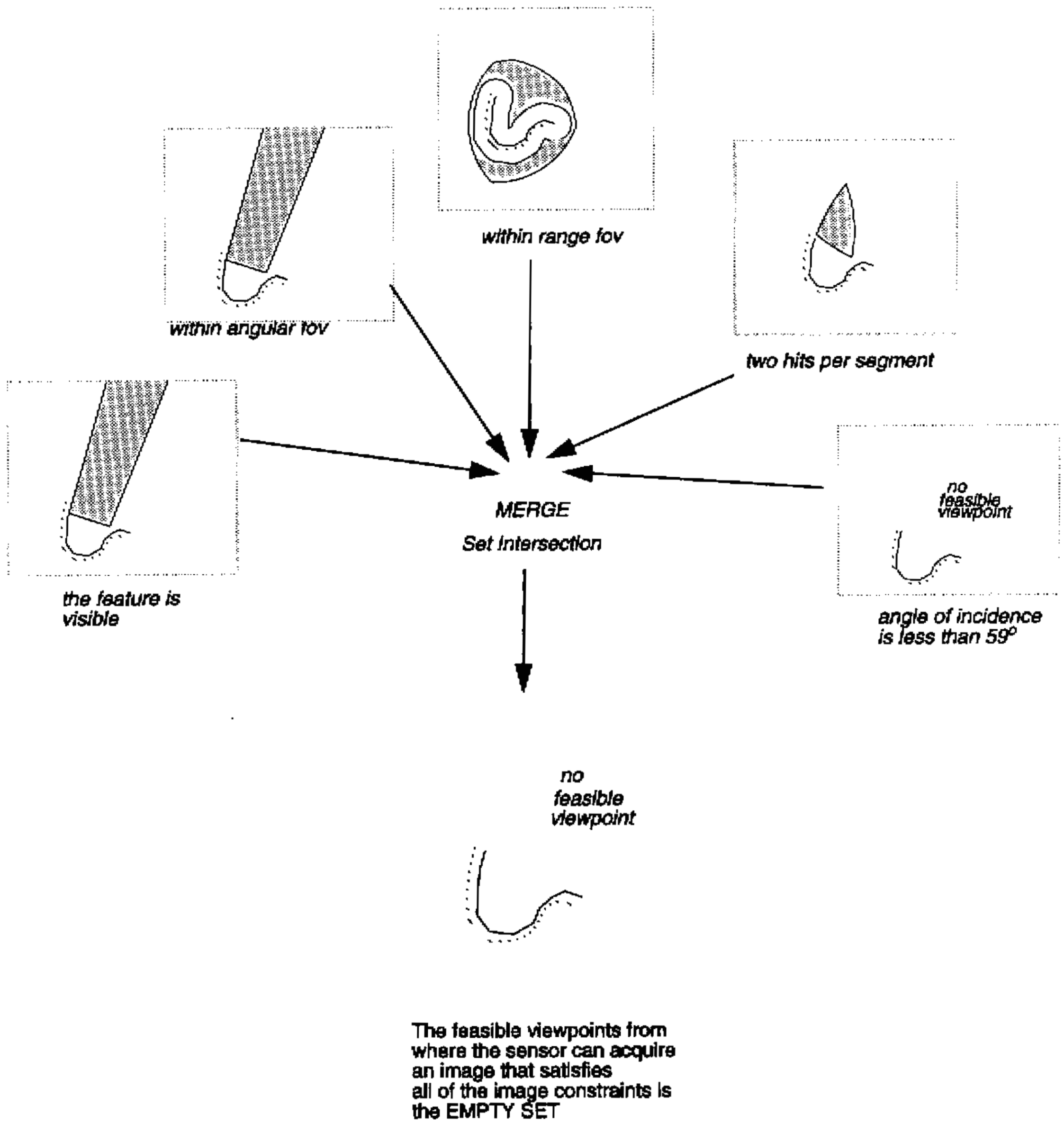


Figure 1.7 The set intersection of all of the points that satisfy each constraint is the set of feasible viewpoints. For this problem, the set is null.

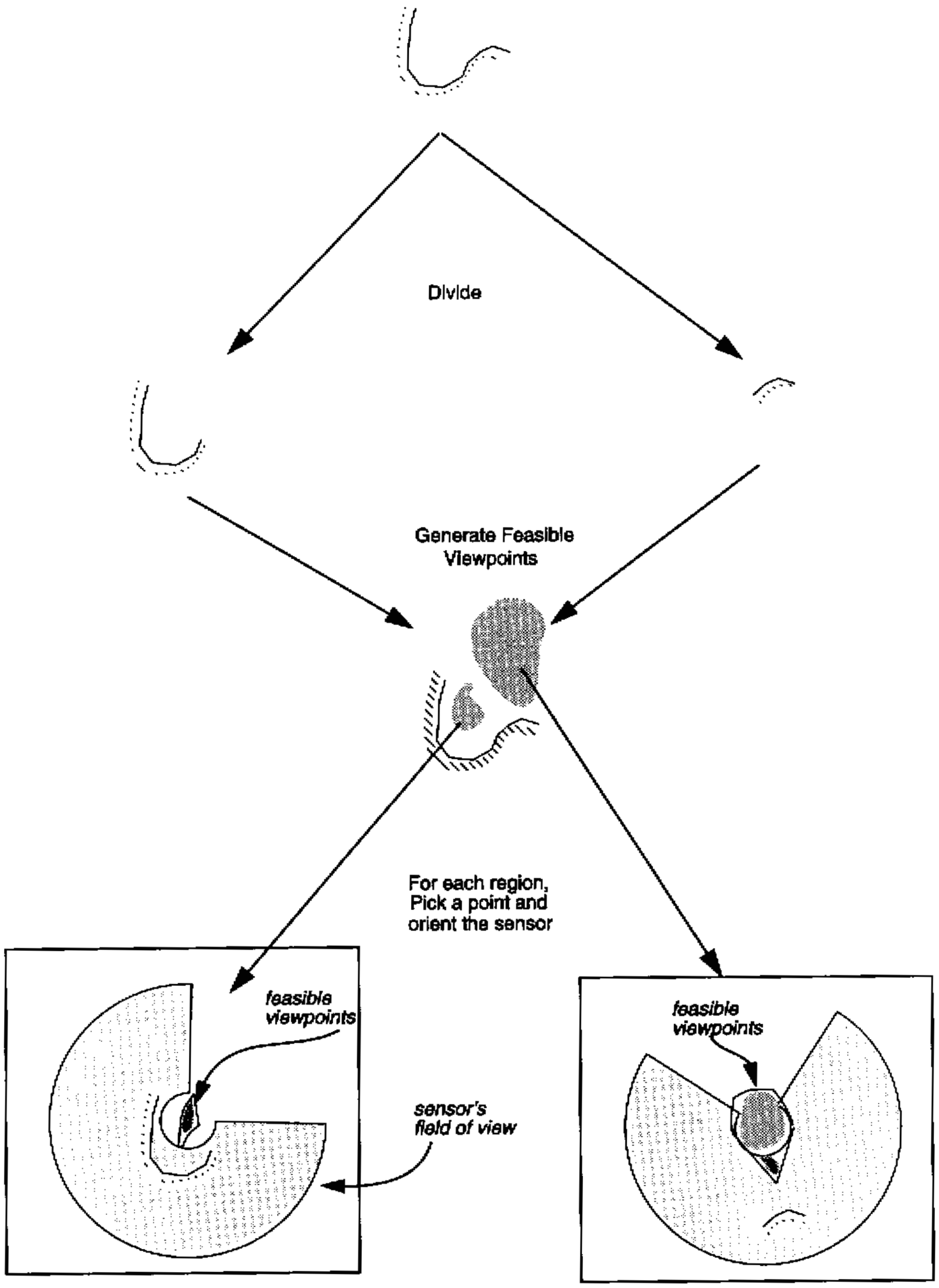


Figure 1.8 The feature is recursively subdivided until for each sub-feature there is at least one viewpoint that satisfies the image constraints. For each sub-feature a viewpoint is selected and the sensor is oriented to enclose the sub-feature in its field of view. The number of images is equal to the number of sub-features.

1.4 Representations and Implementation

This section defines some of the terms and representations that are used throughout the document.

1.4.1 Feature

In the main body of this document, methods are developed to generate the set of feasible viewpoints for a feature that is composed of a single non-intersecting polyline. These same methods are used in Chapter 7 to generate the viewpoints for a feature which is composed of multiple non-intersecting polylines. The rest of this section defines a feature, a polyline and a segment.

A feature is represented by a collection of disconnected and non-intersecting polylines. Each polyline is composed of a sequence of connected segments. The simplest feature is a single polyline which consists of a single segment.

A segment consists of an ordered sequence of points defined by a curve that is geometrically C^0, C^1 and C^2 continuous [38]. The sign of the second derivative is the same for any point along the segment. Each segment can be specified by two end-points one denoted as the *start* point and the other is *finish* point. For a polyline, the segments are connected such that the finish point of one is the start point of the next segment. At each end-point on the polyline at most two segments are connected. By convention a polyline is considered open. That is, there are two end points that can be identified as the start and finish points of the polyline. Figure 1.10 is an illustration of valid and invalid segments, polylines and features.

Every segment and polyline has a front side and a back side. The convention that is used in this thesis defines the *front* side as that which is observed along the path which follows the contour of the feature from the start point to the finish point. The *back* side of the feature is obtained by traversing the feature in the opposite direction. In all of the examples that are developed in this dissertation, the front side of a feature is illustrated by a solid black line and the back side is illustrated by a shaded line or by an arrow that follows the contour of the feature.

A feature can be concave, convex or a combination of both. By definition a *convex object* is one in which for every pair of points the straight line segment that connects them is in the interior of the object. In general, a *concave object* is one that is not convex. In this dissertation, the definition of a concave feature is slightly more restrictive. A *convex feature* is one for which the front side of the feature is convex, on the other hand, a *concave feature* is one for which the back side of the feature is convex. Concave and convex features are illustrated in Figure 1.9.

a) a concave feature b) a convex feature

Figure 1.9 Concave and convex features.

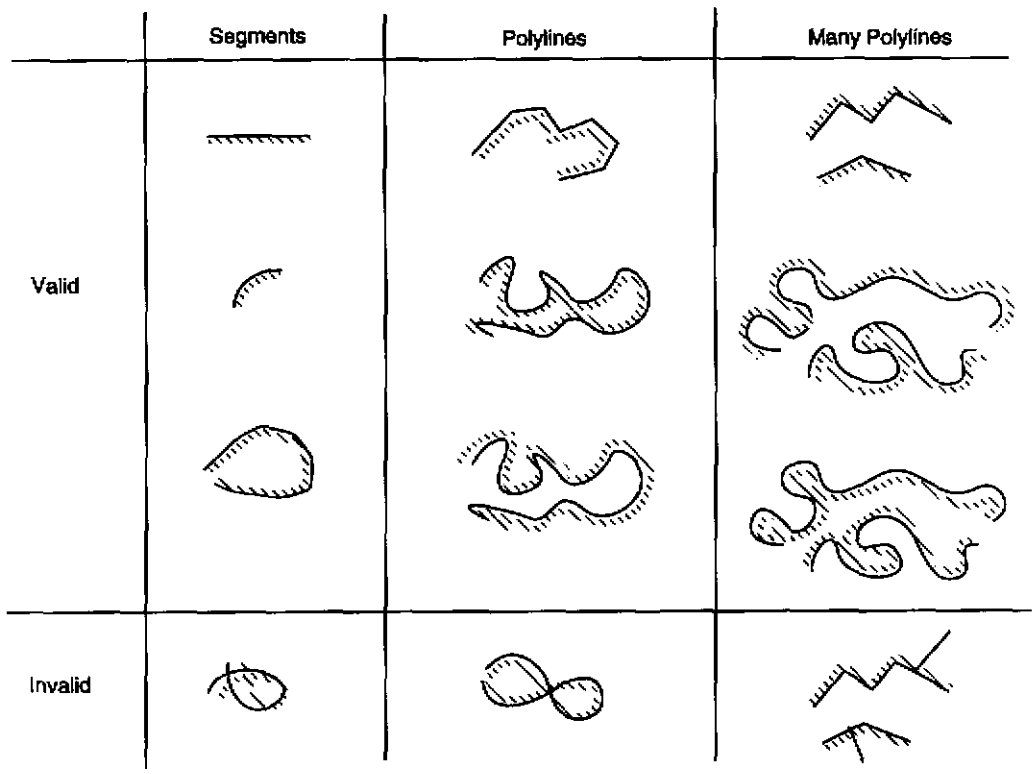


Figure 1.10 Features: a composition of segments and polylines.

1.4.2 Sensor

Algorithms are developed for sensors such as laser range scanners (described in section 1.3.1) that have a wide angular field of view, a limited range of detection, a low angular resolution and a scanning pattern similar to the one illustrated in Figure 1.2. The field of view and scanning pattern of the sensor is characterized by three parameters: *angular field of view* (A_{FOV}), *range field of view* (r_{min}, r_{max}), and *angular resolution* $\Delta\theta$. The *accuracy* of the sensor is characterized by the angle of incidence of the beam [48].

1.4.3 Image quality

The image quality is characterized by two parameters, the spatial resolution which is inversely proportional to the distance between any two consecutively sampled points and the data accuracy.

The density is a function of the distance between the sensor and target, and the angle of incidence of the beam. The closer the sensor is to the target the higher the density of points (and the smaller the distance between consecutive points). Similarly, the greater the angle of incidence the greater the distance between consecutively sampled points.

The accuracy of a data point is also a function of the beam orientation. The larger this angle, the lower the accuracy [48].

The maximum separation between consecutively sampled points and the maximum allowable beam orientation are values that are given as part of the problem.

1.4.4 Feasible viewpoints

The set of feasible viewpoints is the set of points that satisfies each constraint. The collection of points that satisfy each constraint form a set. In this document this set is represented geometrically by a closed region. For implementation purposes, sets that are open are implemented as a region for which one of the boundaries is at "infinity". One advantage of this representation is in its implementation as many set operators (such as *union, intersection,* and *difference*) have a counterpart in computational geometry.

Five operators are used to implement the algorithms: *union, intersection, difference, complement and body sweep*. *Union, intersection* and *difference* are standard boolean set operations. The *complement* operation produces a region which is equal to the set difference between a region that encloses the set of all possible viewpoints and a primitive. The *body sweep* operation produces a region that is equivalent to the union of many instances of one primitive which are closely spaced together along a contour of a given path.

1.4.5 Implementation

The methods to synthesize the set of all possible viewpoints are implemented using a geometric modeling kernel [4]. This kernel provides the necessary structures to build, modify and operate on geometric regions. The feasible regions are formed by two primitives, a block[2] and a circle, and five operators (complement, union, intersection, difference, and body sweep[3]). In this sense, the method that is used to generate the regions is analogous to the *constructive solid geometry* (CSG) method employed to build geometric solid models [38].

Algorithms are developed for curved as well as polygonal features; however, for simplicity and ease of explanation, examples are developed on polygonal features. Most of the figures and illustrations that are presented this document have been generated automatically using the algorithms presented herein. Many of these figures, have been annotated after the fact for illustration purposes.

1.5 Document Organization

The organization of the thesis is as follows.

Chapter 2. This section presents an overview of the most relevant research in the topic of automatic sensor placement. It compares the state of the art with the methods that are developed in this research.

Chapter 3. The viewpoints from where a feature is visible and detectable are developed. A feature is visible if there is an direct line of sight between the sensor and every point along the contour of the feature. A feature is detectable if it is in the sensor's angular and range field of view.

Section 3.1. The viewpoints from where a feature is visible are generated as the intersection of all of

2. A block is the implementation of an infinite *line* which divides the space of all viewpoints into two sets: the set that is in front of the line and the set that is in back.
3. The body sweep generates the area that is swept by a body as it traverses through a given trajectory. The body sweep can be used to calculate the set union of many instances of a body which are located at an infinitesimal distance to each other along a given trajectory.

the half-planes generated by the tangents for every point along the contour of the feature. Principles of convexity and concavity are exploited to reduce the number of operations that are required.

Section 3.2. Within the space in which all the points on the contour of a feature are visible, the projection of a feature onto the image plane is equal to the projection of a straight line onto the image plane. This principle is used to determine the exact positions for which a sensor with a given angular field of view is not able to detect an entire feature.

Section 3.3. A feature is in the sensor's range field of view if every point along the contour of the feature is within a minimum and maximum distance of the sensor. The viewpoints from where the feature is in the sensor's range field of view is generated from a description of the minimum and maximum range of detection of the sensor. A geometric "body sweep" is used to reduce the number of operations and extend the constraint to curved features.

Chapter 4. Two attributes characterize the quality of an image: spatial resolution and data accuracy. A feasible viewpoint is one from where the spatial resolution and data accuracy are within a given threshold. The viewpoints from where the sensor can acquire an accurate and complete image are developed in the following two subsections.

Section 4.1. The viewpoints from where the sensor can acquire an image whose spatial resolution is within acceptable boundaries is generated from a description of the sensor, the feature and the sensor's angular field of view. Three cases are observed: one in which the spacing between consecutively sampled points is a variable which is defined as one-half the length of the segment along the contour of the feature, another one in which the spacing is given by a constant arc-length distance along the contour of the feature and a third in which the spacing is given by a constant chord-length distance along the contour of the feature.

Section 4.2. The accuracy of a data point is affected by the angle of incidence of the beam. By controlling the angle of incidence of the beam it is possible to limit the worst case error in measurement.

Chapter 5. The methods developed in Chapters 3 and 4 are able to obtain the viewpoints from where there is some orientation for which the sensor will be able to detect a given feature. Any of these points is a valid sensor position. Once a viewpoint is selected, the sensor is oriented to enclose the feature in its field of view. The angle at which to orient the sensor is obtained from the spanning angle of the projection of the feature onto the image plane of the sensor.

A single viewpoint may not be sufficient to acquire an accurate and complete representation of a given feature. A method is developed to generate multiple viewpoints from where a sensor can acquire multiple images that collectively represent the feature. The viewpoints are generated by recursively subdividing the feature until there is at least one viewpoint from where the sensor can acquire accurate and complete information of the entire feature.

An example illustrates how to apply the methods developed in chapters 3 and 4 to determine the viewpoints from where a sensor can acquire an accurate and complete image of the interior of a cavity with multiple openings.

Chapter 6. An analysis of each of the constraints generates conditions under which there is a feasible solution. This same analysis is used to determine that the points of inflection and the midpoint of a concave feature are "good" locations at which to subdivide a feature to generate multiple viewpoints.

Chapter 7. A summary of the research, conclusions and future work are presented.

Chapter 2

Background

2.1 Sensor Placement

Careful placement of a sensor and sources of illumination are essential for enabling active perception systems. Currently, there have been three viable approaches for generating feasible viewpoints for sensor placement that have been described in the literature: discretizing the space of all candidate solutions[44], synthesizing the space of all feasible solutions[15] and searching for the optimally best viewpoint in the space of all possible solutions[53].

Sakane constrains the feasible viewpoints to lie on the surface of a discretized gaussian sphere centered about the object. He then uses a Z-buffering and ray casting techniques to identify the subset of the points that are candidate sensor positions [44]. Each cell on the gaussian sphere is analyzed to determine if an image acquired at this location would satisfy all of the image constraints. Coherence and regularity properties are used to group viewpoints together and reduce the amount of required computations. A technique based on the singular value decomposition is used to find the "best" sensor position.

Cowan synthesizes this space from a description of the sensor, the feature and each of the constraints[15]. The viewpoints that satisfy each constraint are combined (set intersection) to generate the space of all feasible solutions.

The third approach is to generate the optimal viewpoint algebraically. Each constraint is expressed as a non-linear equation. Then, using non-linear optimization techniques, the "best" viewpoint is located. Tarabanis does not only find the "best" sensor position, he also obtains the configuration of the sensor that will maximize the constraint [53].

The approach that is adopted in this research is to synthesize the space of all valid viewpoints. This approach has a number of advantages over the numerical optimization technique. The causal relationships between different constraints can be easily visualized and determined from a representation of all feasible viewpoints. This is important when making a trade-off analysis that will require the relaxation of one or more constraints. Furthermore, through this approach one can identify if there is an island of solutions or no solution at all. Optimization methods on the other hand do not provide any feedback of the relationships between the constraints and the parameters, they are unable to quickly determine if there is a solution, they require a starting point and are subject to finding local minima. On the other hand, analytic techniques express the sensitivity of changes in the solution to changes in the parameters.

2.1.1 Angular field of view constraint

A viewpoint is only valid if from that location the entire feature is in the angular field of view of the sensor. Two different approaches have been proposed to generate the set of valid viewpoints The first method uses the smallest circle that completely encloses the feature to generate a set of viewpoints from where the sensor is able to see the entire feature [15]. The second method projects the feature onto a vector oriented in the direction in which the sensor is pointing to determine whether or not the entire feature is visible [53]. The set of viewpoints that is generated by the first approach is a conservative subset of the true set of feasible viewpoints. The second approach requires a procedure to determine the orientation at which the sensor can see the entire feature.

Due to technical limitations, neither of these methods is able to determine viewpoints (position and orientation) that are inside the cavity of a non-convex feature from where the sensor can see the entire feature. These methods are therefore unable to determine the space of valid viewpoints for sensors which have an angular field of view greater than 180^o.

Two methods developed in this thesis overcome this limitation. The first, which is described in section 3.2, generates the complete set of viewpoints from where a sensor with a wide angular field of view is able to see the entire feature . The second method which is described in section 5.1.1 uses the projection of the feature onto a unit circle to verify that the entire feature fits in the angular field of view of the sensor and to orient the sensor to center the feature inside its field of view. Both methods apply to sensors with either a narrow or a wide angular field of view.

2.1.2 Visibility constraints

Visibility has received considerable attention in computer graphics [20], object recognition [29] and sensor placement [14][54]. Most of these methods are interested in computing the appearance of an object from a given location. Few of these methods address the inverse problem, that is to determine the viewpoints from where an object (or a part of the object) is visible.

In computer graphics binary space-partitioning trees (BSP trees) have been used to determine visible surfaces in graphics, to represent arbitrary polyhedra and to generate the volume that encloses an occluded region [11][20]. A BSP tree recursively divides the space of all viewpoints into pairs of subspaces, each separated by a plane of arbitrary orientation and position. The plane corresponds to a surface of the feature.

In computer vision aspect graphs are used to classify the appearance of an object as a parameter of the surfaces that are visible [29]. An aspect graph is constructed by encoding the transitions due to changes in occlusions and relationships between object surfaces. Different views are grouped together based on these codes.

Few methods are able to generate the viewpoints from where a feature is visible. Cowan generates the viewpoints from where a feature is occluded by another convex object by sweeping planes (called boundary supporting planes) about vertices on the feature to points and surfaces on the object. These planes become the boundaries of the occluded region. Tarabanis and Tsai extend this work to arbitrary polygons with holes [54].

Most of the references that are outlined above generate the viewpoints from where a feature will be occluded by another object but they do not generate the viewpoints from where a feature is self-occluded. Another method which is able to generate the viewpoints from where a feature is self-occluded is based on the BSP algorithm. The algorithm extends a plane at each surface and uses it to recursively divide the space of all possible viewpoints into regions from where each surface is visible or not [20]. The method requires an operation for every surface on the contour of a feature and is unsuitable for non-polygonal features. An alternative method presented in section 3.1 exploits properties of concavity and convexity of a feature to generate the space of feasible viewpoints with a minimum number of operations.

2.1.3 Range field of view

A sensor may only be able to detect a feature if it is positioned in a region that is between a maximum and a minimum range of detection. This window, known as the *range field of view*, is determined by the sensing characteristics of the sensor and is device dependent.

The range field of view of a CCD camera is determined by the depth of field. This depends on the distance between the sensor and the feature (focus distance), the aperture and focal length of the lens, and the pixel size. Because of this relationship, the viewpoints from where a feature will be in focus has to be determined iteratively [14][53].

Alternately, the range field of view of a range sensor is constant. The range field of view of a laser range scanner is determined in part by the sensitivity of the detector to an incoming return signal and is adjusted during fabrication. An algorithm developed in section 3.3 sweeps a region that describes the range field of view along the contour of a feature to determine the viewpoints from where every point on the contour of a feature is in the sensor's range field of view.

2.1.4 Spatial resolution

The spatial resolution can be interpreted as either a measurement of the distance between two consecutively sampled points or the magnification of a feature on the image. The magnification of a feature is a function of the sensor's location and optical settings; the distance betwen two consecutively sampled points a function of the sensor location and the angular resolution of the sensor.

Tarabanis et. al. present techniques to analytically determine the complete locus of camera poses and optical settings that satisfy the resolution requirements of a machine vision task[52]. They measure the magnification by the number of pixels that are occupied by a line segment on a camera image thereby interpreting resolution as the magnification of the feature on the camera image.

Cowan generates the viewpoints from where the distance between any two consecutive sampled points does not exceed a maximum allowed separation [14]. He defines the space of valid viewpoints by the set intersection of a circle (defined by the maximum allowed separation and the angular resolution of the sensor) at every point on the feature. The method is only defined for polygonal features and disregards the effects of corners or curvature.

In section 4.1 a method is developed to generate the viewpoints from where the distance between any two consecutive sampled points does not exceed a maximum allowed separation. The viewpoints are generated for an arbitrary polygonal and curved feature and for variable spatial resolution.

2.1.5 Range accuracy

The range accuracy quantifies the difference between the sensor range measurement and the true target distance. For a laser range scanner, this accuracy is proportional to the viewing angle of the sensor. By restricting the maximum viewing angle it is possible to restrict the magnitude of the range error.

Resarch on sensor placement has concentrated primarily on the constraints that will determine the position of a camera. But range accuracy affects range scanners does not affect cameras. Therefore, the issue has not been dealt with in the sensor planning literature.

In section 4.2, methods to synthesize viewpoints that constrain the viewing angle are generated for an arbitrary curved feature. Methods to characterize the range accuracy of a laser range scanner have been developed by Kweon et al. [33] and Sedas and Gonzalez [48].

2.2 Sensor Orientation

A feature cannot be detected if the sensor is not oriented towards the feature. The techniques that are outlined in section 2.1.1 through 2.1.5 describe the set of valid viewpoints independent of the sensor orientation. The following can be used to orient the sensor once a sensor position has been selected.

Method that are used to determine the orientation at which a sensor can see an entire object are unable to determine the orientation of the sensor when the sensor is inside the cavity of a concave object. Anderson [3] proposes to enclose an object by a minimum spanning circle (or sphere). This circle when projected onto the unit circle, projects a minimum spanning cone that completely encloses the sphere. The bisector of this cone is the orientation of the feature. Cowan on the other hand chooses the orientation as the bisector of the angle that is subtended when the sensor is looking at the feature from any point on the contour of the circle.

A method proposed in this thesis projects the feature onto a unit circle centered about the sensor.

Each projection cuts a sector of the circle; the union of these is the minimum spanning sector for the object. The vector that bisects the sensor gives the orientation of the sensor.

2.3 Dynamic Sensor Planning

Stationary cameras may not be adequate in dynamic environments where moving objects can occlude the visibility of the sensors. Abrahams et. al. mount a camera on a manipulator thereby giving the sensor mobility [1]. They calculate the position of the camera by predicting the motion of the moving object, expressing the motion by a swept volume. Then they use the swept bodies as occluding bodies, thereby reducing the dynamic sensor problem to static. If no solution is found then a temporal interval search is performed to find the largest time intervals which can be monitored by a single viewpoint.

2.4 Decision Theoretic Models

There is a cost associated with executing any sensing action (the sensor must be relocated, an image processed, and so forth). Decision theoretic models such as those that are investigated by Hager [23] and Durant-White [6] consider the cost of executing an action when searching for an optimal sensing strategy.

2.5 Searching for Missing Information

One aspect of active perception is to search for missing information. Although a perception system may not have sufficient information to know what it has not seen, it may have sufficient information to determine what information is missing and to identify areas that were occluded, inaccessible or simply have not yet been explored. With this information, a sensor planning system can select the viewpoint from where to acquire missing information.

Kim[31] and Lee and Hann [35] identify features that will eliminate the ambiguity in an object recognition system and use the models of the objects to determine the viewpoint from where the sensor can acquire the missing information. Maver determines from a range image of an object areas that are occluded from the sensor [36]. With this information he determines the next illuminating and viewing directions for a line stripe sensor to acquire the missing information. Xie developed a scheme to plan viewpoints and viewing directions for a mobile robot from a partial description of an environment [58][59]. The description of the environment is built incrementally with each image.

2.6 Reachability

In order for a sensor to observe a feature from a chosen viewpoint, it must be able to reach that viewpoint from its current location. Thus, the set of valid sensor positions are those positions that are reachable by the sensor and from where the image that is acquired satisfies the image constraints [47].

Chapter 3

Visibility and Detectability Constraints

In order for a sensor to detect an object, (i) the sensor must be facing the object, (ii) the sensor's view must be unobstructed, (iii) the object must be in the sensor's angular field of view, and (iv) the object must be within the sensor's operating distance range. Each constraint divides the space of all possible viewpoints into two main categories: those from where the sensor can see the object, and those from where it cannot. The methods to generate viewpoints from where a feature is visible and detectable are developed in this chapter.

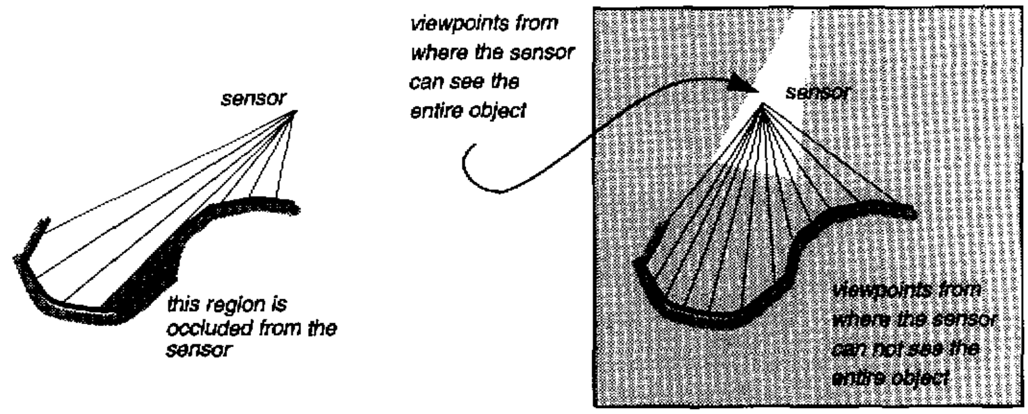
3.1 Visibility Constraint

An object is visible if its features are all facing the sensor. This constrains the location of the sensor since the sensor will only be able to see the feature from positions that are in front of every point of the feature. Although there are many possible sensor positions, only from a few of these is the sensor able to see the entire feature. For example in Figure 3.1-a, the sensor is not able to detect the entire feature from where it is located. The illustration in Figure 3.1-b is a map of the points from where the sensor can see the entire feature.

3.1.1 Generating the valid viewpoints

A feature is only visible from a position from where a sensor can see every point along the contour of the feature. The viewpoints from where all of the points are visible are constrained by the tangents along the curve of the feature.

At every point along the contour of the feature there is a line that is tangent to the feature. This line divides the space of all possible viewpoints into two regions: the points from where the sensor can "see" the point and the points from where it can't. The set intersection of the positions from where the sensor can see each point along the contour of the feature is the region that encloses the viewpoints from where the sensor can see the entire feature. Figure 3.2-a is an illustration of the tangent line and



b) A position from where the sensor cannot sense t he entire object

b) A map of viewpoints from where the sensor can get a clear view of the object

Figure 3.1 Areas from where the sensor can get an un-occluded view of an entire object.

the set of feasible viewpoints for a single point on the contour of the feature. Figure 3.2-b is an illustration of the viewpoints for the entire feature. The light lines represent the tangent lines at different points along the contour of the feature.

If the region from where every point on the contour of the feature is visible is labeled H, then:

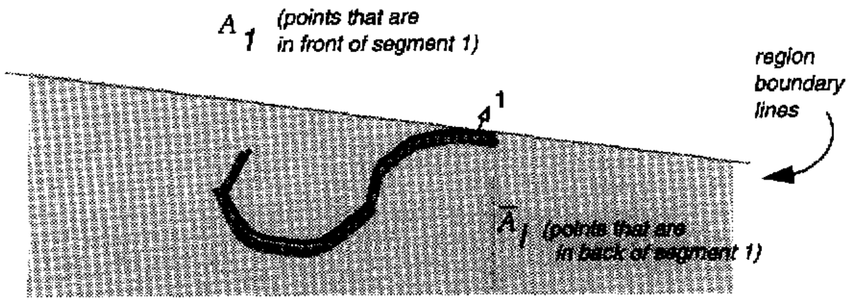
$$H = \bigcap_{i=1}^{n} A_i \qquad [3\text{-}1]$$

A_i is the region from where a point i on the contour of the feature is visible.

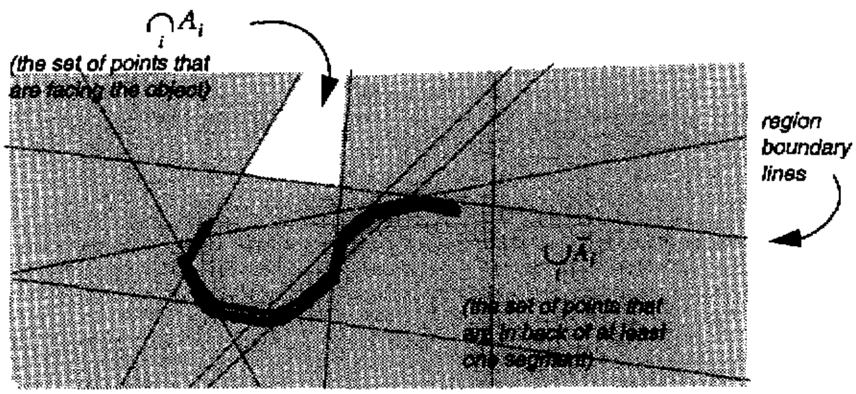
One of the drawbacks of this approach is the number of set operations that are required to determine the viewpoints from where a feature is visible. Fortunately, the piecewise convexity or concavity of a feature can be exploited to reduce the number of operations.

3.1.2 Reducing the number of operations

Geometrically, evaluating equation [3-1] requires a lot of resources. Fortunately there are geometric properties of convexity and concavity that apply to segments of a feature that can be used to reduce the number of operations that are required to evaluate the region from where a feature is entirely visible.



(a) An imaginary line that is colinear with the segment divides the space of view points into a set that is "in front" of the segment and a set that is "in back".



(b) The set of points that are in front of all three segments is the intersection of each set of points that is in front of each segment.

Figure 3.2 Generating the points that are "in front" of an object.

For a concave feature, the points from where the entire feature is visible is enclosed by a region that is bounded by the contour of the feature and two tangent lines at each of the end points of the feature. Geometrically, this region can be expressed in terms of four regions and four operators. On the other hand, for a convex feature, the set of valid points is bounded by just two tangent lines at each of the end points of the feature. This region can be generated with a single intersection operator.

A feature that has a mixture of concave and convex segments can be divided into sub-features that are only one or the other. The intersection of the set of viewpoints from where each of these viewpoints is visible is the set of viewpoints from where the entire feature is visible.

The following sections elaborate on the methods used to generated the viewpoints for concave, convex and mixed features. Dividing a mixed feature into concave or convex segments is discussed in section 3.1.5.

3.1.3 Concave features

A concave feature divides the space of all sensor positions into a convex region that encloses the viewpoints from where every point on the feature is visible and a concave region that encloses the viewpoints from where at least one point on the contour is not visible. This region can be determined as the set intersection of the half-planes defined by the tangent lines at every point along the contour of the feature. Generating the region by geometrically intersecting all of the half-planes is computationally expensive. Fortunately the same region can be expressed in terms of four regions and four operators.

A concave feature and U the space of all sensor positions

The set of viewpoints from where every point on the segment is visible

Figure 3.3

The region H, that encloses the viewpoints from where every point along the contour of a concave feature is visible is given by:

$$H = (A_L \cap A_0 \cap A_n) \cup V \qquad [3\text{-}2]$$

where A_0 and A_n are half-planes defined by the tangent lines at the two end points of the feature, A_L is a half-plane obtained from a line connecting the two end points, and V is the region enclosed by the convex hull of the feature. The proof for equation [3-2] is given next.

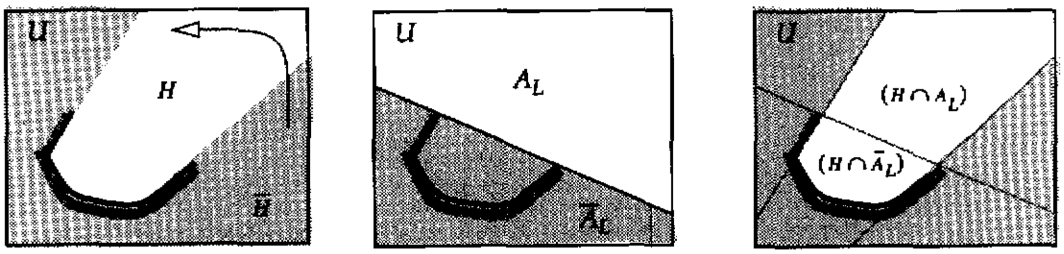
3.1.3.1 Proving that $H = (A_L \cap A_0 \cap A_n) \cup V$.

The proof of the identity is shown by reducing equation [3-1] to equation [3-2].

Let L be the infinite line that passes through the two end points of the feature and l be the segment on this line between the two end points. Line L divides the space of all possible viewpoints into two regions A_L and $\overline{A}_L$. Equation [3-1] can be rewritten as:

$$H = (H \cap A_L) \cup (H \cap \overline{A}_L) \qquad [3\text{-}3]$$

See Figure 3.4.



a) H b) A_L c) $H = (H \cap A_L) \cup (H \cap \overline{A}_L)$

Figure 3.4

The intersection of H and $\overline{A}_L$ defines a region V which is a subset of H. This region is equal to the convex hull of the feature. V then is defined by:

$$V = H \cap \overline{A}_L$$

As will be proved in the sequel,

$$(H \cap A_L) = A_L \cap A_0 \cap A_n \qquad [3\text{-}4]$$

Substituting [3-4] into equation [3-1] yields:

$$H = (A_L \cap A_0 \cap A_n) \cup V \qquad [3\text{-}5]$$

Equation [3-5] can be implemented with three intersections and one union operations. Regions A_l, A_0 and A_n are each obtained by dividing the space of all possible viewpoints by a half-plane. Region V is region enclosed by the convex hull of the feature.

Proof of $(H \cap A_L) = A_L \cap A_0 \cap A_n$:

To prove the identity it is necessary and sufficient to prove that every point in $(H \cap A_L)$ is also a point in $A_L \cap A_0 \cap A_n$ and that every point not in $(H \cap A_L)$ is not in $A_L \cap A_0 \cap A_n$. Proving that a point in $(H \cap A_L)$ is also a point in $A_L \cap A_0 \cap A_n$ is trivial since by definition A_0, A_n and A_L are subsets of $(H \cap A_L)$. The difficult part of the proof is in showing that every point that is not in $(H \cap A_L)$ is also not in $A_L \cap A_0 \cap A_n$.

Assume that w is a point that is not in $(H \cap A_L)$ but is in $(A_L \cap A_0 \cap A_n)$. If t w is not in $(H \cap A_L)$ it is either in $(\overline{H} \cap \overline{A}_L)$, $(\overline{H} \cap A_L)$, or $(H \cap \overline{A}_L)$. If it is in $(\overline{H} \cap \overline{A}_L)$ or $(H \cap \overline{A}_L)$ then it is in $\overline{A}_L$, and it cannot be in $A_L \cap A_0 \cap A_n$ as this contradicts the statement that w is not in $(H \cap A_L)$ but it is in $(A_L \cap A_0 \cap A_n)$.

All that remains to be disproved is that w can be in both $(\overline{H} \cap A_L)$ and $(A_L \cap A_0 \cap A_n)$. Suppose otherwise. This implies that

$$w \in (\overline{H} \cap A_L \cap A_0 \cap A_n) \qquad [3\text{-}6]$$

If $w \in (\overline{H} \cap A_L)$, then there must be at least one point i along the contour of the feature such that $w \in \overline{A}_i$. Adding this relation to equation [3-6] implies that:

$$w \in (\overline{H} \cap A_L \cap A_i \cap A_0 \cap A_n) \qquad [3\text{-}7]$$

For equation [3-7] to be true, w must be in $(A_L \cap A_i \cap A_0 \cap A_n)$. In for order for w to be in this region, the orientation of the vector (N_i) that is perpendicular to the tangent line at a point i must be outside of the range of angles spanned by the vectors that are perpendicular to the tangent line of every other point on the contour of the feature (see Figure 3.9). But, a characteristic of a convex contour is that the angle of each vector that is perpendicular to the tangent line at every point along the contour of the feature lies within the angles spanned by the vectors that are perpendicular to the tangent line of the feature at the two end points. Therefore, $(A_L \cap A_i \cap A_0 \cap A_n)$ is empty in which case equation [3-6] is false, thereby proving that every point that is not in $(H \cap A_L)$ is also not in $A_L \cap A_0 \cap A_n$.

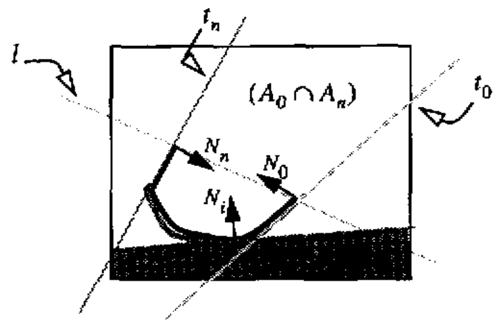


Figure 3.5 The normals of the tangent lines to the feature at a point i and the two end points 0 and n. The normals are labeled N_0, N_i and N_n. The tangent lines t_0, t_i and t_n.

A concave feature divides the space of all sensor positions into a convex region which encloses the viewpoints from where the sensor is facing the feature and a concave region which encloses the viewpoints from where the sensor is not facing at least one of the points along the contour of the feature. The boundary that separates these two regions is composed of the feature and two rays which are tangent to the feature at its end points Every point in the region is in front of every tangent line on the contour of the feature. Any point that is not in this region is behind at least one of the tangent lines on the contour of the feature and is not a visible point.

A concave feature and U the space of all sensor positions

The set of viewpoints that are "in front" of every segment that describes the object

Figure 3.6

Suppose U is the set of all possible viewpoints, bounded by an arbitrarily large region, second V is the region that is enclosed by the convex hull of the feature. Let A_L denote the set of viewpoints that are facing a line l that connects the two end points of the feature. Let A_0 and A_1 be the viewpoints that are facing each of the two lines that are tangent to the feature. These quantities are illustrated in

Figure 3.7. The set of valid viewpoints is thus given by:

$$U \cap (A_L \cup V) \cap A_1 \cap A_0 \qquad [3\text{-}8]$$

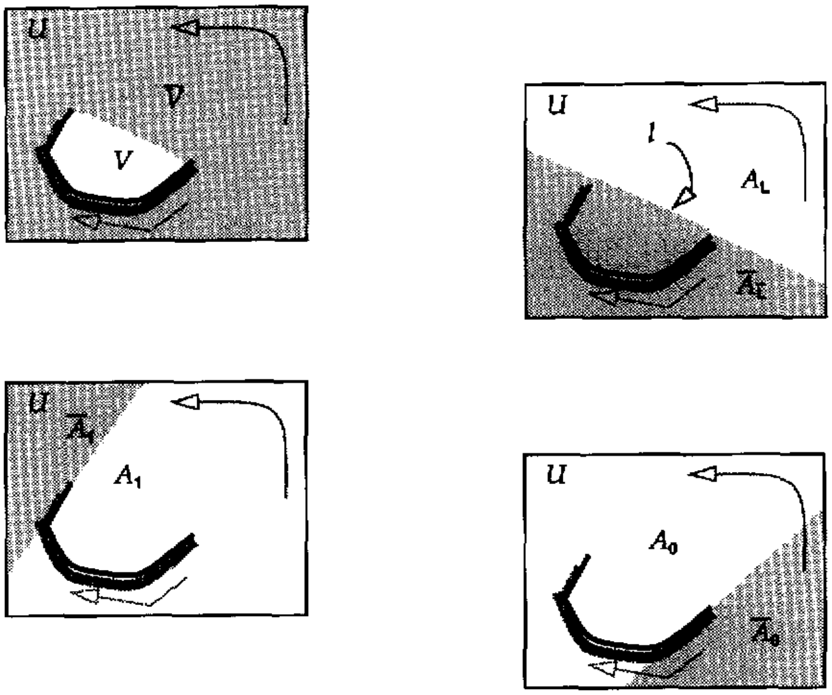


Figure 3.7 An illustration of the different regions that decompose the set of all possible viewpoints.

3.1.3.2 Spiralling concave features

The method described by equation [3-2] fails when the feature is a concave spiral. A feature is a *concave spiral* if for some point in the cavity of the feature the projection of the feature onto a unit circle centered about that point is the unit circle. In contrast, the projection of a concave feature that is not a spiral is only a sector of this circle.

A concave spiral feature can be divided into sub-features that are not concave spiral. The viewpoints from where the entire feature is visible is the intersection of the viewpoints from where each of the non-spiral concave sub-features is visible.

3.1.4 Convex features

A convex feature also divides the space of all possible sensor positions into two regions. However, unlike a concave feature, the region is bounded by two tangent lines at each end point of the feature (Figure 3.8). The region from where a convex feature is visible is given by

$$A_1 \cap A_0 \qquad [3\text{-}9]$$

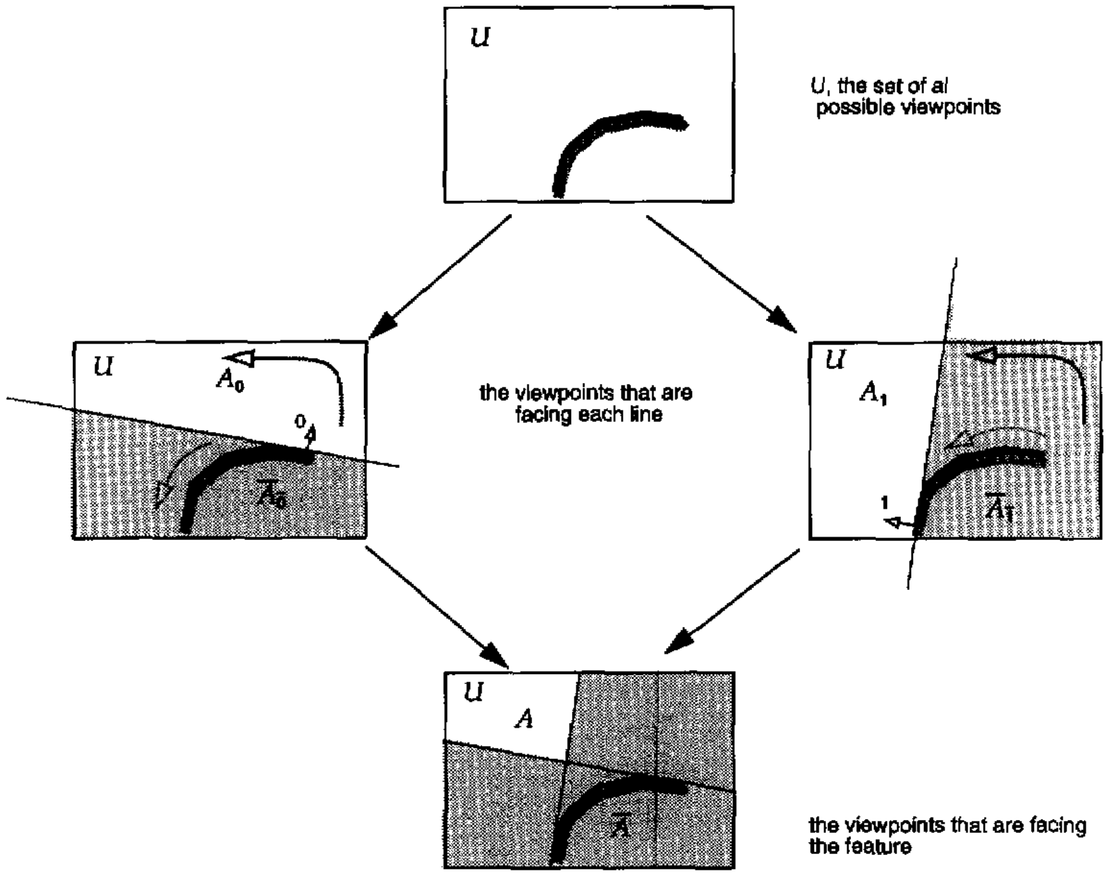


Figure 3.8 The viewpoints that are facing a convex object are the intersection of two regions generated from two tangent lines at each of the endpoints.

In order to prove that $(H = \bigcap_{i=1}^{n} A_i) = A_n \cap A_0$, it is necessary to prove that $\bigcap_{i=1}^{n} H_i$ and $A_n \cap A_0$ are subsets of each other. The proof compares each tangent half-plane A_i against the region specified by equation [3-9] and shows that $(A_n \cap A_0 \cap A_i) = (A_n \cap A_0)$. The proof assumes a polyline feature but also applies to curved features.

For each point i along the contour of the feature there is a tangent line which defines a half-plane. The inside of this half-plane corresponds to the convex side of the feature at point i. Let the tangent lines be denoted t_i, and the corresponding half-planes A_i.

The proposition then states that

$$(H = \bigcap_{i=1}^{n} A_i) = A_n \cap A_0 \qquad [3\text{-}10]$$

where, A_0 and A_n, the sets of viewpoints that are facing the two tangent lines at the end points of the feature.

Let p_0 and p_n denote the endpoints of the feature. Let q denote the point of intersection of the tangent lines t_0 and t_n. Due to the convexity of the feature, every tangent line t_i (i =1 to n-1) intersects the lines t_0 and t_n at two points that are within the respective segments p_0q and p_nq (Figure 3.9). As a result, the region $A_n \cap A_0$ is a subset of A_i (i = 1 to n), and thus $(A_n \cap A_0 \cap A_i) = (A_n \cap A_0)$.

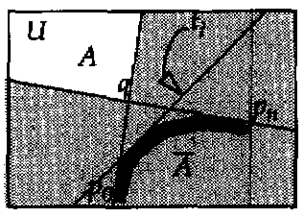
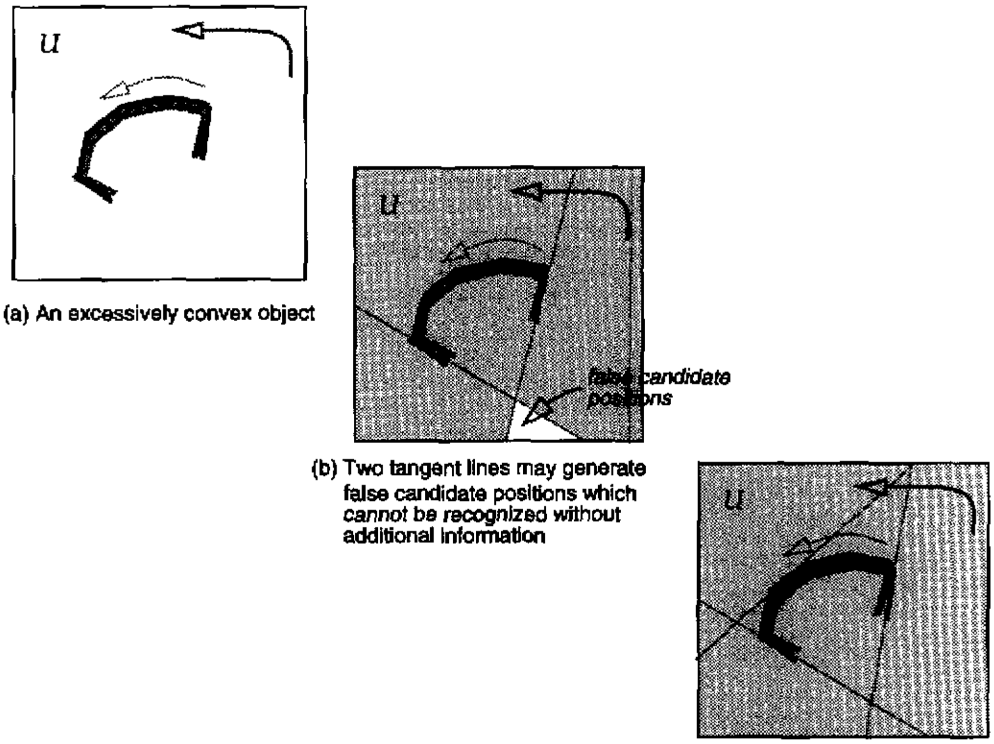


Figure 3.9 The tangent line t_i intersects the lines t_1 and t_n within the segments p_0q and p_1q.

3.1.4.1 Excessively convex or spiralling convex features

A sensor is unable to see an entire convex feature when the shape of this feature is such the orientation angles of every vector normal to the contour of the feature span an angle that exceeds 180^o. A feature with these characteristics is called an *excessively convex* feature and when the spanning angle exceeds 360^o, the feature is called a *spiralling convex* feature.

A sensor is not able to simultaneously observe the front and back of a feature. In order for a sensor to see a point, the angle between the normal vector and a vector from the sensor to the point must be less than or equal to 90°. When the angle that is spanned by the normal vectors is greater than 180°, there is at least one pair of distinct points on the contour of the feature for which the surface normals at these two points are facing in opposite directions. The angle between the normal vector and at least one of the vectors between the sensor and each of the points is greater than 90°.



(a) An excessively convex object

(b) Two tangent lines may generate false candidate positions which cannot be recognized without additional information

(c) A third segment tangent to the feature provides sufficient information to resolve the ambiguity.

Figure 3.10 A third segment defined at a point on the contour of a feature can generate the set of candidate positions that cancel out any false candidate positions that may have been generated.

If a feature is spiralling or excessively convex, it is possible to define a collection of points on the contour that divides the feature into sub-features that are not excessively convex. A tangent line at each of these points divides the space of all viewpoints into regions from where each point is visible.

The set intersection of these regions with the valid viewpoints of the two tangent lines (defined at the end points of the feature) is empty. This illustrates that there is no viewpoint from where every point on the feature is visible (Figure 3.10-c).

3.1.5 Mixed concave and convex features

The algorithms that apply to an entirely concave or entirely convex feature do not apply directly to a feature that is partially concave and partially convex. They do apply however to sub-features that are either concave or convex. The valid viewpoints for a mixed concave and convex feature is obtained by intersecting the viewpoints that are obtained from segments of the feature that are either concave or convex. See Figure 3.11.

The points at which the feature should be divided are those points at which the curvature of the feature changes directions. For a polygonal feature the points of inflection are vertices, for a curved object, the points of inflection are those where the second derivative of the contour is zero.

3.1.6 Summary

In order to detect a feature, the sensor has to be placed in a position that is facing every point of this feature. The set of valid viewpoints can be obtained from the intersection of the viewpoints from where each of the points on the feature is visible. Each of these regions is identified by a line tangent to the feature that divides the space of all possible viewpoints into a set from where the point is visible and a set from where it is not.

The number of operations required to find the set of valid viewpoints can be substantially reduced. When the feature is concave, the set of viewpoints is bounded by the contour of the feature and two lines that are tangent to the feature at the feature's two end points. When the feature is convex, the set of viewpoints is the intersection of the viewpoints from where two tangent lines at the end points of the feature are visible. When the feature is part concave and part convex it can be divided into entirely concave or entirely convex features.

Excessively concave/convex and spiralling concave/convex features have important geometric characteristics. If a convex feature is excessively convex there is no viewpoint from where the sensor can detect the entire object. If a concave feature is excessively concave or spiralling concave, there

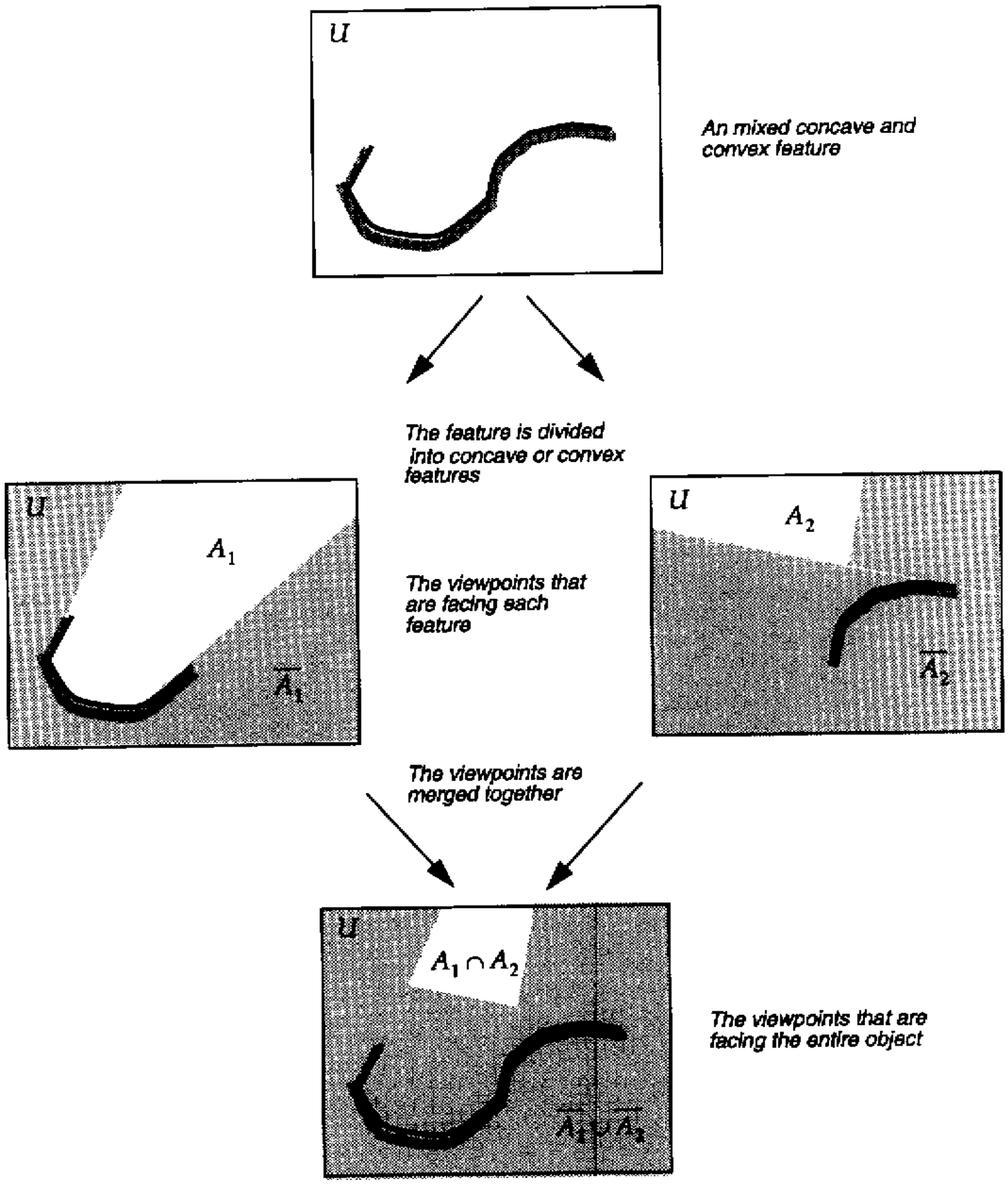


Figure 3.11 The viewpoints that are facing a mixed concave and convex feature.

may be valid viewpoints. To generate these viewpoints, the feature must be decomposed into sub-features that are not excessively concave. The set intersection of the viewpoints for each sub-feature is the region from where the entire feature is visible.

3.2 Angular Field of View Constraint

In order to be seen, a feature must be in the sensor's field of view. If the feature is not in the sensor's field of view, it may be because the angular field of view is too small, because the sensor is too close to the feature, or because the sensor is not pointing in the right direction. In the following sections a method is derived to generate the locations from where an entire feature will fit in the sensor's angular field of view provided that the sensor is properly oriented. A method to properly orient the sensor is developed in Chapter 5.

3.2.1 The angular visibility arcs

Whether or not a sensor will be able to detect a complete object depends on the sensor's position, the angular field of view, the features that are visible, and the geometry of the object. Because of the way these elements are interrelated, it is not easy to determine where to place the sensor so that it can see the entire object.

Figure 3.12 illustrates a series of maps that describe the positions from where a sensor will be able to see an object[1]. Each map has been generated by spanning space of all sensor orientations and at discrete intervals comparing the minimum angular field of view required to detect the feature (M_{FOV}) against the angular field of view of the sensor. Three maps (Figure 3.12-a, b & c) have been generated by thresholding the M_{FOV} against different sensor angular field of view. The map in Figure 3.12-d is a contour plot that displays the boundaries of regions that have a constant M_{FOV}.

The boundary of a region along which the M_{FOV} is constant is formed of connected arc segments (Figure 3.12-d). These arc segments, labeled *angular visibility arcs*, enclose a collection of viewpoints from where the sensor will not be able to see the entire object. From any point on the curve, the minimum field of view required for the sensor to see the entire feature is equal to the angular field of view of the sensor. From any point that is enclosed by the curve, the angular field of view needed to see the feature exceeds the angular field of view of the sensor.

Suppose that the feature to be detected is a single straight line segment of length L and that the

1. The emphasis of this thesis is to determine the viewpoints from where a sensor can detect a feature, for illustration purposes, the entire object provided a better example to illustrate the point.

angular field of view of the sensor is α and $\alpha < 180^o$. The boundary of the region at which the M_{FOV} is constant is a circle that is defined by a radius R and a position x_c, y_c such that[2]:

$$C = \begin{cases} R = \left|\dfrac{L}{2Sin(\alpha)}\right| \\ (x_c, y_c) = (\dfrac{L}{2}, RCos(\alpha)) \end{cases} \quad [3\text{-}11]$$

From any point on the circumference of this circle, the angular field of view required to see the line segment is equal to α[3]. From any point that is inside this circle the angular field of view required to see the line is greater than α while for any point that is outside this circle it is less than α. See Figure 3.13.

The visibility region for a feature that is composed of multiple curved and straight segments is not generated by an intersection or union operation of the circle over every point along the contour of the feature[15]. Instead the sensor's angular field of view is a collective property of the feature and cannot be derived from each independent point. Thus, while the individual points of a feature may fit within the sensor's angular field of view, the feature as a whole may not. This suggests that the region that describes the set of feasible viewpoints is derived from the feature as a whole and not as a collection derived from sub-components of the feature.

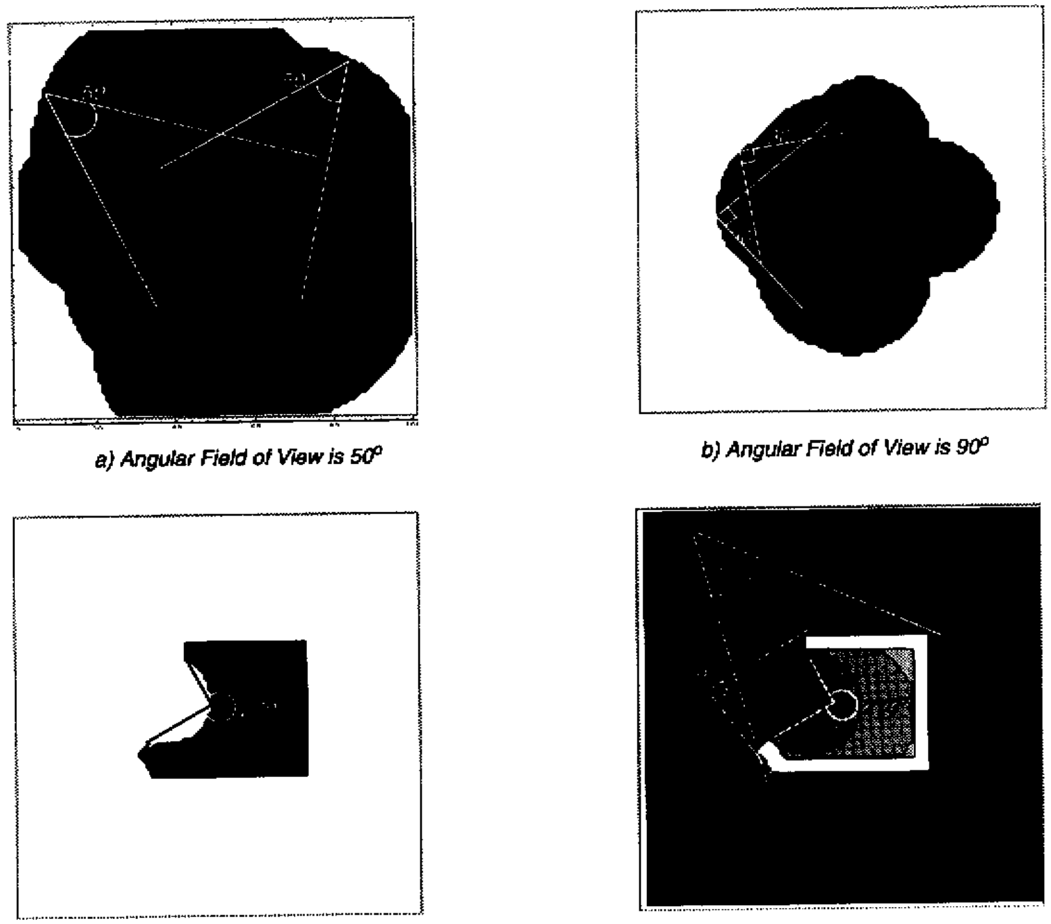
Work in the area of sensor placement has overcome this limitation by defining the set of feasible viewpoints as those that are outside a circle that completely encloses the feature [15]. The dimensions of this circle are a function of the dimensions of the smallest circle that completely encloses the feature and the angular field of view of the sensor. While this approach generates a region of feasible viewpoint for a sensor with a narrow field of view, it is unable to produce the exact region and to generate even a single viewpoint that lies inside the cavity of a concave feature.

3.2.2 The set of feasible viewpoints

The sum of the projection of any partition of a feature onto an image plane is equal to the projection of the two end points of the feature. If every point on the feature is visible, the points are ordered in such

2. Without the absolute value of R, Equation [3-11] would only be valid for an angular field of view that is less than 180^o. With the absolute value, the equation is valid for α less than or α greater than 180^o. Note at $\alpha = 180^o$, R goes to infinity.

3. This is a well known property of a circle.



a) *Angular Field of View is 50°*

b) *Angular Field of View is 90°*

c) *Angular Field of View is 270°*

d) *Boundaries of regions with a constant Angular Field of View*

Figure 3.12 Places from where a sensor will be able to see an entire object.

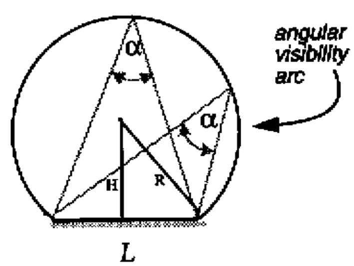


Figure 3.13 The angular visibility arc of a line segment L, due to the sensor's angular field of view α. ($\alpha < 180^0$)

a way that the sign of the magnitude of the projection of any segment of the feature onto the image plane is the same for any segment of the feature[4]. Therefore, *Statement 3-1: From any point that is inside the region from where the entire feature is visible, the minimum angular field of view required to detect the entire feature is equal to the minimum angular field of view that is required to see the two end points given that the sensor is oriented towards the feature.*

The importance of this observation is that for any arbitrary feature the viewpoints from where a sensor can enclose an entire feature within its angular field of view is equal to the set of viewpoints from where the sensor can enclose a line segment that connect the two end points of the feature.

Therefore, if the feature is not a straight line, but rather a complex feature composed of curved and straight arc segments, the angular visibility arc is still a circle. However, the center and radius of the circle is defined by the chord connecting the two end points of the feature. The two end points are termed the *predominant features of projection* and the chord between the two end points is termed the *equivalent feature of projection* (Figure 3.14).

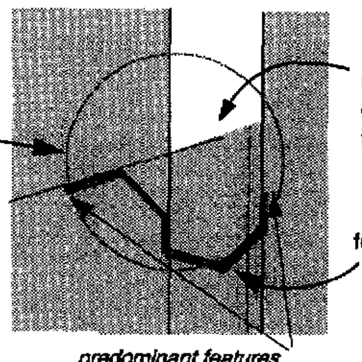


Figure 3.14 Within the region where the entire feature is visible, the end points of a feature define the *predominant features of projection*

3.2.2.1 Proof of statement 3-1

Let the feature be a sequence of consecutive points labeled $0, 1, \dots n$. Let w be a point in the space from where the entire contour is visible. Let $\vec{w}_i$ and $\vec{w}_j$ be two vectors connecting points i and j which are on the contour of the feature to w. The angle between the two vectors is given by:

4. The magnitude of the projection is defined as the arc-length of a segment times the cosine of an angle formed between the segment and the surface of projection.

$$\phi_{i,j} = \operatorname{acos}\left(\frac{\vec{w}_i \bullet \vec{w}_j}{|\vec{w}_i||\vec{w}_j|}\right) \qquad [3\text{-}12]$$

From point w, the points that are along the contour of the feature are ordered in such a way that for any three ordered points i, j, and k that lie on the contour of the feature,

$$\phi_{i,k} = \phi_{i,j} + \phi_{j,k} \qquad [3\text{-}13]$$

Equation [3-13] suggests the following recursive formula:

$$\phi_{i,j_{m+\Delta t}} = \phi_{i,j_m} + \phi_{j_m,j_{m+\Delta t}} \qquad [3\text{-}14]$$

initialized with $i = 0$ and $m = 0$. Equation [3-14] is iterated until $m = n$, the last end point on the boundary of the feature. For an ordered sequence of points equation [3-12] is always positive, therefore, $\phi_{0,n}$ is the smallest angular field of view needed to see the entire feature from point w.

Therefore, the viewpoints from where a sensor can enclose an entire feature within its angular field of view is equal to the set of viewpoint from the sensor can see the two end points of the feature assuming that the sensor is facing towards the feature.

3.2.3 Generating the viewpoints

The set of positions from where the sensor can see the entire feature is given by the geometric combination of three regions: a region V that is the half-plane that contains the set of points from where the line that connects the two end points of the feature is visible, a region C that is defined from viewpoints from where this segment is visible, and a region F that encloses the viewpoints from where every point on the contour of the feature is visible (Figure 3.15). How these regions are combined depends on whether the angular field of view of the sensor is less than, equal, or greater than the 180^0. In the remainder of this section, each of these cases is considered separately.

3.2.3.1 When $\alpha < 180^0$

If $\alpha < 180^0$, the region A_{FOV} that defines the set of positions from where the feature can be enclosed in the sensor's angular field of view is given by:

$$A_{FOV} = F \cap (V - C) \qquad [3\text{-}15]$$

where F is the set of points from where every point on the contour of the feature is visible, V is the set of points that are in the half-plane that is facing the line that is connecting the two end points of the feature and C is the circle defined by equation [3-11] that encloses the viewpoints from where the angular field of view required to detect the feature exceeds the angular field of view of the sensor. The four regions C, V, F and A_{FOV} are illustrated in Figure 3.15.

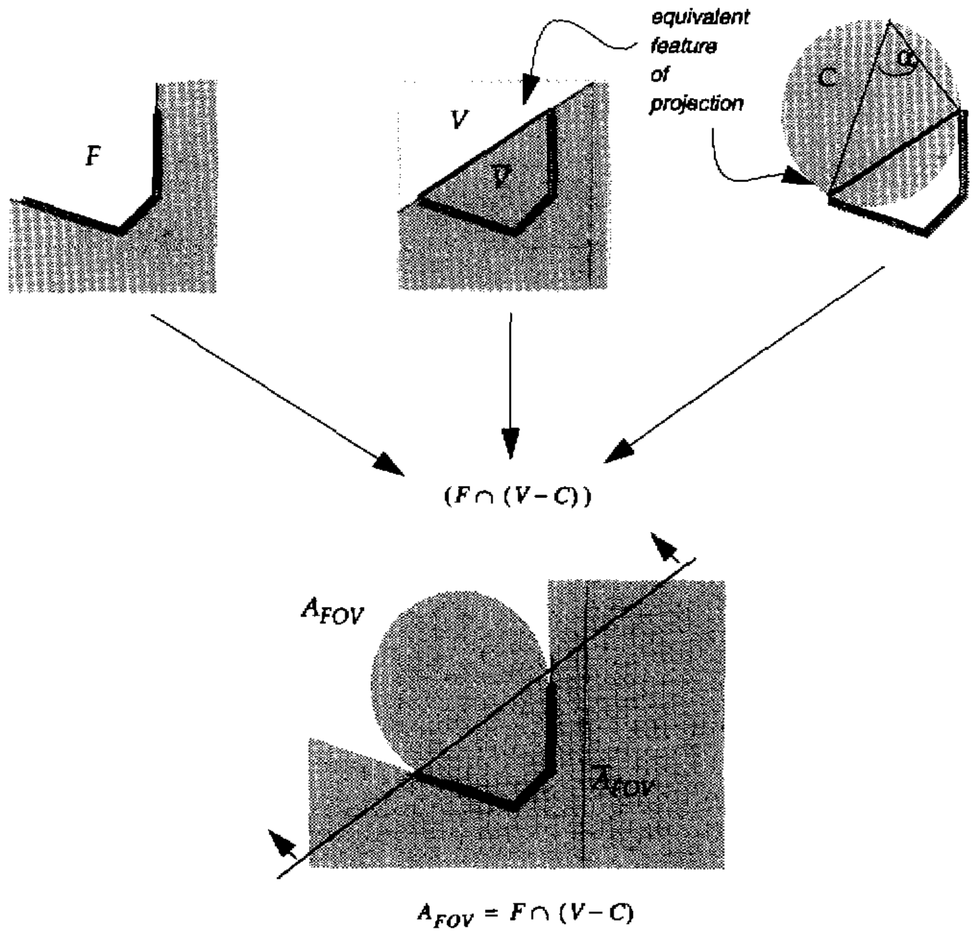


Figure 3.15 Viewpoints from where the sensor can see the object for $\alpha < 180^0$.

In order to prove equation [3-15] it must be shown that from any point $w \in A_{FOV}$, the feature is visible and the angular field of view required to detect the entire feature is less than the angular field of view of the sensor. It must also be shown that from any point $w \notin A_{FOV}$ either the feature is not visible or the angular field of view required to sense the entire feature exceeds the angular field of view of the sensor. Figure 3.15 is used to illustrate the proof.

First, it is shown that from any point $w \in A_{FOV}$, the entire feature is visible and the angular field of view required to see the entire feature is less than or equal to the angular field of view of the sensor. From equation [3-15], $A_{FOV} = F \cap (V - C)$, thus $A_{FOV} \subset F$. Since F encloses the set of viewpoints from where the entire feature is visible, from any point $w \in A_{FOV}$ the feature is visible.

$(F \cap \bar{V})$ is the set of points that are enclosed by the convex hull of the feature and $(F \cap V)$ is the set of points that are not enclosed by the convex hull of the feature. From any point $w \in (F \cap V)$ the angular field of view required to see the entire feature is less than 180°.

$F \cap (V - C)$ can be rewritten as $(F \cap V \cap \bar{C})$. C encloses the set of points for which the angular field of view required to sense the feature is greater than or equal to the angular field of view of the sensor. The angular field of view required to sense the feature from any point that is not in C is less than the angular field of view of the sensor. This completes the first part of the proof.

It is now shown that from any point $w \notin A_{FOV}$, either the feature is not visible or the angular field of view required to sense the feature exceeds the angular field of view of the sensor. If $w \notin A_{FOV}$, then $w \notin (F \cap V \cap \bar{C})$. By DeMorgan's theorem, $w \in (\bar{F} \cup \bar{V} \cup C)$. If $w \in \bar{F}$, then the point is not visible.

If $w \in \bar{V}$, then either the point is $(\bar{F} \cap \bar{V})$ or in $(F \cap \bar{V})$. If $w \in (\bar{F} \cap \bar{V})$, the feature is not visible from w. If $w \in (F \cap \bar{V})$, the viewpoint is in the convex hull of the feature. From any point in the convex hull of a feature, the angular field of view required to see the entire feature is greater than 180°, which is greater than the angular field of view of the sensor.

Finally, from any point $w \in C$, the minimum angular field of view needed to see the entire feature is greater than the angular field of view of the sensor. This completes the proof.

3.2.3.2 When $\alpha = 180°$

If $\alpha = 180°$, the region A_{FOV} that defines the set of positions from where the feature can be enclosed in the sensor's angular field of view is given by:

$$A_{FOV} = F \cap V \quad [3\text{-}16]$$

where again F is the set of points from where every point on the contour of the feature is visible and V is the set of points that are in the half-plane that is facing the line that is connecting the two end points of the feature. The three regions V, F and A_{FOV} are illustrated in Figure 3.16.

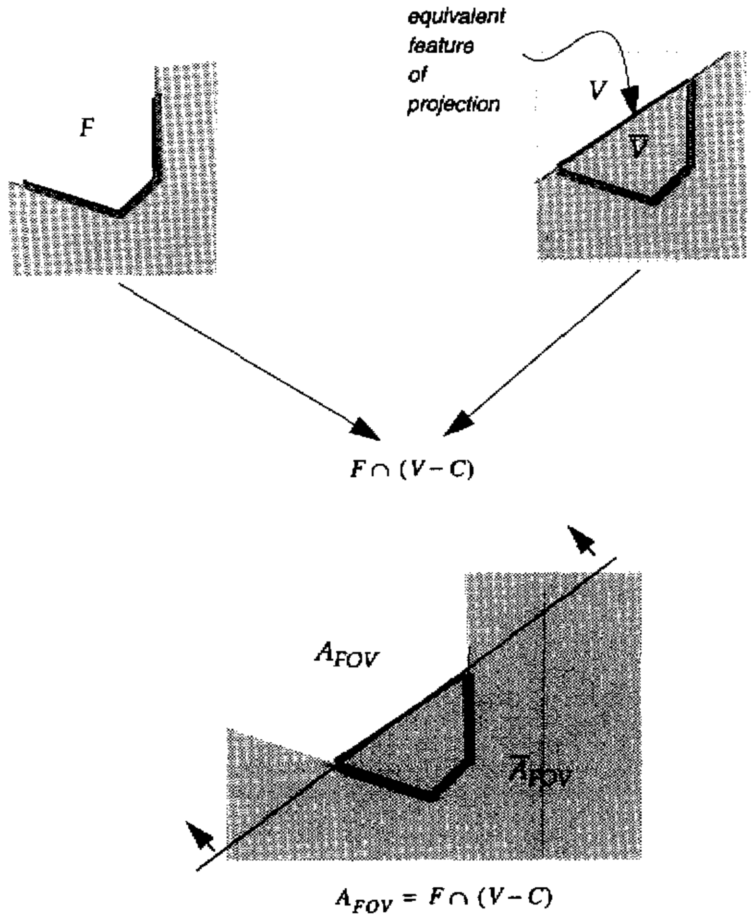


Figure 3.16 The viewpoints from where the sensor can see the object $\alpha = 180°$.

As before, it suffices to show that from a point $w \in A_{FOV}$ the feature is visible and within the angular field of view of the sensor and that for a point $w \notin A_{FOV}$ the feature is either not visible or the angular field of view required to see the feature exceeds the angular field of view of the sensor.

From equation [3-16], $A_{FOV} = F \cap V$ therefore any point $w \in A_{FOV}$ is also in F, the set of viewpoints from where the feature is entirely visible. Since w is not in $(F \cap \overline{V})$ [which is the set of points that is enclosed by the convex hull of a concave feature], the angular field of view required to see the entire feature is less than 180^0, the angular field of view of the sensor. The argument applies to every

$w \in A_{FOV}$.

If a point $w \notin A_{FOV}$, then either w is not visible in which case $w \notin F$, or w is in the cavity of the convex hull enclosing the feature (given by $F \cap \bar{V}$). In either case, $w \notin F \cap V$. The argument applies to every $w \notin A_{FOV}$.

3.2.3.3 When $\alpha > 180^o$

If $\alpha > 180^o$, the region A_{FOV} that defines the set of positions from where the feature can be enclosed in the sensor's angular field of view is given by:

$$A_{FOV} = F \cap (V \cup C) \qquad [3\text{-}17]$$

F is the set of points from where every point on the contour of the feature is visible, V is the set of points that are in the half-plane that is facing the line that is connecting the two end points of the feature and C is a circle that encloses the viewpoints from where the line segment connecting the two end points is not visible. The four regions C, V, F and A_{FOV} are illustrated in Figure 3.17.

As before, in order to prove equation [3-17] it must be shown that from any point $w \in A_{FOV}$, the feature is visible and the angular field of view required to detect the entire feature is less than the angular field of view of the sensor. It must also be shown that from any point $w \notin A_{FOV}$ either the feature is not visible or the angular field of view required to sense the entire feature exceeds the angular field of view of the sensor.

From equation [3-17], $A_{FOV} = F \cap (V \cup C)$. Therefore, a point $w \in A_{FOV}$ is also in F so that from any point w, the feature is entirely visible.

Equation [3-17] can be rewritten as $A_{FOV} = (F \cap V) \cup (F \cap \bar{V} \cap C)$ If $w \in (F \cap V)$, w is the space where the feature is visible and the angular field of view required for the sensor to see the feature is less than or equal to 180^o. If $w \in (F \cap \bar{V})$, w is in the region enclosed by the convex hull of the feature where the angular field of view required to see the entire feature is greater than 180^o.

If $w \in C$, the angular field of view required to sense the entire feature is less than or equal to the angular field of view of the sensor. The minimum angular field of view required to sense a feature from any point in C is less than the angular field of view of the sensor when the angular field of view is

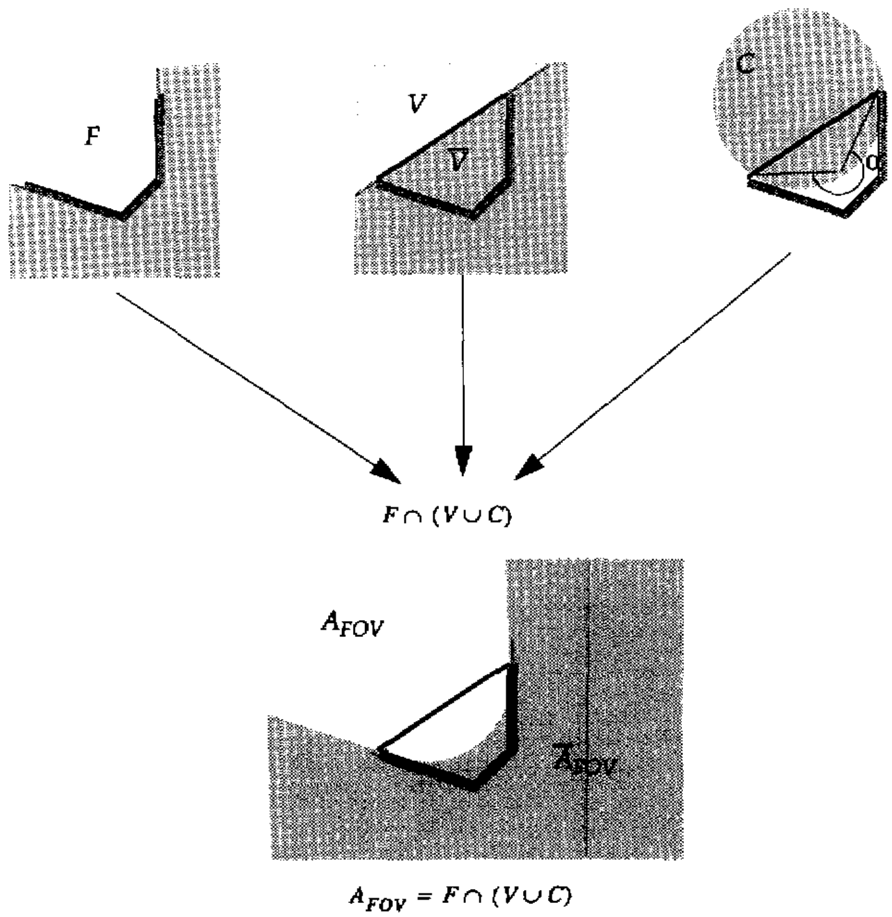


Figure 3.17 he viewpoints from where the sensor can see the object $\alpha > 180^o$ **.**

greater than 180^0. Region C is a circle defined by equation [3-11]:

$$C = \begin{cases} R = \left|\dfrac{L}{2Sin(\alpha)}\right| \\ (x_c, y_c) = (\dfrac{L}{2}, RCos(\alpha)) \end{cases}$$

When $\alpha > 180^0$, the angle subtended from a point $v \in F \cap \bar{V}$ on the circumference of the circle is constant equal to α. From any point $v \in F \cap \bar{V}$ that is inside the circle the angular field of view the subtended angle is less than α. Therefore, showing that $A_{FOV} \subset F \cap (V \cup C)$. The argument applies to any point $w \in A_{FOV}$.

What remains to be shown is that from any point $w \notin A_{FOV}$ either the feature is not visible or the angular field of view required to sense the entire feature exceeds the angular field of view of the sensor. If $w \notin A_{FOV}$, then $w \in (\bar{F} \cup (\bar{V} \cap \bar{C}))$ which can be rewritten as $w \in (\bar{F} \cup (F \cap \bar{V} \cap \bar{C}))$ If $w \in F$, the feature is not visible. If $w \in F \cap \bar{V}$, the point is inside the convex hull of the feature. From any point in this region, the angular field of view required to see the entire feature exceeds 180^0. Since $w \in F \cap \bar{V} \cap \bar{C}$, the angular field of view required to see the entire feature is also greater than the angular field of view of the sensor.

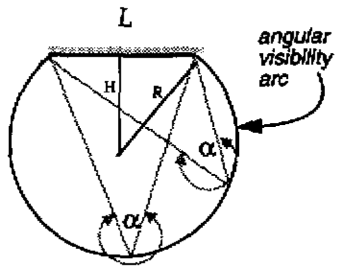


Figure 3.18 The angular visibility arc of a line segment L, due to the sensor's angular field of view α. ($\alpha > 180^0$)

3.2.4 Summary

A valid viewpoint is the position and orientation from where the sensor will be able to enclose a target object within its angular field of view. Searching for a valid viewpoint is divided into searching for a sensor position and once this is found, computing the sensor orientation. By decoupling sensor position from sensor orientation, the task of finding a viewpoint is simplified.

The positions from where there is at least one orientation in which the object is in the sensor's angular field of view is bounded by an *angular visibility arc*. This arc is defined in terms of the sensor's angular field of view and a line segment. The straight line is shown to be an *equivalent feature of projection* that is obtained from the projection of the feature onto the image plane.

In summary, the region from where the sensor can see the entire feature assuming that the sensor is properly oriented is given by:

$$A_{FOV} = \begin{cases} V - C & \text{if } \alpha < 180^0 \quad \text{(Figure 3.15)} \\ V & \text{if } \alpha = 180^0 \quad \text{(Figure 3.16)} \\ V \cup C & \text{if } \alpha > 180^0 \quad \text{(Figure 3.17)} \end{cases} \qquad [3\text{-}18]$$

where U is the region that describes the set of all possible sensor positions, V is the region that is in front of the line connecting the predominant features of projection, and C is the region enclosed by the angular visibility arc of the sensor.

The visibility arc that defines the boundary from where a sensor will be able to see an entire object is a circle with radius R and center (x_c, y_c) given by:

$$C = \begin{cases} R = \left| \dfrac{L}{2Sin(\alpha)} \right| \\ (x_c, y_c) = \left(\dfrac{L}{2}, RCos(\alpha)\right) \end{cases} \qquad [3\text{-}19]$$

where L is the line segment connecting the two end points of the feature.

The appropriate translations and rotations apply to equation [3-19] L when L is not horizontal.

3.3 Range Field of View Constraint

Spot laser range scanners, line stripe range sensors and other range sensing devices are tuned to detect objects that are within a bounded distance from the sensor. The distance at which a sensor can detect an object is called the sensor's *detection range* or *range field of view*. Only the points of an object that are within the detection range of the sensor can be detected.

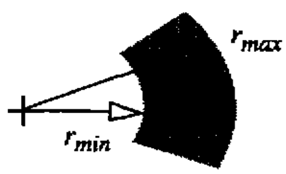


Figure 3.19 The field of view of a laser range scanner. r_{max} and r_{min} are the boundaries of the detection range of the sensor.

3.3.1 Viewpoints in the detection range of the sensor

The detection range of a sensor is bounded by a minimum distance and a maximum distance. It can be modeled by two concentric circles, A and B, centered about the sensor. The radius of A is the maximum distance that can be detected by the sensor (r_{max}) and the radius of B is the minimum distance that can be detected by the sensor (r_{min}).

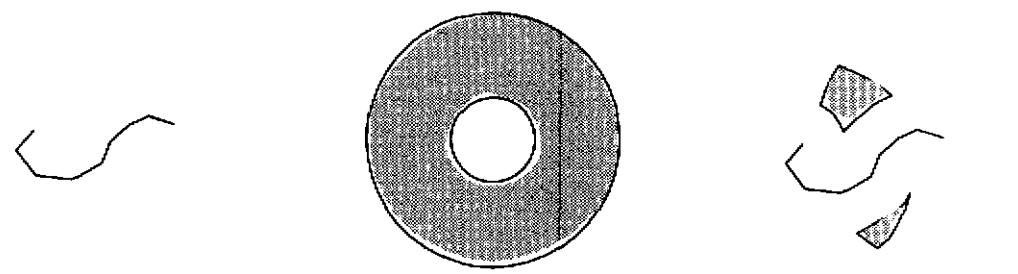
A point p can only be detected if it is at a distance between r_{min} and r_{max} from the sensor. In other words, the point can only be detected if it is in the region spanned by the two concentric circles A and B centered at the sensor. This region D_s, spanned by the two circles A_s and B_s is defined by:

$$D_s = (A_s - B_s) \qquad [3\text{-}20]$$

where s is the location of the sensor.

From the point of view of the feature, the viewpoints from where a sensor can see the entire object comprise the set of viewpoints that are a distance between r_{min} and r_{max} from every point on the feature. The region from where a single point is visible is the intersection of the same two circles (A and B) centered on the point. The viewpoints from where the sensor will be able to see the entire object is the intersection of these regions spanned over every point on the feature.

Figure 3.20 illustrates the viewpoints from where a sensor with a limited detection range is able to detect a feature. The feature is illustrated in Figure 3.20-a. The detection range $D = A - B$ illustrated in Figure 3.20-b. The region of valid viewpoints, which is the intersection of the regions D_p taken at every point p on the contour of the feature are illustrated in Figure 3.20-c. The shape of this region depends on the shape of the feature and the range of detection of the sensor.



(a) A feature

(b) viewpoints in the detection range of the sensor as given by $D_p = (A_p - B_p)$

(c) viewpoints from where the sensor will be able to detect the entire feature

Figure 3.20 The viewpoints from where a sensor will detect the entire feature.

3.3.2 Finding the viewpoints

Suppose p is a point on the feature, A_p is a circle of radius r_{max} that bounds the maximum detection distance and B_p is a circle of radius r_{min} that bounds the minimum detection distance both centered about p. The locus of sensor positions at which the complete feature will be within range distance of the sensor is given by:

$$D = \bigcap_p (A_p - B_p) \qquad [3\text{-}21]$$

where $p = 0, 1, \ldots n$ are all of the points on the contour of the feature.

To implement equation [3-21] requires equation [3-20] be evaluated for every point on the contour of the feature. This is unrealistic due to the number of computations that this would entail. Selecting few sample points in order to reduce the number of operations will produce false viewpoints from where the sensor will not be able to detect the object.

These problems can be eliminated by rewriting equation [3-21]. Equation [3-21] can be rewritten as:

$$D = \bigcap_i A_i - \bigcup_j B_j \qquad [3\text{-}22]$$

From this equation, the valid viewpoints can be determined by generating the set intersection of all A regions and then subtracting from it the union of all the B regions.

3.3.2.1 Polygonal features

If the feature is a polygonal feature, the set intersection of all A regions is equal to the set intersection of the A regions taken only at the vertices of the polygon. It can be shown that the intersection of the A regions along a straight line segment is equal to the intersection of two A regions taken at each of the end points of the segment. Therefore, the intersection of the A regions taken along every point on the contour of the feature is the intersection of the A regions taken only at the vertices that describe this contour.

The set union of all the B regions taken along the contour of any two dimensional feature is equal to the area obtained by sweeping a circle of radius r_{min} across the contour of the feature. The swept area is equal to the union of the B circles spanned across every point.

The following sequence is an illustration of the steps in generating the field of view for the polygonal feature shown in Figure 3.20. Each step of the algorithm is annotated by illustrations.

Step 1. To find the set intersection of all of the A regions located at every vertex of the feature

- Place a circle of radius r_{max} at each vertex

- Compute the set intersection of all of the circles

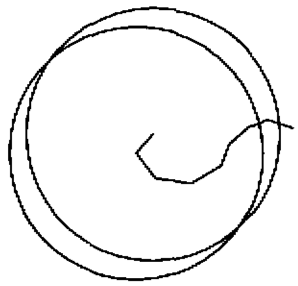


Figure 3.21 The second iteration in generating the A region by placing one A region per vertex.

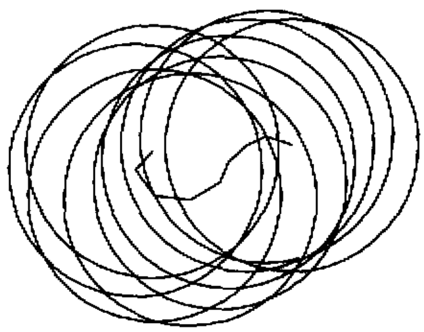


Figure 3.22 Placing one A region per vertex just prior to applying the set intersection to all of the regions.

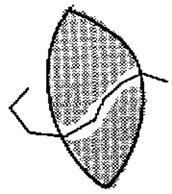


Figure 3.23 The intersection of the A regions.

Step 2. To obtain the union of all of the B regions, sweep a circle of radius r_{min} region about the contour of the feature.

- Place a circle of radius r_{min} on a vertex

- Sweep the circle to the next vertex on the feature

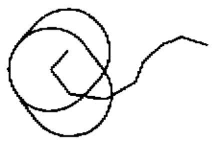


Figure 3.24 The first two iterations in the sweeping of the B regions

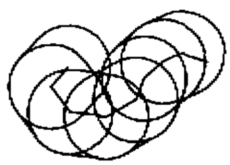


Figure 3.25 The B regions have been swept. This scene is just prior to the merging of all regions.

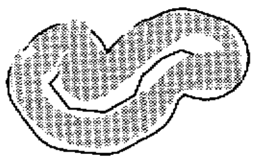


Figure 3.26 The B region, after applying the set union operation on all B regions.

Step 3. Subtract the B region from the A region to produce the viewpoints from where the sensor can see the entire object.

Figure 3.27 The set of valid viewpoints.

3.3.2.2 A curved feature

If the feature is composed of curved segments, it is not possible to generate the intersection of all A regions (equation [3-22]) directly as the intersection of the A regions is no longer the intersection of only a few vertex points. Linearizing the feature in order to use this algorithm is undesirable since it will produce viewpoints from where the feature is not visible and eliminate valid viewpoints. Fortunately, it is possible to rewrite [3-22] in a form that allows the intersection of A regions to be substituted by a union operation. This operation can be implemented by sweeping a region (function of A) along every point on the contour of the feature.

By DeMorgan's theorem ($\bigcap_i A_i = (U - \bigcup_i \bar{A}_i)$), equation [3-22] can be rewritten as:

$$D = (U - \bigcup_i \bar{A}_i) - \bigcup_j B_j \qquad [3\text{-}23]$$

where U is the set of all possible viewpoints and $\bar{A}_i$ is the set of viewpoints in U that are not in region A_i.

The set of valid viewpoints from where a curved feature is visible is obtained by subtracting the areas described by $\bigcup_i \bar{A}_i$ and $\bigcup_j B_j$ from the set of all possible viewpoints (U). $\bigcup_i \bar{A}_i$ is obtained by sweeping an area ($U - A$) over the contour of the feature. Similarly, $\bigcup_j B_j$ is obtained by sweeping an area B over the contour of the feature.

3.3.3 Summary

Range sensors are tuned to detect objects that are within a bounded distance from the sensor. Only features that are located within these boundaries can be detected.

The viewpoints from where a single point on the feature can be detected is bounded by two concentric circles centered on the point. The viewpoints from where the entire object is visible is the intersection of the viewpoints from where each point can be detected.

To reduce the number of computations that are required, the region can be obtained by sweeping two regions derived from the two concentric circles along the contour of the object. The swept area contains the viewpoints that will locate the sensor in a position in which the object is within the range of detection of the sensor.

Chapter 4

Image Quality Constraints

Data density and data accuracy are characteristics that describe the quality of an image. The density (or spatial resolution) of the image describes how closely spaced the sample points are from each other. In a dense image, the points are spaced closely together, whereas in a sparse image they are far apart. The accuracy is a measure of the error between the range distance returned by the sensor and the true distance from the sensor to each point.

The density and accuracy of the points in an image depend on the characteristics of the sensor, the geometry of the features, and the sensor's location and viewing angle. The density of an image, which is a measure of the number of target points per target surface area, decreases with the viewing angle of the sensor and with the distance between the sensor and the feature. The accuracy which depends on the magnitude of the return signal decreases with the viewing angle of the sensor.

Valid viewpoints from where the sensor will acquire a dense and accurate image can be generated from a geometric description of the features, a description of the sensing characteristics of the sensor, and a description of the limits that characterize a dense and accurate image. These limits constrain the spacing between consecutive target points and the viewing angle of the sensor.

The following sections develop a method to generate viewpoints from where a sensor can obtain an accurate and dense image.

4.1 Spatial Resolution Constraint

The spatial resolution of an image is a direct function of the sensor's angular resolution, the viewpoint and the geometry of the object. The angular resolution of the sensor determines the angular distance between two consecutive sample points. The angular resolution of the sensor together with the distance between the sensor and the target, and the viewing angle determine the spatial resolution of the image.

The challenge is to develop a strategy to determine the viewpoints from where a sensor will acquire data that satisfies given spatial resolution requirements. The requirements are given either as a constraint on the number of points sampling every portion of the feature or as a constraint on the distance between any two target points.

The density and uniformity of an image can be controlled by selecting the position and viewing angle of the sensor that satisfies the spatial resolution constraints. The set of valid viewpoints is obtained from the geometry of the object, the angular resolution of the sensor and the spatial resolution constraint. Figure 4.1 is an illustration of the parameters that define the problem: the angular resolution of the sensor $\Delta\theta$, the distance between the sensor and a target point r_θ and the spacing between two points ΔY. Figure 4.2 is an example which illustrates a set of valid viewpoints. The valid viewpoints have been selected as the points from where the sensor can acquire two target points for every segment that defines the contour of a polygonal object. Other criteria for density could have been considered such as requiring the maximum distance (actual or arc length) to be constant for any pair of consecutively sampled points in the image.

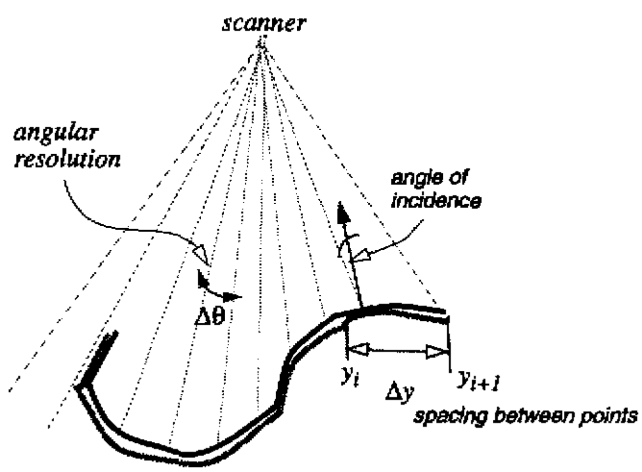


Figure 4.1 A sensor scanning a target.

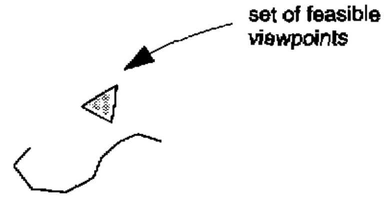


Figure 4.2 The viewpoints from where a sensor with a small angular field of view can detect at least two points on every segment that composes the contour of the feature.

A method to generate the viewpoints from where a sensor can acquire an image that satisfies a set of spatial resolution constraints is derived. It is assumed that the angular resolution of the sensor, the geometry of the feature and the conditions that satisfy the spatial resolution of an image are given. Three not necessarily inclusive conditions that are observed in this chapter are:

- that at least two points sample every segment on the contour of a (polygonal) feature.
- that the spacing between any two sample points does not exceed a maximum (or minimum) allowable limit
- that the arc length between two sample points does not exceed a maximum (or minimum) allowable limit

For ease of explanation, the discussion is initially limited to polygonal features; subsequently the generalization to arbitrary two dimensional objects is provided.

4.1.1 The valid viewpoints

The viewpoints from where the sensor will acquire an image that satisfies the spatial resolution constraint is obtained by finding the viewpoints from where the maximum distance between every pair of points on the contour of the feature is less than or equal to a *maximum allowable distance*, Y. Every pair of points on the contour of the feature defines a region of valid sensor positions. From any point in this region, the distance between two consecutively sampled points that fall between the original pair of points will be less than or equal to Y. The set intersection of all of these regions is the set of viewpoints from where the acquired image will satisfy the resolution constraint. The distance between any two consecutively sampled points contained in an image that is acquired from a valid sensor position will not exceed the maximum allowable distance.

Given any two points P_i and P_{i+1},which are separated by a distance Y, and the angular resolution of a sensor, $\Delta\theta$, the viewpoints from where the sensor can be oriented such that it strikes both target points (with two consecutive scanning beams) is defined by a circle. Only viewpoints that are inside the circle are valid viewpoints[1].

From any point on the contour of this circle, the sensor can be oriented such that two consecutive target points coincide with P_i and P_{i+1}. Since the distance between P_i and P_{i+1} is equal to Y, any point on the contour of the circle satisfies the viewpoint constraint. A sensor that is located inside the circle can be oriented such that it samples two points that are between P_i and P_{i+1}. The distance between these two points is clearly less than Y. A sensor that is located at a point outside of the circle the sensor will be unable to sample two points that lie between P_i and P_{i+1}. Therefore, viewpoints that are outside of the region enclosed by the circle will not satisfy the constraint.

Figure 4.3 is an illustration of the circle that defines the set of allowable viewpoints for any pair of points. The set of feasible viewpoints is labeled S. The radius R and center x_c and y_c of the circle are a function of the parameters Y, $\Delta\theta$, and P_i:

$$R = \frac{Y}{2\ Sin(\Delta\theta)} \qquad [4\text{-}1]$$

and

$$(x_c, y_c) = \left(\frac{Y}{2} + P_i, R\ Cos(\Delta\theta)\right) \qquad [4\text{-}2]$$

1. The circle and the parameters for the circle are developed from the definition of spatial resolution in section A.3.

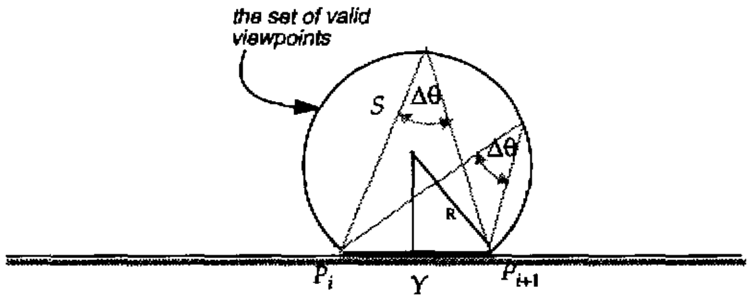


Figure 4.3 The boundary of allowable viewpoints.

4.1.2 Generating the set of all valid positions

The set of allowable sensor positions is obtained by generating a set of feasible sensor positions for every pair of points (separated by a distance Y) that are on the contour of the feature. The intersection of these regions is the set of viewpoints from where the distance between any two consecutively sampled points in the image is less than or equal to Y.

Suppose that S_i is the set of valid viewpoints for two consecutive target points P_i and P_{i+1} which are separated by a distance Y. Then the set of possible viewpoints for the entire feature is given by:

$$S = \bigcap_i S_i \qquad [4\text{-}3]$$

If the feature is a line segment and the length of the segment is greater than Y, equation [4-3] can be evaluated by taking the intersection of the regions at only two extreme points evaluated at points P_0 and P_{L-Y} where 0 and $L-Y$ are values measured in the arc-length on the contour of the feature.

If the length of the segment is less than Y then the distance between any two consecutively sampled points is always less than Y. However, unless the sensor is inside a circle computed for $Y = L$, there is no guarantee that the feature will even be detected.

Consider the example illustrated in Figure 4.4. The feature in figure Figure 4.4-a is a line of length $L > Y$. Figure 4.4-b is the two regions S_0 and S_{L-Y} located at points S_0 and S_{L-Y}. Figure 4.4-c is the set of viewpoints from where the distance between any two consecutively sampled points is less than or equal to Y.

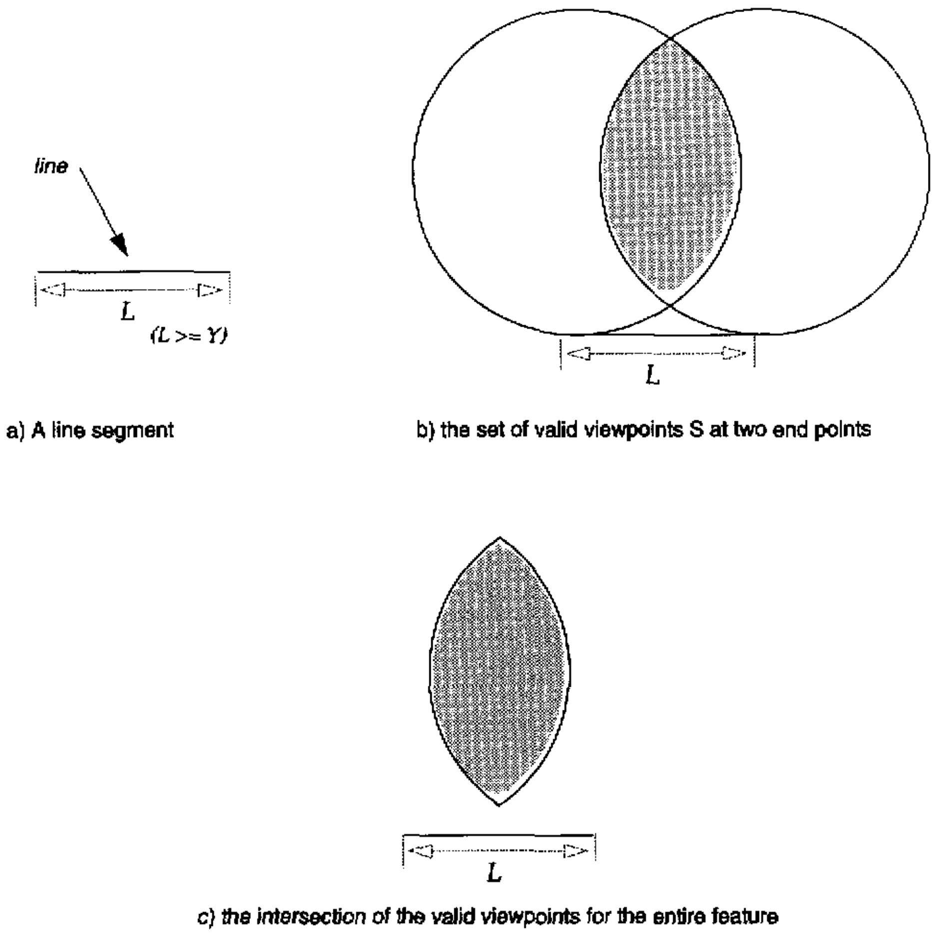


Figure 4.4 A sequence to generate the viewpoints for a single line segment. The region of viewpoints is the intersection of the spatial resolution constraint at every point on the feature. Since the feature is a straight line, the region is the intersection of two regions evaluated at points P_0 and P_{L-Y}.

4.1.2.1 Example: Generating viewpoints that enable two hits per segment

Acquiring two points per segment is important as that is the minimum number of samples that are required to reconstruct the lines of a polygonal feature. The algorithm described by equation [4-3] can be used to generate the viewpoints from where the sensor will be able to acquire at least two points for every segment on the contour of a polygonal feature.

To guarantee that the image will contain at least two target points per segment, the value of Y must be at least one half of the length of each line segment. Y can be chosen to be a constant (equal to 1/2 the length of the smallest segment) or a variable that depends on each segment. Choosing Y to be variable increases the space of valid positions.

The following algorithm generates the viewpoints from where the image is guaranteed to contain at least two target points per segment. The algorithm is developed assuming that Y is a variable. The appropriate substitution can be done if Y is a constant.

Step 1. For each segment

- $Y = L/2$
- *Generate a circle at each end point (P_0 and P_{L-Y}) with radius and center given by equations [4-1] and [4-2].*

Step 2. Evaluate the set intersection of all of the regions (equation [4-3]). The result is the set of viewpoints that satisfy the constraint for every segment on the contour of the feature.

Figure 4.5 is an illustration of various steps in the process. Figure 4.5-b is the space of candidate viewpoints for only two segments. Figure 4.5-c is the set intersection of these two spaces. Figure 4.5-d illustrates the valid viewpoints for the entire feature. The region is obtained by the set intersection of the candidate viewpoints for every segment on the contour of the feature.

The size and shape of the region depends in part on the radius of each circle that constrains the set of feasible viewpoints. As shown by equation [4-1], the radius of this circle is inversely proportional to the angular resolution of the sensor. Reducing the angular field of view increases the radius of the circle and should increase the space of allowable sensor positions. Figure 4.6 is a comparative study of the set of valid sensor positions for different angular resolutions. As the angular resolution of the sensor decreases the space of valid sensor positions increases.

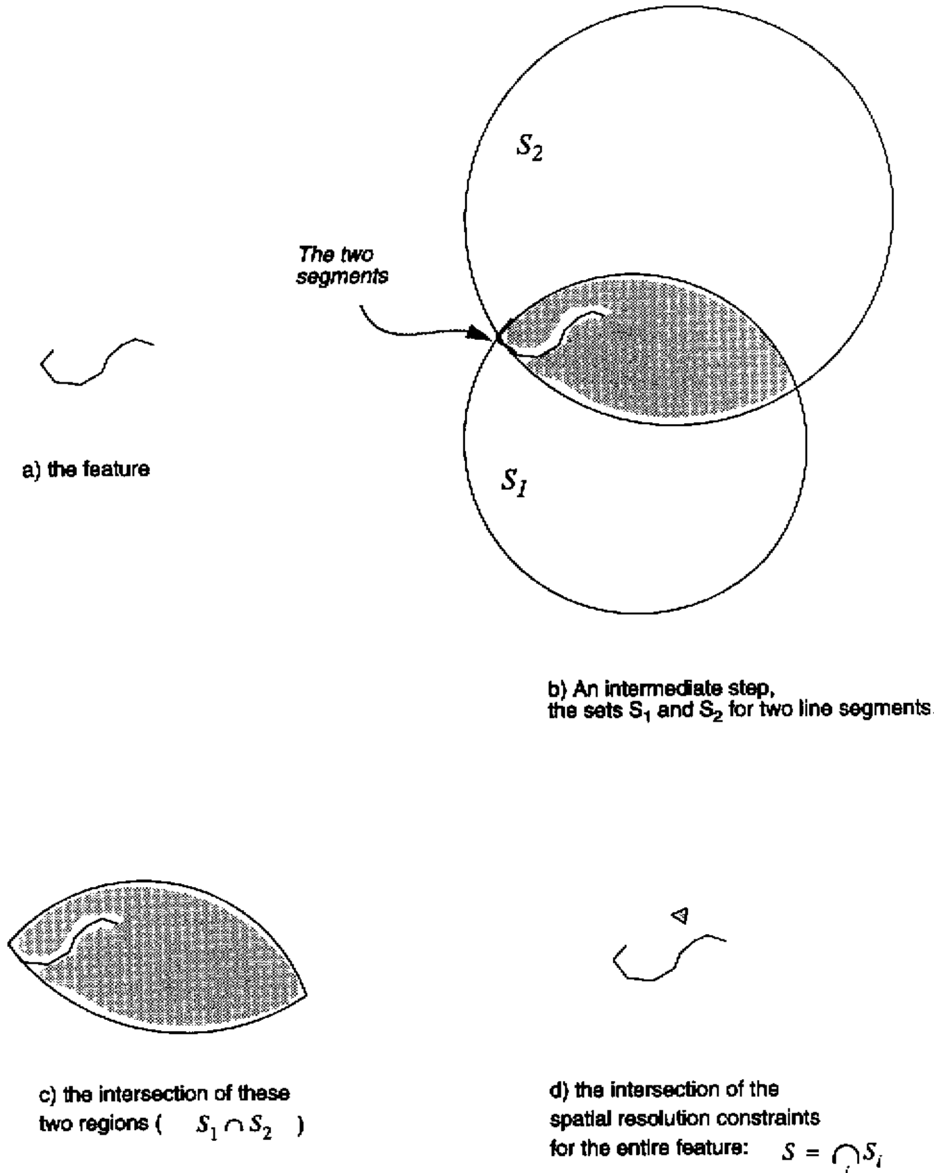


Figure 4.5 Several steps in the generation of the viewpoints that satisfy the spatial resolution constraint of a feature. ΔY is defined at each feature.

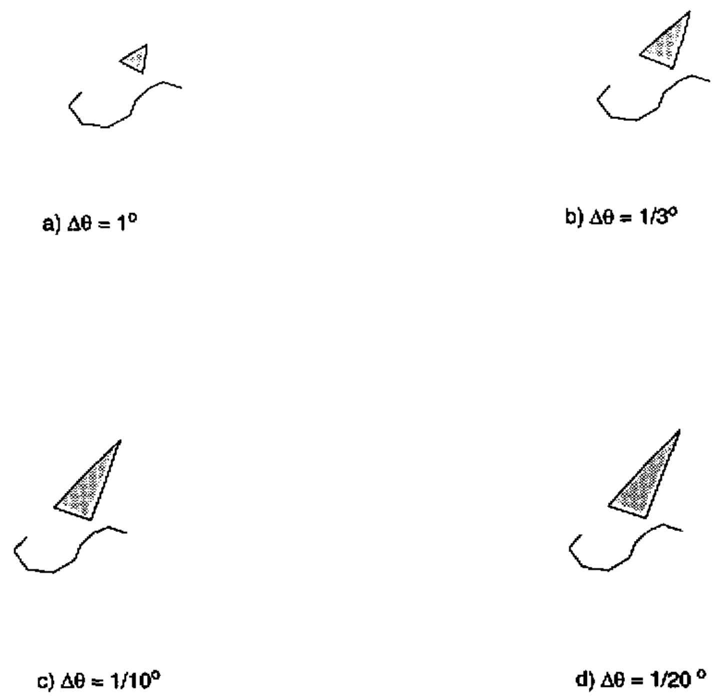


Figure 4.6 A comparison of the viewpoints that enable two hits per feature generated for different values for the angular resolution of the sensor, $\Delta\theta$.

The size of the region that encloses the feasible viewpoints increases with the size of Y, the maximum allowable separation between two consecutive target points. Figure 4.7 is a comparative study of the size of the region to different sizes of Y. The size of Y was controlled by setting it to be equal to L or equal to $L/2$ for each segment. As expected, from equation [4-1] the space of valid viewpoints increases with Y.

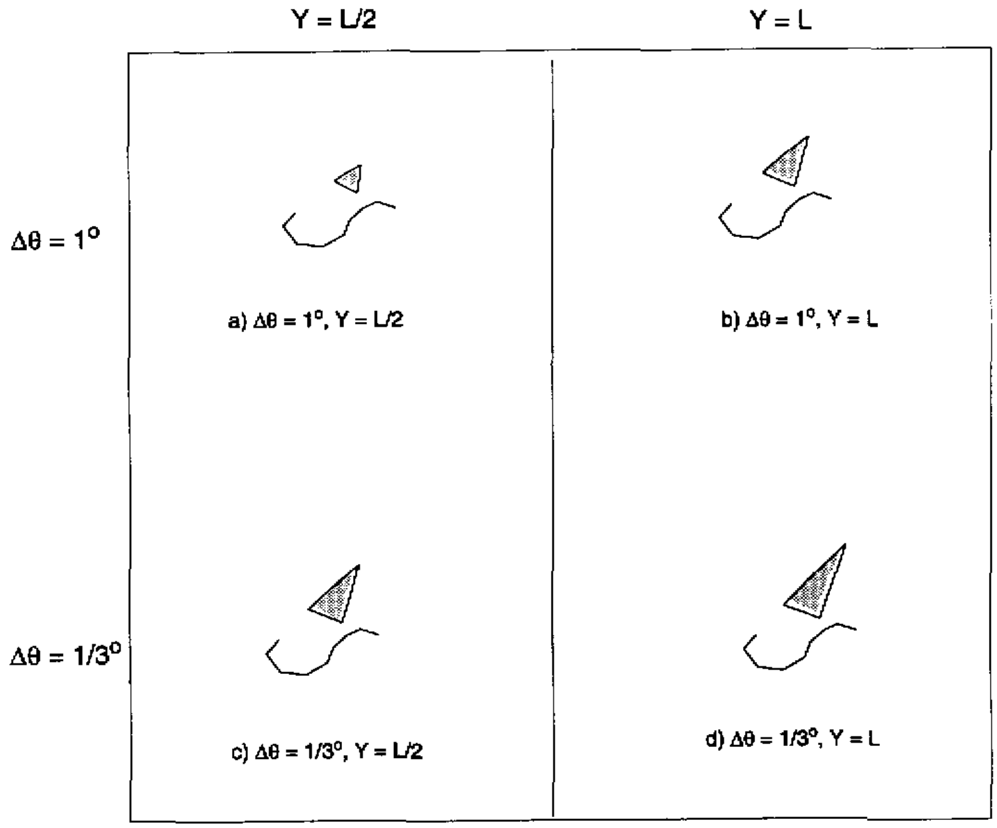


Figure 4.7 A comparison of the viewpoints that enable two hits per feature against the viewpoints that enable only one hit per viewpoint generated for different values of Δθ.

4.1.3 Viewpoints that maintain a constant distance between target points

In many applications it is important to maintain a a certain consistency and uniformity in the resolution of a range image. The criteria used to define consistency may be "that there are at least two target points per segment" (section 4.1.2), maintain a uniform chord distance between two consecutively sampled target points, or maintain a constant arc length distance between consecutively sampled points.

In general, criteria like this one can be used to define a distance between two consecutive target points. This distance defines the pair of points that constrain the placement of the sensor. The region that encloses the valid points is defined by the position, distance and orientation of these two points.

In this subsection, the method discussed in section 4.1.2 is generalized for arbitrary two dimensional curves. The method proposes to locate a second point for every point on the contour of the feature such that the distance between the two points satisfies an image constraint. These two points define a region that encloses the set of candidate viewpoints. The intersection of the candidate viewpoints is the set of viewpoints that satisfy the image constraint for every pair of points on the feature (equation [4-3]).

Whereas the conceptual ideas are simple, the implementation may be complex and require substantial computational resources. Finding a second point for every point on the contour may not be straightforward for complex curves and may require numerical algorithms to compute the second point. Second, computing the intersection of the candidate viewpoints for every point on the contour of the surface is computationally expensive unless, as in section 4.1.2, some regularity can be found that will simplify the operations.

The purpose of this section is to describe the method to generate the set of candidate viewpoints for an arbitrary object. Investigation of mechanisms to reduce the number of operations is beyond the scope of this dissertation but may be considered for future work.

4.1.3.1 The parametric description of a curve

An arbitrary two dimensional curve can be defined by two parametric functions[20][19]:

$$x = x(s) \qquad [4\text{-}4]$$

$$y = y(s) \qquad [4\text{-}5]$$

where s is the arc-length of the curve (Figure 4.8).

Two arbitrary points P_{s1} and P_{s2} on the contour of the feature are defined by:

$$P_{s1} = (x(s1), y(s1)) \qquad [4\text{-}6]$$

$$P_{s2} = (x(s2), y(s2)) \qquad [4\text{-}7]$$

The chord distance, i.e. the absolute distance between the two points is given by:

$$\Delta P = P_{s2} - P_{s1} \qquad [4\text{-}8]$$

The arc-length distance is the distance between the two points following the contour of the feature. The arc-length distance is given by:

$$\Delta s = s2 - s1 \qquad [4\text{-}9]$$

If the curve is not parameterized by the arc-length but rather by an arbitrary parameter t, then the arc-length is defined by[51]:

$$s = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx(t)}{dt}\right)^2 + \left(\frac{dy(t)}{dt}\right)^2} \qquad [4\text{-}10]$$

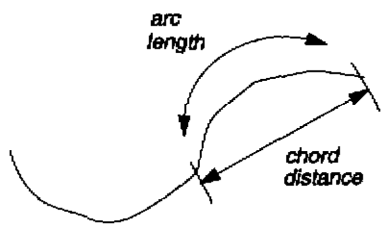


Figure 4.8 The distance between two points on a curve.

4.1.3.2 The two points are separated by a chord distance ΔP

If the maximum allowable distance between points is the chord length, then the consecutive sampled point is the intersection of the curve with a circle of radius ΔP centered about the point P_i. A point P_{i+1} located at a distance ΔP from a point P_i will be located on the circumference of a circle with radius ΔP centered about point P_i. A way to find the intersection of two curves is to express one in implicit form the other in parametric form. By substituting the parametric curve into the implicit curve, one solve for the independent variable of the parametric curve.

For example, the implicit equation of a curve is given by:

$$(x - x(s_i))^2 + (y - y(s_i))^2 = (\Delta P)^2 \qquad [4\text{-}11]$$

where $x(s_i)$ and $y(s_i)$ are the coordinates of the point P_i.

Substituting equation [4-7] into [4-11] yields:

$$(x(s_{i+1}) - x(s_i))^2 + (y(s_{i+1}) - y(s_i))^2 = (\Delta P)^2 \qquad [4\text{-}12]$$

Then, solve for parameter S_{i+1}.

For example, consider the parametric equation of a line[42]:

$$x = x_0 + ft \qquad [4\text{-}13]$$

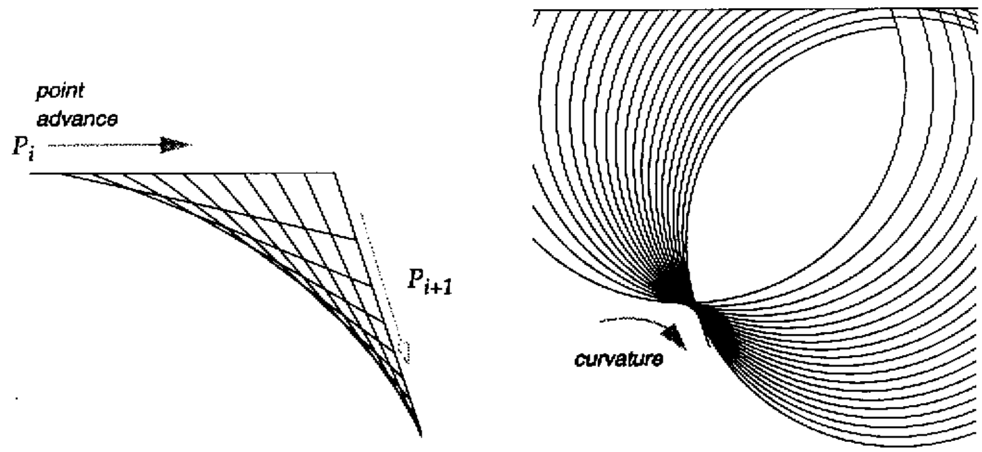
$$y = y_0 + gt \qquad [4\text{-}14]$$

where t is a normalized parameter $0 \le t \le 1$, $f = x_2 - x_0$, and $g = y_2 - y_0$ and x_0, y_0 and x_2, y_2 are the two end points of the segment.

Substituting equations [4-13]and [4-14] into equation [4-12] and solving for parameter s yields[9]:

$$t = \frac{f(x_i - x_0) + g(y_i - y_0) + \sqrt{\{\Delta P^2 (f^2 + g^2) - [f(y_i - y_0) - g(x_i - x_0)]^2\}}}{(f^2 + g^2)} \qquad [4\text{-}15]$$

Figure 4.9 is a trace of the chord connecting points P_{i+1} to P_i. The illustration shows the interaction that occurs at the corner of two segments when P_i is such that L - P_i is less than the maximum allowable distance, Y, where L is the length of the segment. Figure 4.9-a point P_i is advanced along a line segment. The location of point P_{i+1} is computed by using equation [4-15]. The chord between the two points has been drawn to illustrate the change in position and orientation of the segment. Figure 4.9-b is a trace of the spatial resolution constraint about the corner. Corners are further discussed in section 4.1.3.4.



a) the chords

b) a trace of the spatial resolution constraint

Figure 4.9 A trace of the chord for a corner angle equal to 45°

4.1.3.3 The two points are separated by an arc-length distance Δs.

If the curve is parameterized by arc-length, then the evaluation is straight forward. The point P_{i+1} located at an arc-length distance Δs from a point P_i is given by the coordinates:

$$\begin{array}{l} x(s_i + \Delta s) \\ y(s_i + \Delta s) \end{array} \qquad [4\text{-}16]$$

If the curve is not parameterized by arc-length, then the normalized parameter t must be solved from the parametric equation of the curve.

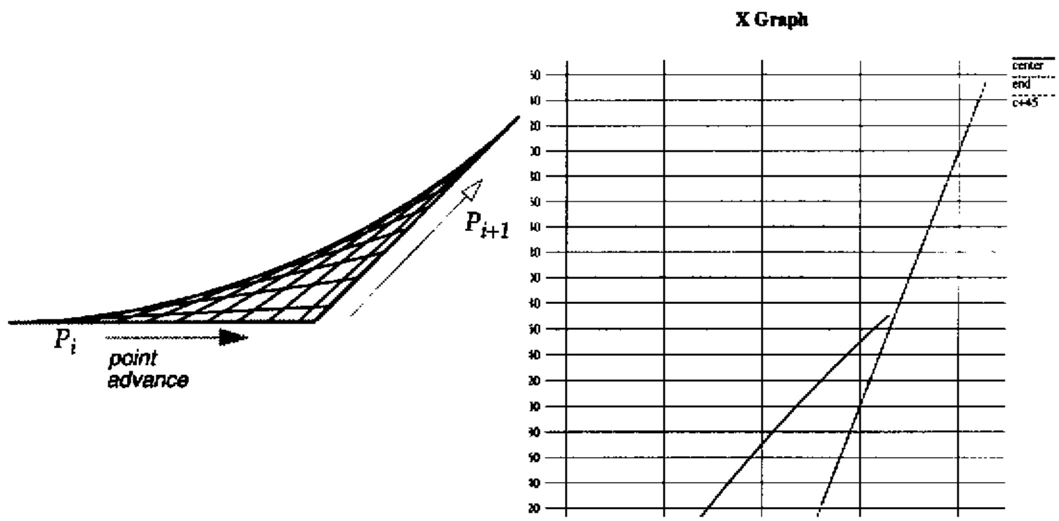
4.1.3.4 Corners and points of high curvature

When the point P_i reaches the end of the segment, the next point that is at a distance ΔP from P_i on the adjoining segment. The same procedures as were described above are used to find the point.

Figure 4.10 through Figure 4.12 illustrate the chord as it changes in position and orientation along two segment. Note that if the angle between the two segments is greater than 90^o, then the distance between two points will be greater than or equal to ΔP. The problem arises when the angle between the two segments is less than 90^o. There is a large discontinuity (gap) between the end of one segment and the point that is sampled.

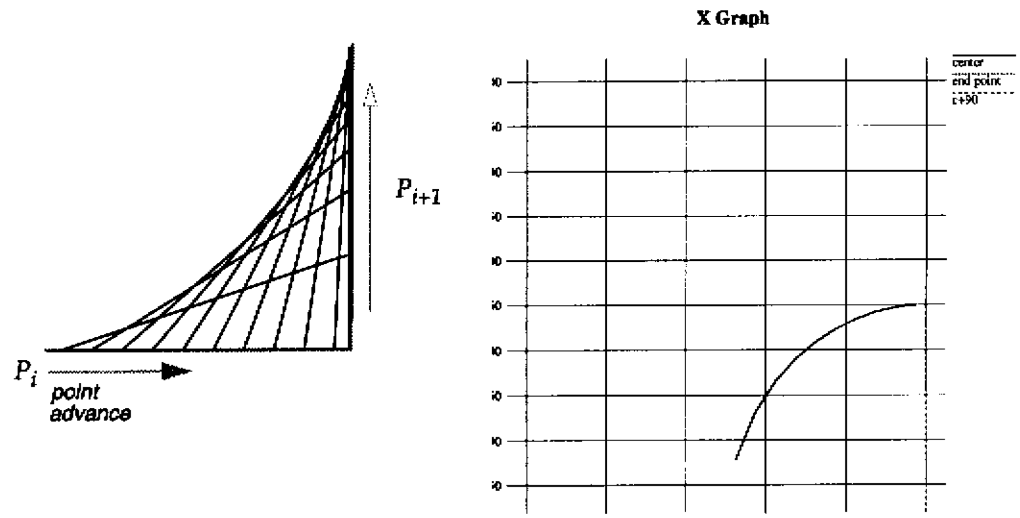
The region of valid points is the intersection of the spatial resolution constraint illustrated in Figure 4.3. In previous chapters (section 3.3.2.2) it was stated that the intersection of a shape across consecutive points along a trajectory could be solved by sweeping the complement of the shape occupied by the constraint along the contour of the feature. This is true for simple geometric descriptions of the constraint and for a simple translational or rotational motion.

The spatial resolution constraint follows the contour of the chords for every point on the curve. Around a corner, the trajectory that the spatial resolution constraint travels includes simultaneous translation and rotation. Further analysis is required to determine if the area swept by the constraint can be generalized as in the previous examples.



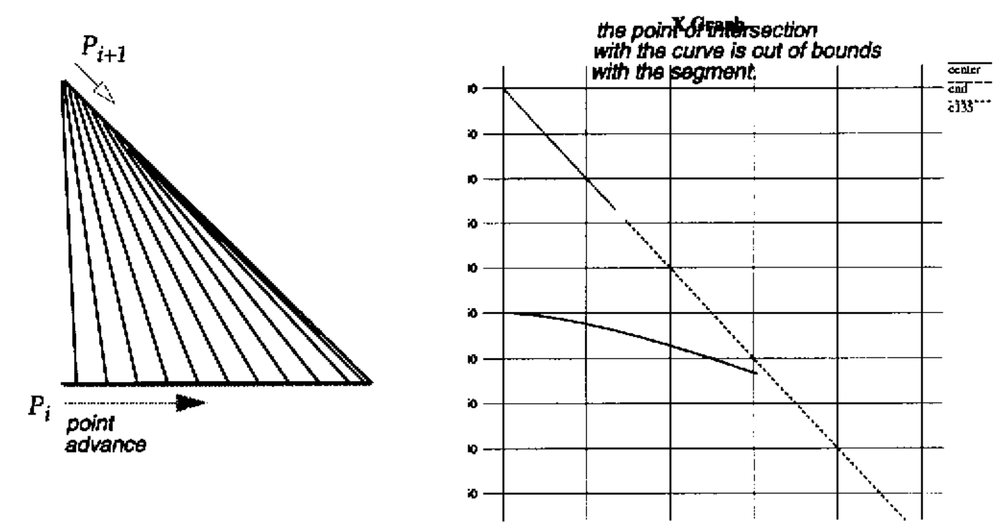
a) the chords b) a trace of the center of the chord

Figure 4.10 A trace of the chord for a corner angle equal to 90°.



a) the chords b) a trace of the center of the chord

Figure 4.11 A trace of the chord for a corner angle equal to 90°.



a) the chords b) a trace of the center of the chord

Figure 4.12 A trace of the chord for a corner angle equal to 135°.

4.1.4 Limiting the density of points

In many applications it may be desirable to limit the spatial resolution of the image by limiting the minimum distance between any two sampled points. The valid viewpoints are those from where the distance between any two sampled points is greater than the minimum allowable distance.

Suppose that S is the region of viewpoints that satisfy the constraint that the distance between two consecutively sampled points (P_θ and $P_{\theta+\Delta\theta}$) is less than or equal to Y. $\bar{S}$ is the region of viewpoints that satisfy the constraint that the distance between the two points is greater than Y. Then, the viewpoints from where the distance between any two sampled points is greater than Y is given by:

$$S = \bigcap_i \bar{S}_i \tag{4-17}$$

which is equal to the complement of the union of all S_i

$$S = \overline{(\bigcup_i S_i)} \tag{4-18}$$

4.1.5 Discussion

The space of candidate sensor positions depends on the maximum allowable distance, Y, which can be either a constant measure between two consecutively sampled points or a variable. As a variable, it may be a function of the arc-length of each segment on the contour of the feature, or of the maximum allowable arc-length distance between two consecutively sampled points.

Imposing a constraint on the chord distance results in a difficult evaluation of the spatial resolution constraint. Although the size and shape of the constraint remains constant its placement is variable in a non-linear way. When the chord length is constant the arc-length is variable, particularly at the corners and points of high curvature. Therefore, a constant chord distance will not guarantee a uniform sampling.

Uniform sampling can be obtained by maintaining the arc-length distance between any two points constant. However, if the arc-length is constant the chord length may be variable. This changes the shape of the spatial resolution constraint.

It is unknown without further analysis whether the area that is swept by the spatial resolution constraint around the contour of an arbitrary feature can be parameterized.

4.2 Range Accuracy Constraint

The accuracy of a range sensor is a measure of its ability to repeatedly measure the true target distance. It is measured by the mean and standard deviation of the error between range and true target distance.

The accuracy and dynamic range of a laser range scanner is affected by the intensity of the return signal. When a beam strikes a target, only part of the energy that is emitted is reflected back to the sensor. The intensity of the return signal is proportional to the angle of incidence of the beam[40].

Experimental calibration studies have demonstrated that the accuracy of a laser range scanner decreases proportionally with the angle of incidence of the beam [33][48]. This is illustrated in Figure 4.13 which contains a plot of the standard deviation of the range measurement error of the Cyclone Range Scanner [48]. The plot was obtained by measuring the distance to a known target. The axes of the plot are the standard deviation of the range error (vertical axis) versus target orientation (horizontal axis).

It is possible to control the accuracy of the data by restricting the viewing angle of the sensor. Since the range accuracy is proportional to the angle of incidence, restricting the viewing angle will restrict the maximum range error. This angle, labeled the *maximum allowable angle of incidence*, is selected from a calibrated plot of range accuracy versus target orientation (i.e. Figure 4.13).

4.2.1 Valid viewpoints

The set of feasible sensor positions is constrained by the maximum allowable angle of incidence. A cone spanned along every point on the contour of the feature restricts the possible sensor locations. The width of the cone is equal to twice the maximum allowable incidence angle; the orientation, which is given by the vector that bisects the cone, is perpendicular to the tangent of the feature. The angle of incidence of the beam at any point inside this region is less than or equal to the minimum allowable incidence angle.

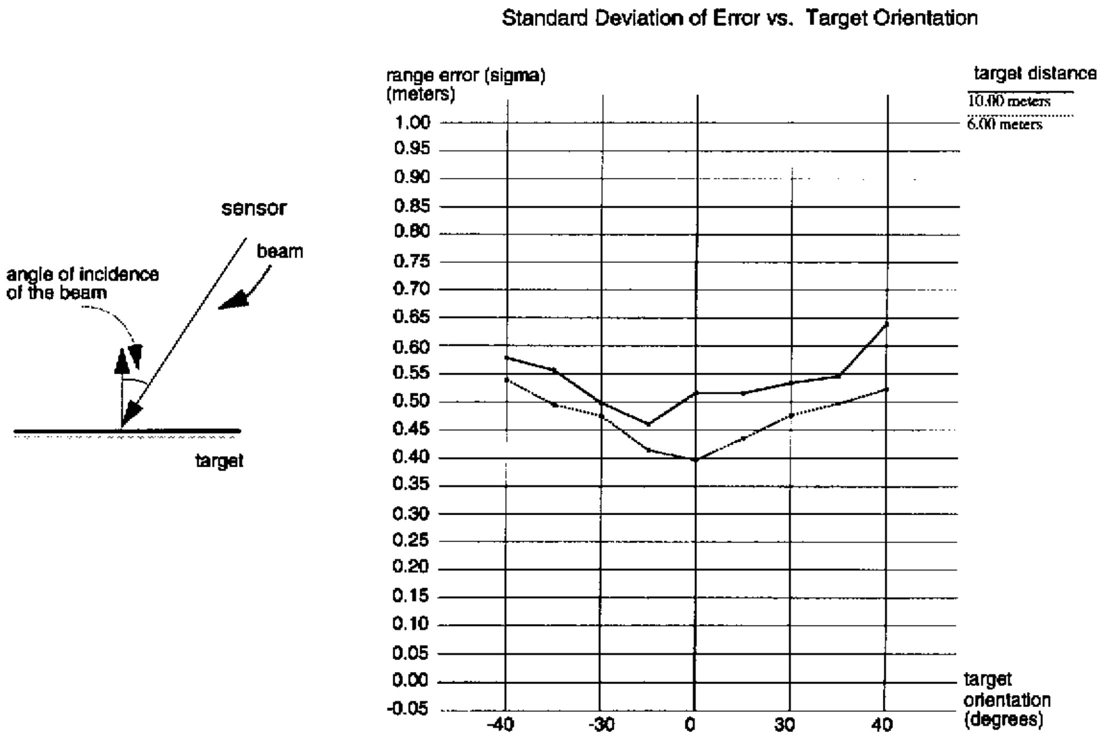


Figure 4.13 The standard deviation of the range error vs. target orientation for the Cyclone Laser Range Scanner [48].

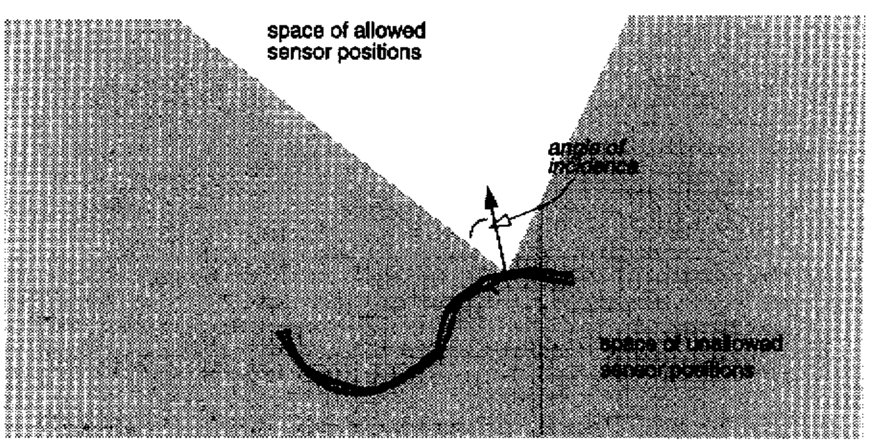


Figure 4.14 A sensor scanning a target and the region of allowed sensor positions.

The set that defines the space of valid sensor positions is the intersection of the cone spanned at every point along the contour of the feature. If A_i is the region that encloses the set of valid sensor positions about an arbitrary point i, then

$$Z = \bigcap_i A_i \qquad [4\text{-}19]$$

is the region that encloses the set of valid sensor positions for the entire feature. From any point in this region, the angle of incidence of a beam oriented to any point on the contour of the feature is less than or equal to the maximum allowable incidence angle.

When the feature is composed of straight line segments, the region Z (equation [4-19]) is equal to the set intersection of the regions A_i evaluated at the end points of each segment. This can be proved by showing that this region is a member of the cones that span the contour of the feature. The orientation of each cone is defined so that the vector that bisects the cone is perpendicular to the tangent of the segment.

The tangent of a curve is not defined at a corner where there is a C^1 (first derivative) geometric discontinuity. Suppose that the beam does not strike the corner point, rather, it strikes a point that is infinitely close to the corner at one segment or the other. Under this assumption, the set of valid viewpoints at a corner point is the intersection of two cones each of which is perpendicular to one of the two segments connected by the corner.[1]

If the feature is a curved surface, the set of allowable sensor positions can be obtained by sweeping the complement of A_i along the contour of the feature. Applying De-Morgan's theorem to equation [4-19] yields:

$$Z = U - \bigcup_i \bar{A}_i \qquad [4\text{-}20]$$

The term $\bigcup_i \bar{A}_i$ is evaluated by geometrically sweeping the area of $\bar{A}$ over the contour of the feature.

4.2.2 Generating the set of valid viewpoints

The algorithm to generate the viewpoints for a polygonal segment is illustrated in this section.

Step 1. Select the maximum allowable angle of incidence, Θ *from the calibration of range error versus target orientation (i.e. Figure 4.13)*

1. If the curve is discontinuous (C0 geometric discontinuity), then a different kind of error is observed. This error is noted as the *"mixed pixel effect"* [24].

Step 2. For every segment j that defines the contour of the feature:

*A spanning cone of width (2 * Θ) is placed at each end point of the segment. The cones are oriented such that the vector bisecting each cone is perpendicular to the segment.*

The region Zj is the set intersection of these two cones for a segment j.

Step 3. Z, the set of valid viewpoints, is the intersection of the Z regions that were generated for each segment. Thus

$$Z = \bigcap_j Z_j \qquad [4\text{-}21]$$

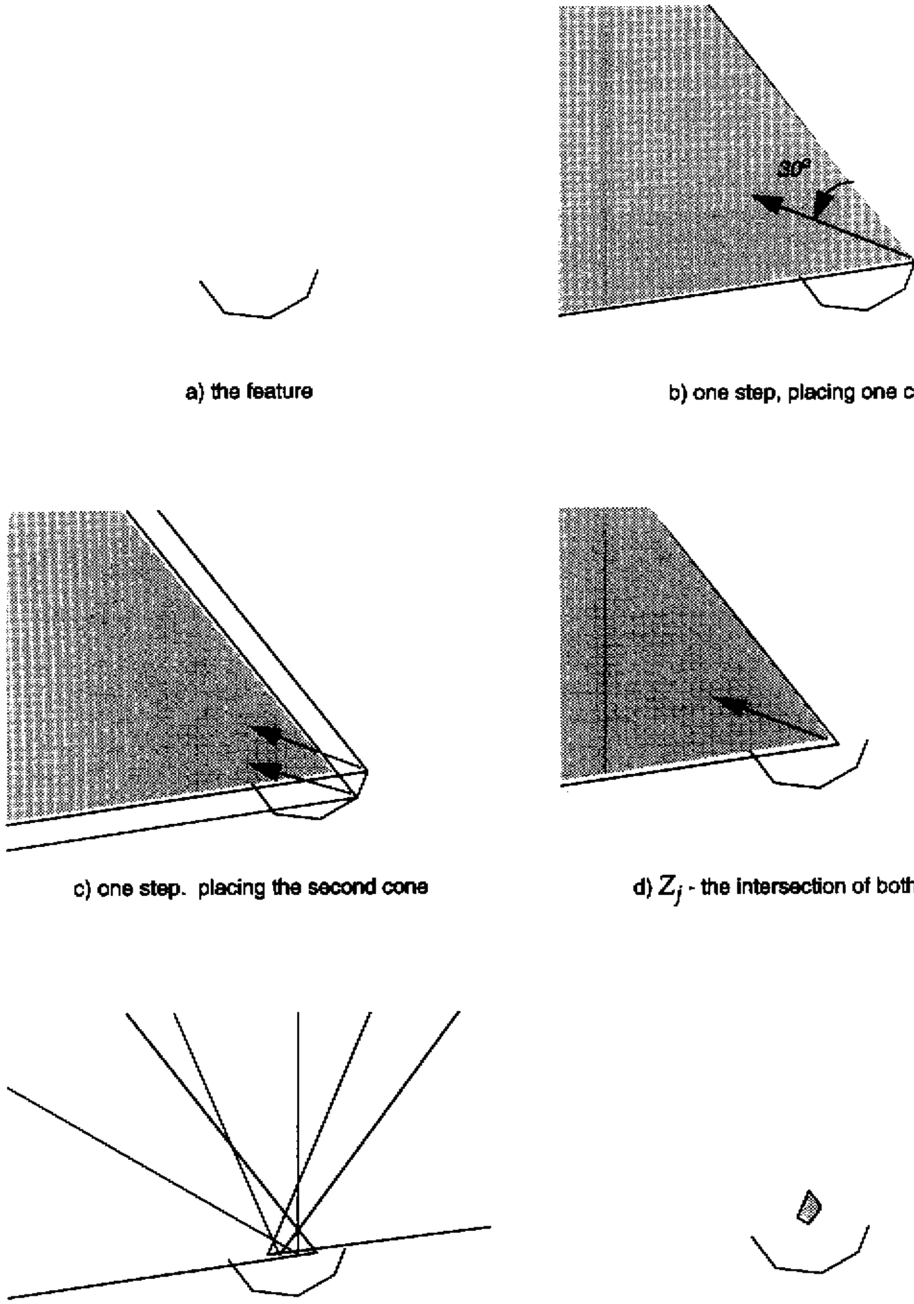
Figure 4.15 is a graphical illustration of this algorithm.

4.2.3 Summary

The accuracy of the data acquired by a laser range scanner is proportional to the angle of incidence of the beam. The accuracy of the acquired data can be controlled by constraining the location of the sensor. The maximum allowable angle of incidence is selected from a calibrated plot of the range error versus angle of incidence.

The maximum allowable angle of incidence defines a cone-region that restricts the placement of the sensor relative to every point on the contour of the feature. The angle of the cone is twice the maximum allowable angle of incidence. The orientation of the cone is such that the vector that bisects the cone is colinear with a vector that is normal to the contour of the feature.

The region of possible sensor locations is the intersection of this region computed at every point. If the feature is a line segment, this computation can be done by finding the intersection of the cone at the endpoints of every segment. If the feature is a curve, the computation can be done by sweeping a region given by the complement of the cone along the contour of the feature. Sweeping is only done through the piecewise differentiable sections of the contour of the feature.



a) the feature

b) one step, placing one cone

c) one step. placing the second cone

d) Z_j - the intersection of both cones

e) Z_j for each segment

f) Z - the intersection of all regions

Figure 4.15 A sequence of steps to generate the viewpoints that satisfy the angular field of view constraint.

Chapter 5

Sensor Placement

Automatic sensor placement to acquire accurate and complete information is a process that requires two steps. The first step is to select a position that satisfies all of the visibility, detectability, accuracy and density constraints (chapters 3 and 4). Once a position has been selected, the second step is to orient the sensor so that the feature is in the sensor's angular field of view (section 5.1).

Multiple images must be acquired when a single position is not sufficient to acquire accurate and complete information of an object. The viewpoints are generated by recursively dividing the feature iuntil for each sub-feature there is at least one viewpoint from where the sensor can acquire accurate and complete information. This strategy for generating multiple viewpoints is discussed in section 5.2.

Many applications such as modeling the interior of a building, a corridor or an engine block require a sensor to image the interior of a cavity. An example is developed in section 5.3 which uses the methods discussed so far to determine the viewpoints from where a sensor can acquire a complete and accurate image of a cavity with multiple openings.

5.1 Sensor Orientation

Once a position has been selected, the sensor is oriented so that the entire feature is contained within the angular field of view of the sensor. The proper sensor orientation is obtained from the projection of the feature onto a unit circle centered about the sensor.

This method has a number of advantages. Unlike other approaches which project the feature onto a vector that is aligned with the orientation of the sensor, the method given in this dissertation is invariant to the actual orientation of the sensor [53]. Unlike other methods that enclose a feature in a minimum spanning circle, the method described in this document is able to determine the proper sensor orientation for sensor locations that are inside the cavity of a concave feature [53][13].

5.1.1 The minimum spanning sector

The *minimum spanning sector* of a feature is the smallest sector (with apex at the sensor) that will completely enclose the feature (Figure 5.1-b). The minimum spanning sector is obtained by projecting the feature onto a unit circle centered about the sensor. The projection of a feature onto the unit circle cuts a sector. The *spanning angle* of this sector is equal to the minimum angular field of view that is required to see the feature. If this angle is less than or equal to the angular field of view of the sensor, then it is possible to orient the sensor so that the entire feature is in its angular field of view.

The nominal orientation of the sensor (so that the feature is enclosed in its field of view) is given by the vector that bisects the sector. At this orientation the field of view of the sensor will enclose the entire feature. If the spanning angle of the minimum spanning sector is less than the angular field of view of the sensor, the angular field of view of the sensor will enclose the object from many different directions. These orientations are deviations from the nominal and are limited by the difference between the spanning angle of the sector and the angular field of view of the sensor.

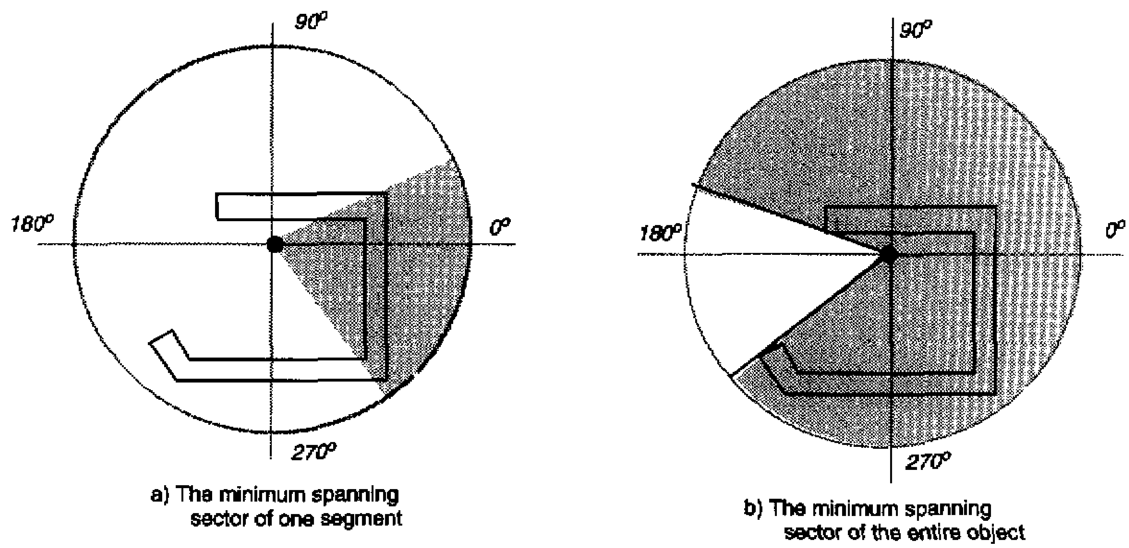


Figure 5.1 The minimum spanning sector of a two dimensional object.

The minimum spanning sector of a concave polygonal object is obtained by projecting every segment on the contour of the object onto a unit circle centered about the sensor. Each of these projections cuts a sector of the circle (Figure 5.1-a). The set union of all these sectors forms the minimum spanning sector of the entire object (Figure 5.1-b)

5.1.2 Generating the minimum spanning sector

Constructing the minimum spanning sector of an object requires a facility that can merge sectors that overlap and keep track of those that do not. Sedas presented [49] a convenient computer representation to manage, manipulate and combine a collection of two dimensional sectors without the need for a geometric modeler. Each sector is recorded by its starting and ending angle on the perimeter of the unit circle. When a new sector is inserted in the list it is compared against all other sectors in the list. Any overlapping sectors are merged together. If the new sector does not overlap any of the sectors on the list, it is simply inserted in the right place on the list.

If the sensor is located inside the convex hull of the feature, the spanning angle of the projection of the feature is greater than 180^0. Otherwise, the spanning angle is less than or equal to 180^0. One way to construct the spanning sector without checking the location of the sensor is to divide the feature into segments for which the sensor is outside the convex hull of each of these segments. When the feature is a polygon, the most straightforward approach is to project every segment on the contour of the feature.

5.2 Visual Task Decomposition

The strategy to place a sensor so that it can acquire a complete and accurate image of a particular feature may fail because no single viewpoint exists. In such cases it may be possible to acquire more than one image to fulfill the objectives. The viewpoints are chosen so that the images collectively give an accurate and complete description of the feature.

The sequence of views can be determined by recursively partitioning the feature until there is at least one viewpoint from where each sub-feature can be sensed. The strategy used in partitioning the feature determines the efficiency and optimally of the solution. At the limit, this algorithm may produce a number of views equivalent to spanning the complete object.

In this section, the strategy of partitioning a feature is shown to be a feasible strategy to be used when a single image will not be sufficient to generate a dense, accurate and complete representation of the feature.

5.2.1 An algorithm for task decomposition

The following algorithm is used to generate the views that will acquire complete and accurate information of a complex feature. The algorithm is called recursively. A list is returned that contains the sets of feasible viewpoints. Each set of viewpoints corresponds to the locations from where that feature is visible.

Generate Views (feature)

Step 1. Generate the feasible viewpoints for the feature

Step 2. If none exist,

a. Select a point on the contour of the feature where the feature will be divided

b. divide the feature into two sub-features (fa and fb)

c. return Append (Generate Views (fa) , Generate Views (fb))

If there is a viewpoint

a. return (set of viewpoints)

The number of views are defined in part by step 2-b, "*select a point on the contour of the feature where the feature will be divided*".

5.2.2 An example for multiple viewpoints

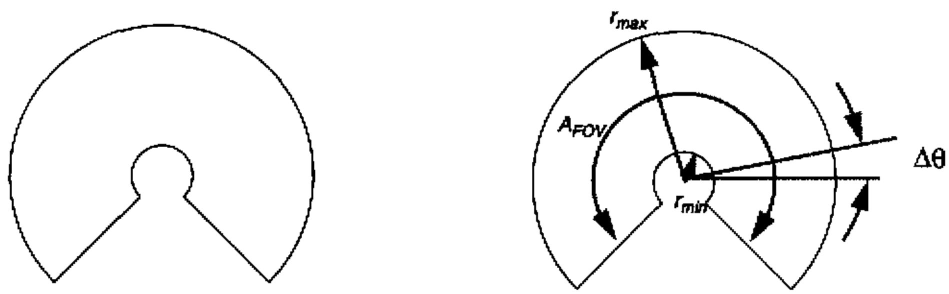
The following is a repeat of the example that was considered in section 1.3.

A laser range scanner is used to scan a polygonal feature. Due to the characteristics of the sensor and the feature, the sensor is only able to acquire accurate and complete information if it is positioned at a limited number of locations. The task is to synthesize the viewpoint from where the sensor can acquire accurate and complete information.

How to synthesize viewpoints for an arbitrary feature has been discussed in sections 1.3 and chapters 3 and 4. *In this example the technique to generate multiple viewpoints by subdividing the feature is* shown to produce a collection of viewpoints from where a sensor can acquire a collection of dense and accurate images that collectively contain complete information of the feature.

The feature used in the first example is illustrated in Figure 5.2. The characteristics of the sensor are illustrated in Figure 5.3. The sensor characteristics and image properties are summarized in Table 2.

Figure 5.2 The feature.



a) The sensor **b) the characteristics of the sensor**

Figure 5.3 The parameters that describe a sensor.

Table 2: The sensor characteristics and other parameters that define the image constraint.

A_{FOV}	r_{min}	r_{max}	$\Delta\theta$	*minimum #hits/ segment*	*Angle of incidence*
270^{o}	1.00	5.00	1^{o}	2	59^{o}

In the sequence in Figure 5.4, the viewpoints that satisfy each of the constraints are synthesized. The set intersection of these viewpoints (illustrated in Figure 5.5) is null, therefore there is no location from where the sensor can acquire a single image that will satisfy all of the constraints. The feature is recursively subdivided until there is at least one viewpoint for each sub-feature. Once the viewpoints have been generated, the system selects one of the feasible viewpoints for each feature. The sensor is placed at that location and oriented so that the entire feature is in the field of view of the sensor (Figure 5.6).

5.2.3 Discussion

In a second experiment, the feature was subdivided at points that divided the contour of the feature in half. The "maximum allowable distance between two consecutively sampled points" was readjusted at every iteration to guarantee that at least two points stuck every segment of the sub-features. This operation effectively increased the spatial resolution at every iteration. The experiment reached a point in which the spatial resolution requirement was the determining factor of whether or not there was a feasible viewpoint.

This experiment led to the following interesting observation. Where the sensor is placed to acquire at least two points per segment that defines the contour of the feature is a global constraint that is not or should not be affected by the subdivision of a feature. Therefore, portions of the feature that do not belong to the sub-feature must be considered to calculate the viewpoints that satisfy the spatial resolution constraints.

On another front, whether the algorithm can produce the minimum number of views that are required depends on the strategy that is used to select the point at which the feature is divided. If the goal is to produce an optimal number of views, then some form of look-ahead search algorithm is required.

Alternately, the description of the regions that satisfy each of the constraints can be used to determine a strategy to partition the feature. Each constraint generates a region of candidate viewpoints. By observing the sensitivity of the constraint it should be possible to infer the characteristics of the feature, that if removed would allow more candidate viewpoints that would satisfy the constraint.

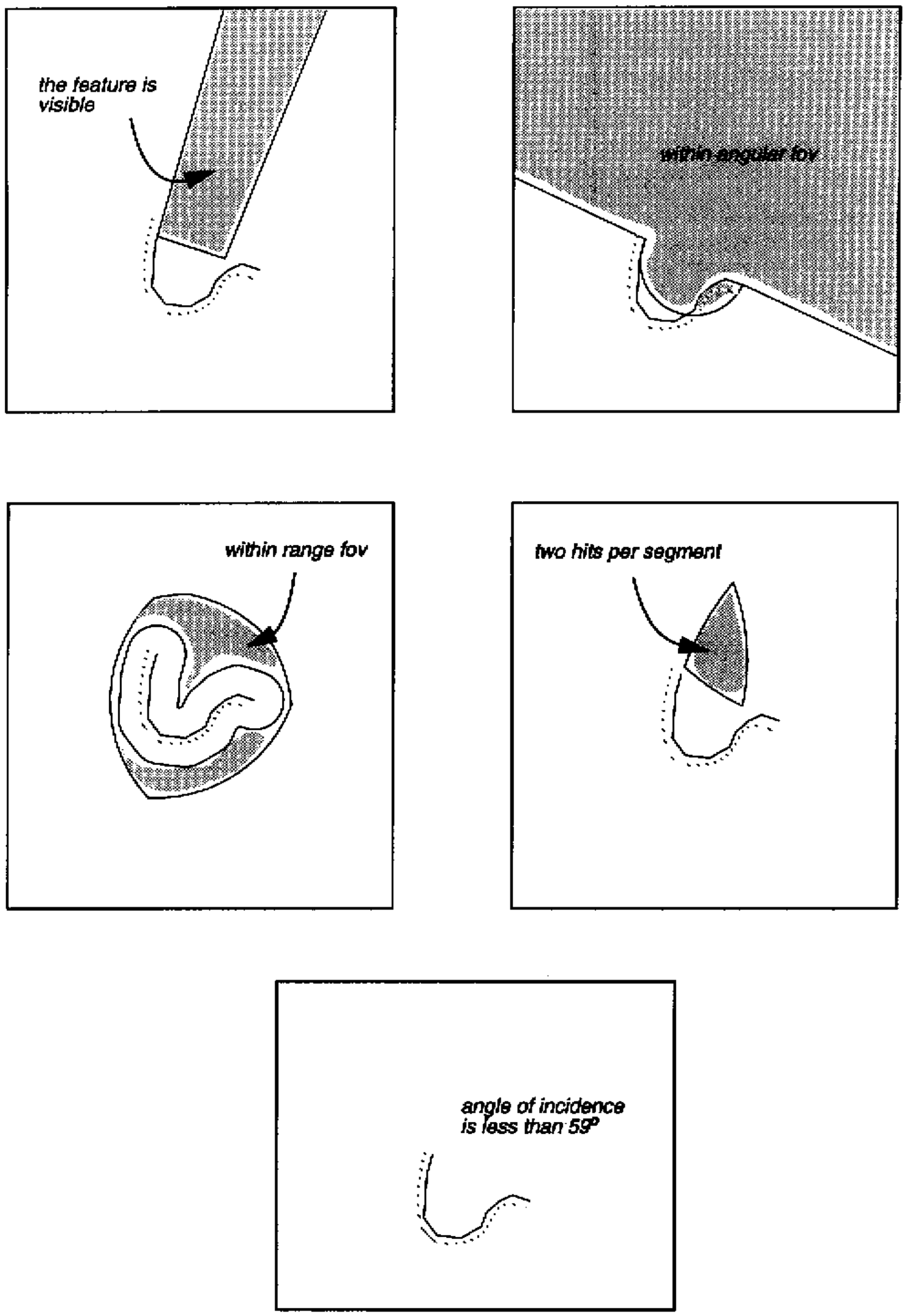


Figure 5.4 Viewpoints for a problem with no solution.

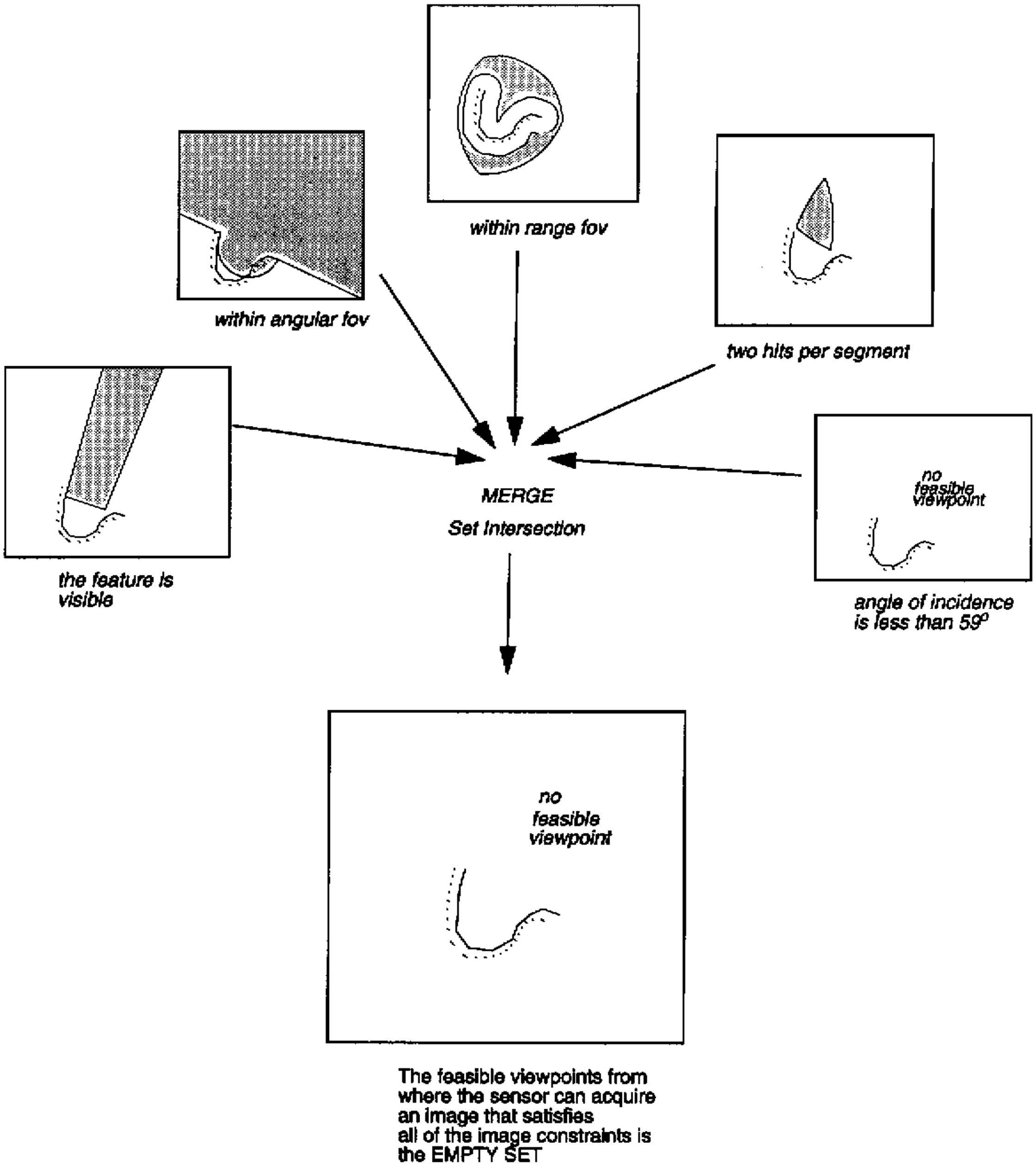


Figure 5.5 The set intersection of all of the points that satisfy each constraint is the set of feasible viewpoints. For this problem, the set is null.

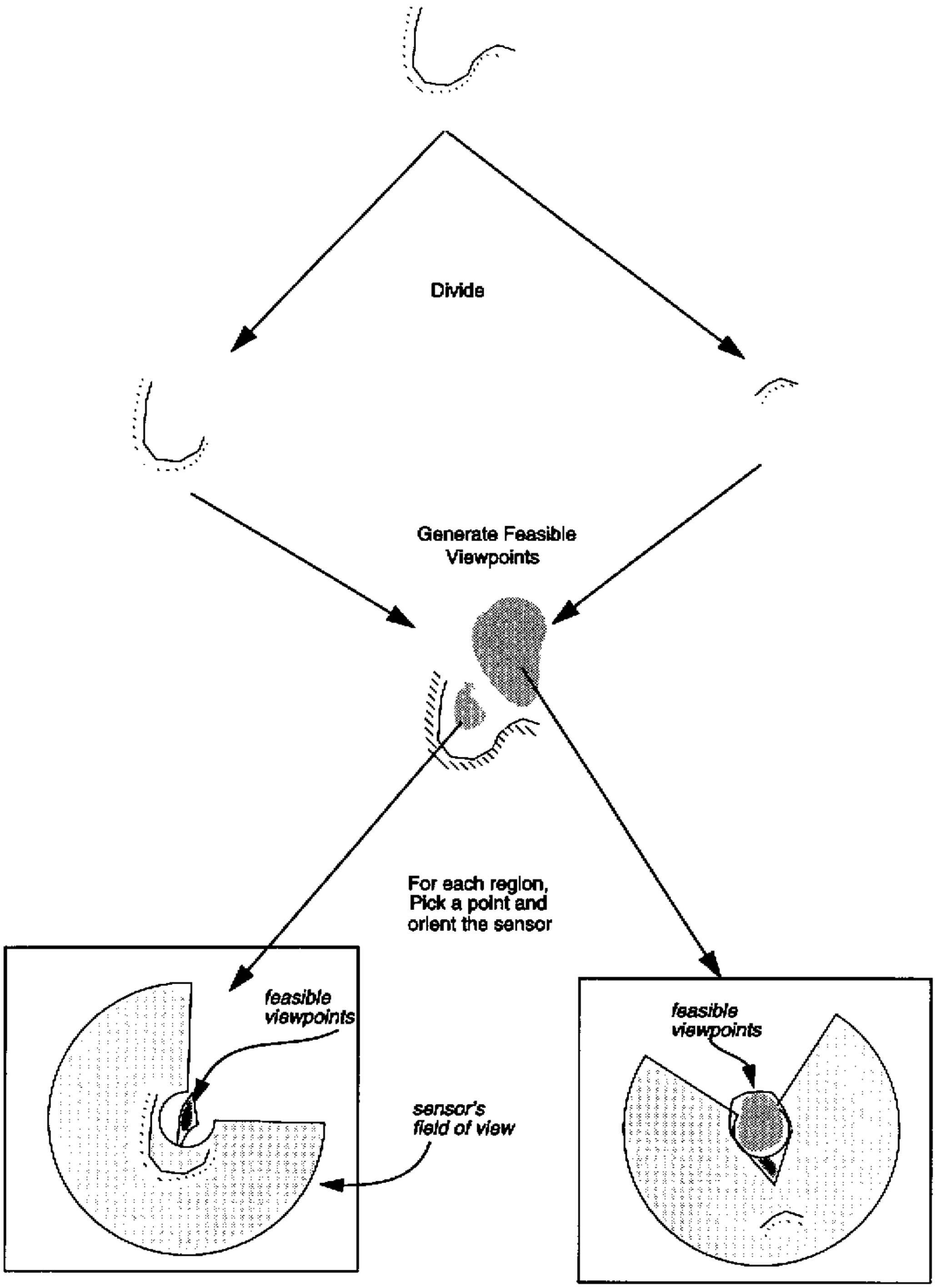


Figure 5.6 The feature is recursively subdivided until for each sub-feature there is at least one viewpoint that satisfies the image constraints. For each sub-feature a viewpoint is selected and the sensor is oriented to enclose the sub-feature in its field of view. The number of images is equal to the number of feature.

5.2.4 Summary

A feature subdivision algorithm is presented as a technique to generate the views that are needed to acquire a complete and accurate description of a feature. A feature is recursively subdivided until there is set of views from where every sub-feature is visible.

Whether the algorithm produces the minimum number of views depends on the strategy that is used to select the point at which the feature is divided. The method employed in this research is sufficient but not optimal. Algorithms that generate the minimum number of views are recommended as future work.

5.3 Sensor Placement Within a Cavity

A broad class of robotic agenda is to sense the interior of objects like rooms in buildings, openings in buried waste tanks, mine corridors, lava tubes on planets, engine cylinders with wall openings and microscopic cavities in the human body. In a simplistic form, these cavities can be represented by a polygonal boundary with multiple openings. Figure 5.7 typifies this class of problems.

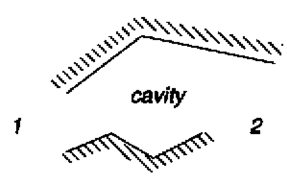


Figure 5.7 A cavity.

Because opening number 2 is larger than opening number 1, it may be conceived that is the best place where to position the sensor. Contrary to what might be expected, the comparative study in section 5.3.2 indicates that for certain sensor configurations entrance number 1 is the favorable location.

5.3.1 Synthesizing viewpoints

The cavity illustrated in Figure 5.7 is composed of two sub-features - the upper feature and the lower feature. The techniques developed in chapters 3 and 4 can be used to determine the viewpoints that will satisfy the visibility, range field of view, accuracy and spatial resolution constraint for each sub-feature. The set intersection of these viewpoints is the set of viewpoints that satisfy these same constraints for both features. This is possible because the visibility, range field of view, accuracy and spatial resolution constraints are evaluated at single points along the contour of the feature.

The angular field of view constraint is global. The set of positions from where a sensor can see the entire feature is not equal to the set of viewpoints from where the sensor can see only one of the features. Therefore, the set of feasible viewpoints cannot be generated from each independent sub-feature.

A set of viewpoints from where the sensor can see the entire feature is generated at the entrance to every cavity. The two end points of the entrance are used to define the equivalent feature of projection about which the constraint is evaluated (section 3.2). For each opening there is a set of viewpoints from where the sensor can see the entire feature. This region is intersected with the set of viewpoints that satisfy the range field of view, visibility, spatial resolution and accuracy constraints to determine the viewpoints from where the sensor can see the entire feature and satisfy every one of the constraints. Since the sensor can be positioned at either entrance, the set of feasible viewpoints is given by the union of the feasible viewpoints generated for each entrance.

The algorithm is illustrated in Figure 5.8 through Figure 5.11. The viewpoints that satisfy the constraints of range field of view, visibility, spatial resolution and accuracy are generated for each sub-feature (see Figure 5.8 and Figure 5.9). These viewpoints are then combined (set intersection) to determine the viewpoints that satisfy the same constraints for the entire feature Figure 5.10.

Next, the set of feasible viewpoints from where the sensor can see the entire feature are generated for each opening. These viewpoints are combined (set intersection) with the set of feasible viewpoints that satisfied each of the other constraints to determine the viewpoints from where the sensor can view the entire feature Figure 5.11 and satisfy each of the constraints.

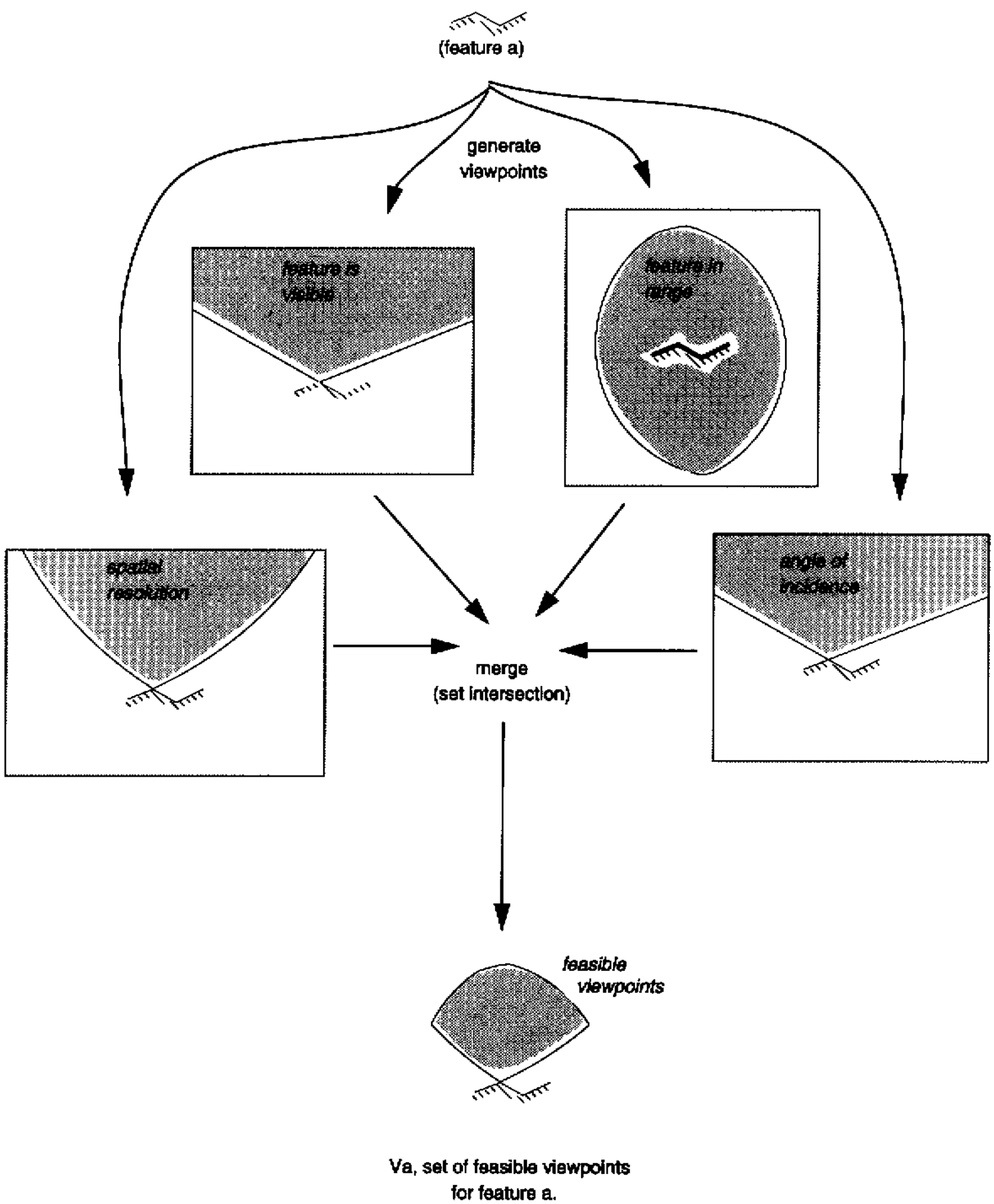


Figure 5.8 Generating the viewpoints for feature a.

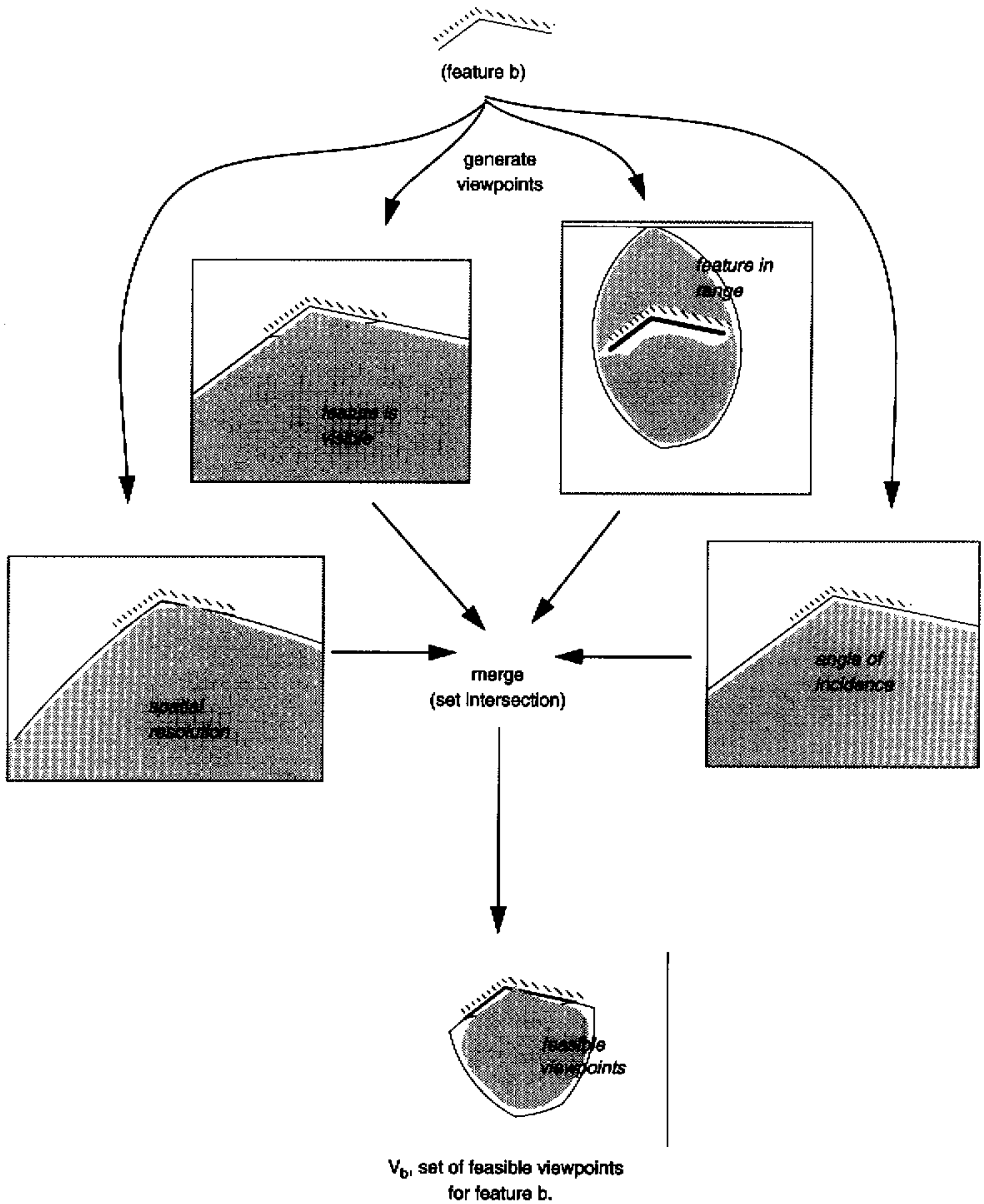


Figure 5.9 Generating the viewpoints for feature b.

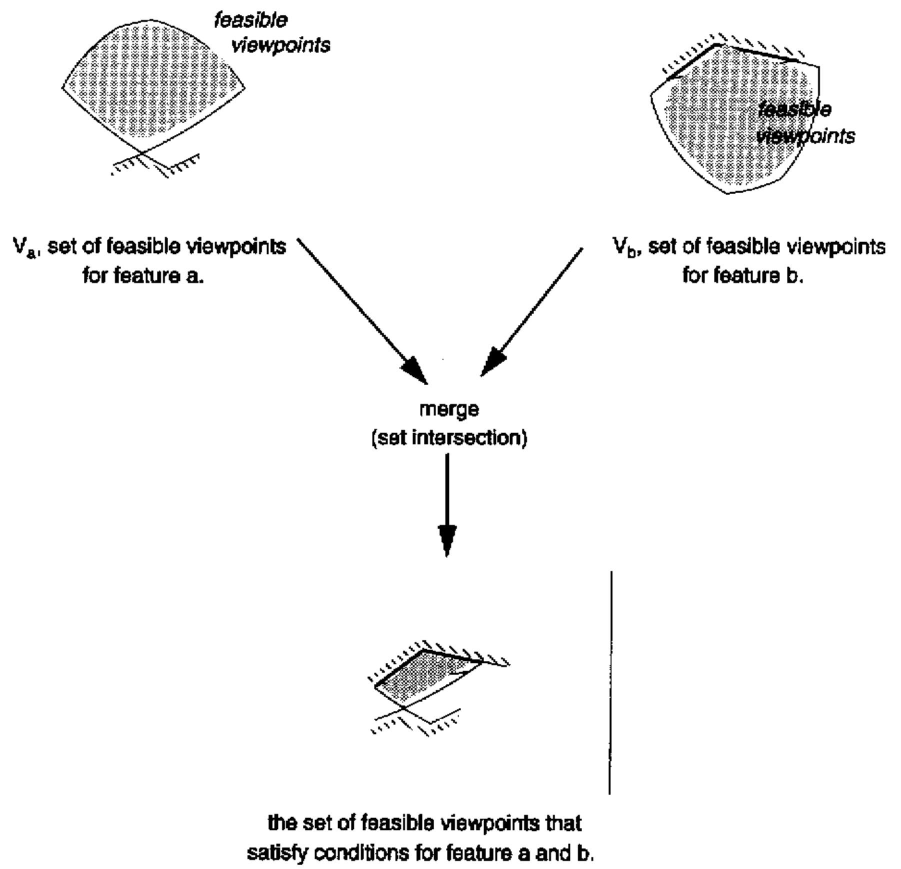


Figure 5.10 Generating the viewpoints that satisfy the constraints for both features simultaneously.

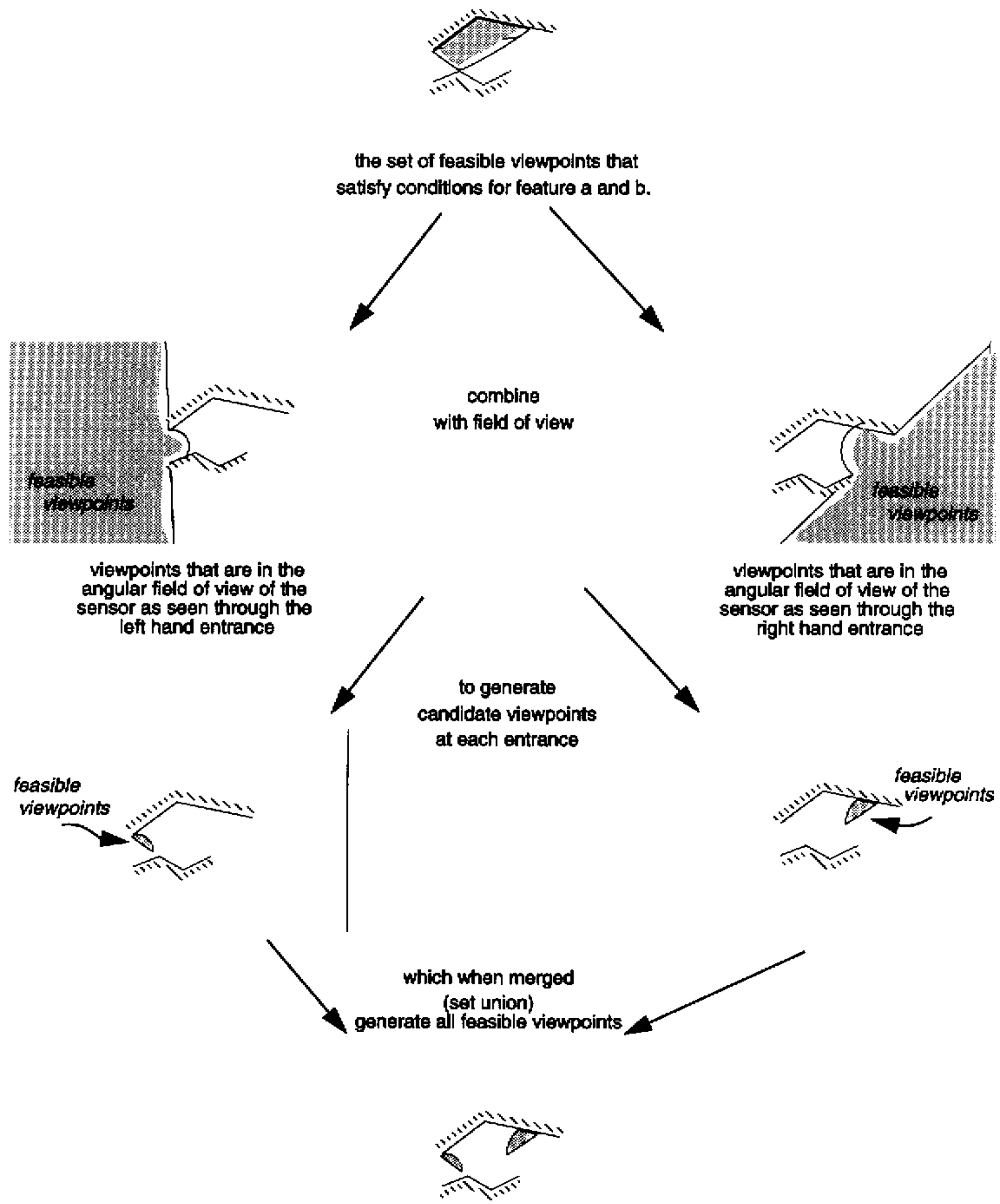


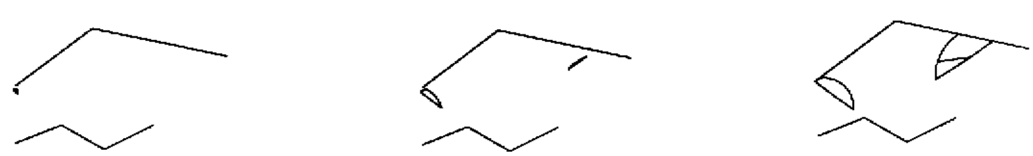
Figure 5.11 Generating the set of feasible viewpoints from the angular field of view constraint evaluated at each entrance.

5.3.2 Comparative study

Without further analysis, one might be inclined to position the sensor near the wider opening, in the hope of obtaining a less obstructed view of the entire feature. However, a trade-off analysis illustrates that, in fact, the "best" location is near the narrower opening.

In this section viewpoints from where a sensor can acquire an accurate and complete image of the interior of a cavity are generated for different sensor configurations. The parameters used in the experiment are sensor orientation, angular field of view and viewing angle. In Figure 5.12, the viewpoints are generated for a sensor with three different angular field of view. The angular resolution of the sensor is set to 1°, the maximum allowable angle of incidence is set at 90°. Several observations can be made. As illustrated in Figure 5.12 (a), (b) and (c), as the angular field of view increases so does the regions of valid viewpoints. When the angular field of view is small, there is a valid region near the narrow opening of the cavity but none near the wider opening. As the angular field of view increases, the valid region near the wider opening increases at a faster rate than that near the narrow opening.

This rate of increase is due to the following reasons. The region near the wide opening that contains the set of valid viewpoints is larger than the region that is closest to the narrowest opening. However, this region is also deeper into the cavity and located further away from the opening. Consequently, although the set of feasible viewpoints may be larger, a wider angular field of view is also required for a sensor to see the entire feature from any of these feasible positions.



a) $A_{FOV} = 200^{\circ}$ b) $A_{FOV} = 250^{\circ}$ c) $A_{FOV} = 280^{\circ}$

Figure 5.12 Space of feasible solutions vs. angular field of view. (The angular resolution $\Delta\theta = 1^{\circ}$, Accy = 90°)

In Figure 5.13, the viewpoints are generated for a sensor with an angular field of view of 250° and an angular resolution of 1°. The difference between the two examples is the maximum allowable angle of incidence which is 80^o for the example in Figure 5.13-a and 90^o for the example in Figure 5.13-b. As illustrated in Figure 5.12 (a) and (b), there is no feasible viewpoint when the angle is 80^o.

Figure 5.13-b illustrates the situation when the maximum angle of incidence is 90^o; there is a solution, which is not the case when the maximum angle of incidence is 80^o. When the maximum angle of incidence is 90^o, the accuracy constraint is equivalent to the visibility constraint [Chapter 6]. Therefore, when the maximum angle of incidence is 80^o, the set intersection between the valid viewpoints generated by the accuracy constraint and those generated by the rest of the constraints is null. This is expected since the set of feasible solutions generated by the accuracy constraint concentrate in a region that is along a line that is perpendicular to the chord connecting each of the two end points of the upper and the lower sides of the feature. For this example, the set of feasible viewpoints due to the accuracy constraint is somewhere in the middle of the cavity whereas the set of feasible viewpoints for all other constraints is near the openings.

a) Accy = 80^o b) Accy = 90^o

Figure 5.13 Space of feasible solutions vs. accuracy. (The angular resolution AFOV = 250^o, $\Delta\theta = 1^o$)

Figure 5.14 is an illustration of the feasible regions generated for sensors with different angular field of view and angular resolution characteristics. The different results are displayed in a table. Horizontally, the angular resolution remains constant. Vertically, the angular field of view remains constant.

As the angular field of view increases (horizontal), the space of feasible viewpoints increases. As the angular resolution decreases (vertical) the space of feasible sensor positions decreases.

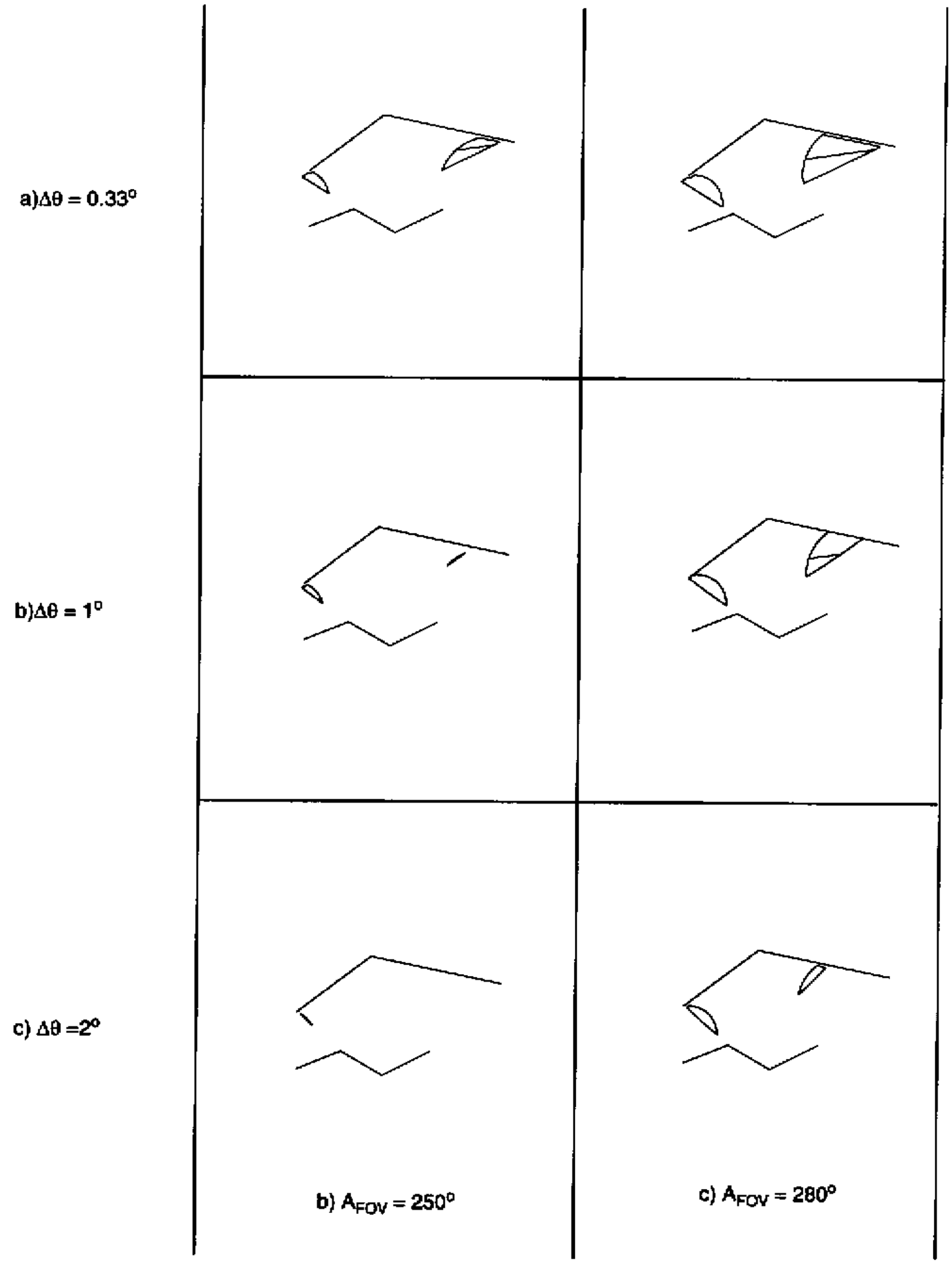


Figure 5.14 Space of feasible solutions vs. angular field of view and angular resolution.

5.3.3 Discussion

Because of the locality of certain constraints (visibility, range field of view, accuracy and spatial resolution) a complex problem can be solved by parts. Thus, regardless of the number of openings in a cavity, it is possible to generate the set of feasible viewpoints.

The angular field of view constraint is global as it depends on all of the points on the contour of the feature. Therefore, the angular field of view constraint cannot be computed by parts. To compute the minimum angular field of view required to see a feature, the feature is connected by a virtual line segment. This segment is only used to determine the viewpoints from where the minimum angular field of view that is required to sense the feature is less than the angular field of view of the sensor. It does not play any other role and is not used to determine the set of viewpoints that satisfy the other four constraints.

Chapter 6

Trade-off Analysis

The methods that have been developed in this dissertation can be used to position a mobile sensing platform at a location from where the sensor is able to acquire accurate and complete information. These methods can also be used to design a sensor and to specify the visual task. One of the current limitations in this technology is the amount of computation time that may be required to determine the "best" sensor position and the "best" sensor design parameters. Heuristics are needed to quickly evaluate the alternatives.

The shape of the region that encloses the set of feasible viewpoints depends on the characteristics of the sensor, the geometry of the feature and the characteristics of an acceptable image. An understanding of how these parameters interact may provide the insights needed for developing heuristics with which one can accomplish some of the following tasks.

1. To be able to quickly evaluate whether or not there is a solution

2. To be able to coarsely position the sensor

3. To reduce the number of computations

4. To increase the space of feasible solutions by changing the parameters of the sensor, the feature and the image quality specifications.

The purpose of this study is to develop the basis for such heuristics through a trade-off analysis of the constraints. Three aspects are examined in this analysis.

1. Dependencies between two or more constraints. Constraints that are a subset of another constraint can be eliminated without affecting the accuracy of the result.

2. Sensitivity of a constraint to the control variables and to the geometry of the feature. This determine how changes in the parameters of the sensor affect the shape of the region that

encloses the feasible solutions.

3. Quick evaluation of a candidate solution. An approximate solution can be used to determine whether or not there is a solution and to identify the overall location of the solution.

6.1 Summary of the Constraints

In this section the algebraic equations that evaluate whether from a candidate sensor position the visibility, angular field of view, range field of view, spatial resolution and accuracy constraints are satisfied are defined. These equations set the groundwork that is needed to complete the trade-off analysis. The viewpoints that satisfy each constraint are members of the set of feasible sensor positions that are contained in the regions which are generated by the methods described in chapters 3 and 4.

Some of the symbols that are used in this section are:

w a point.

$\vec{w}_i$ a vector between points w and i, where i is a point on the contour of the feature.

$\vec{N}_i$ the vector that is normal to the contour of the feature at point i.

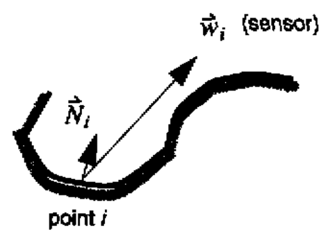


Figure 6.1 The vectors $\vec{w}_i$ and $\vec{N}_i$

6.1.1 Visibility

From a point w the entire feature is visible if the following relation holds true:

$$\operatorname{acos}\left(\frac{\vec{w}_i \bullet \vec{N}_i}{|\vec{w}_i||\vec{N}_i|}\right) \le 90^o \qquad [6\text{-}1]$$

for any point i along the contour of the feature (see Figure 6.2).

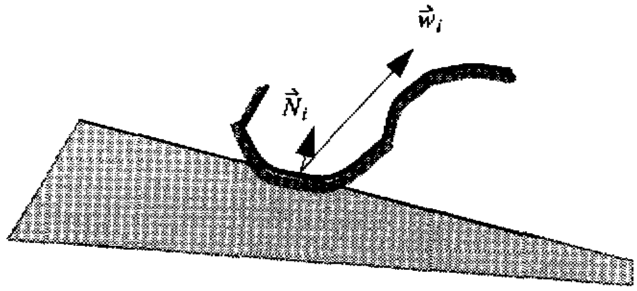


Figure 6.2 Viewpoints that satisfy the visibility constraint.

Whether or not there is at least one viewpoint that satisfies the constraint depends on the shape of the feature and the angle that is spanned by the orientation of each normal vector. As discussed in section 3.1, a concave feature has at least a set of solutions which lies in the convex hull of the feature. If the angle spanned by the orientation of the normal vectors is less than 180°, then the set of feasible positions is unbounded, otherwise it is bounded (see Figure 6.3).

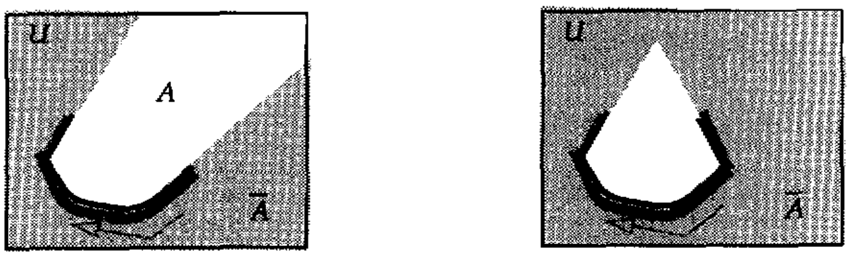


Figure 6.3 The set of viewpoints that are in front of every segment that describes the contour of a concave feature.

If the feature is convex, the set of feasible viewpoints is enclosed by the regions generated by two half-planes defined at the end points of the feature. This region is unbounded if the angle spanned by the orientation of the normal vectors is less than 180° and null otherwise. When the feature is convex, the solutions tend to concentrate along a line that is perpendicular to the chord connecting the two end points of the segment. (see Figure 6.4)

A convex feature for which the orientation of the normal vectors exceeds 180°.

Figure 6.4 The set of viewpoints that are in front of every segment that describes the contour of a convex feature.

6.1.2 Angular field of view

A point w is feasible if from this point, the minimum angular field of view required to see the entire feature is less than the angular field of view of the sensor.

From any feasible position, the minimum field of view required to sense the entire feature (labeled M_{FOV}) is given by:

$$M_{FOV} = \begin{cases} \operatorname{acos}\left(\dfrac{\vec{w}_0 \bullet \vec{w}_n}{|\vec{w}_0||\vec{w}_n|}\right) & if\ (w \notin B) \\ \\ 2\pi - \operatorname{acos}\left(\dfrac{\vec{w}_0 \bullet \vec{w}_n}{|\vec{w}_0||\vec{w}_n|}\right) & if\ (w \notin B) \end{cases} \qquad [6\text{-}2]$$

where $\vec{w}_0$ and $\vec{w}_1$ are two vectors connecting the sensor to the start (0) and finish (n) points of the feature and $|\vec{w}_0|$ and $|\vec{w}_1|$ are the magnitudes of these vectors. B is the region enclosed by the convex hull of the feature.

The minimum field of view required to see the entire feature is directly proportional to the distance between the two end points, it is inversely proportional to the distance between w and the two end points. The closer the two points are to each other, the smaller the M_{FOV} that is required. Conversely, the closer w is to the two end points the larger the angular field of view that is required. Another relationship that is not as apparent is that the M_{FOV} decreases when the sensor is at a location that is closest to the line perpendicular to the chord connecting the two end points.

6.1.3 Range field of view

A point w is feasible if from this point, the distance between this point and every point on the contour of the feature is within the range of detection of the sensor. If $\vec{w}_i$ is the vector from w to a point i on the contour of the feature, and r_{max} and r_{min} are the maximum and minimum distances at which the sensor can detect an object, the feature is within the range field of view provided:

$$r_{min} \leq |\vec{w}_i| \leq r_{max} \qquad [6\text{-}3]$$

The set of feasible viewpoints depends on the shape of the feature and on the values of r_{max} and r_{min}. Decreasing r_{min} and increasing r_{max} are two ways to increase the set of feasible viewpoints. Two other ways are to reshape the feature to reduce the distance between any two points on the contour of the feature and to only sample a portion of the feature.

In the following sections the effect of the range field of view constraint is analyzed by observing the effect of each part of the inequality independently.

6.1.3.1 $|\vec{w}_i| \leq r_{max}$

A point w is feasible if from this point:

$$|\vec{w}_i| \leq r_{max} \qquad [6\text{-}4]$$

Equation [6-4] defines a circle around every point i on the contour of the feature. Only points that lie in every one of these circles are candidate viewpoints. In other words, the set of points that satisfy these constraints are the positions that are contained in the intersection of these circles.

There is a feasible viewpoint only if the distance between every pair of points on the contour of the feature is less than or equal to r_{max}. If the distance between any two points is greater than $2*\, r_{max}$, then the two circles will not intersect and there is no sensor position. (see Figure 6.5)

Whether or not there is a solution depends on r_{max} and on the maximum distance between every pair of points along the contour of the feature. Figure 6.5 is an illustration of the set of feasible regions generated for different configurations of r_{max}. The distance between the two end points is equal to d.

For a feature composed of a single straight line, there is a feasible viewpoint if $d \le r_{max}$ (see Figure 6.5-c).

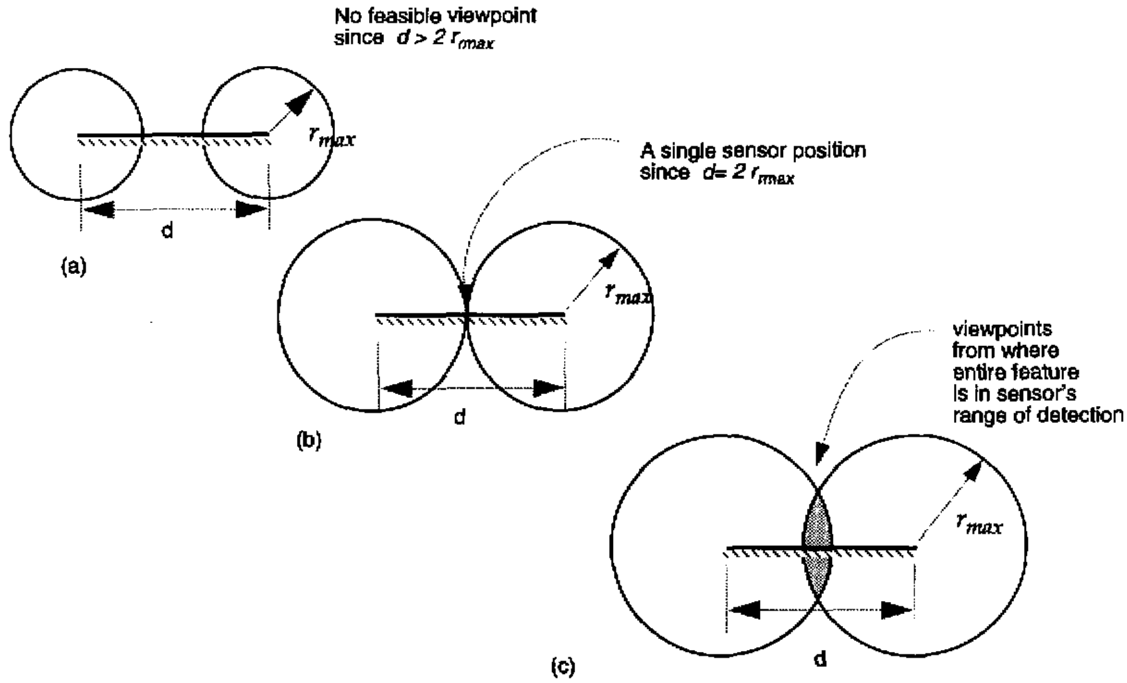


Figure 6.5 The range field of view constraint evaluated on a line segment.

The space of feasible viewpoints is the intersection of the circles placed at every point along the contour of the feature and the region from where the feature is entirely visible. The space of feasible viewpoints depends not only on the sensor's range of detection, but on the geometry of the feature as well. When a feature is concave and the distance between any two points is less than twice r_{max}, then there is a set of viewpoints in the space from where the entire feature is visible (see Figure 6.5). On the other hand, when a feature is convex the intersection of the circles measured at every point along the contour of the feature is not null provided that the distance between any two points on the contour of the feature is less than or equal to twice the maximum distance between any pair of points. However, when the maximum distance between any two points is less than twice r_{max}, the viewpoints lie in the space from where the entire feature is not visible. It is not until the radius of the circles is greater than twice r_{max} that there is a set of viewpoints that is also in the space from where the entire feature is visible (see Figure 6.7). The set of feasible solutions is concentrated along the line that is perpendicular to the chord that connects the two end points.

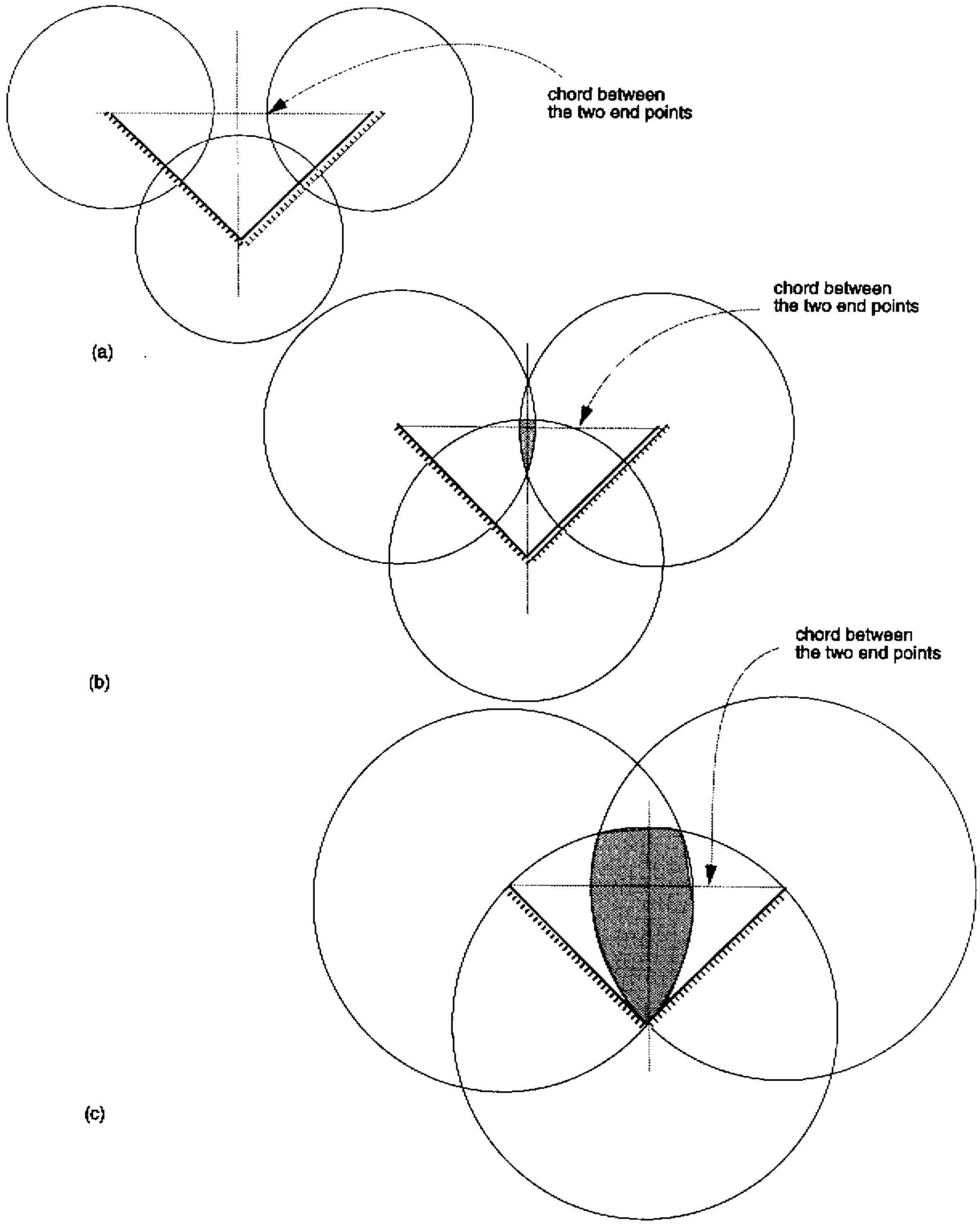


Figure 6.6 The range field of view constraint evaluated on a concave feature.

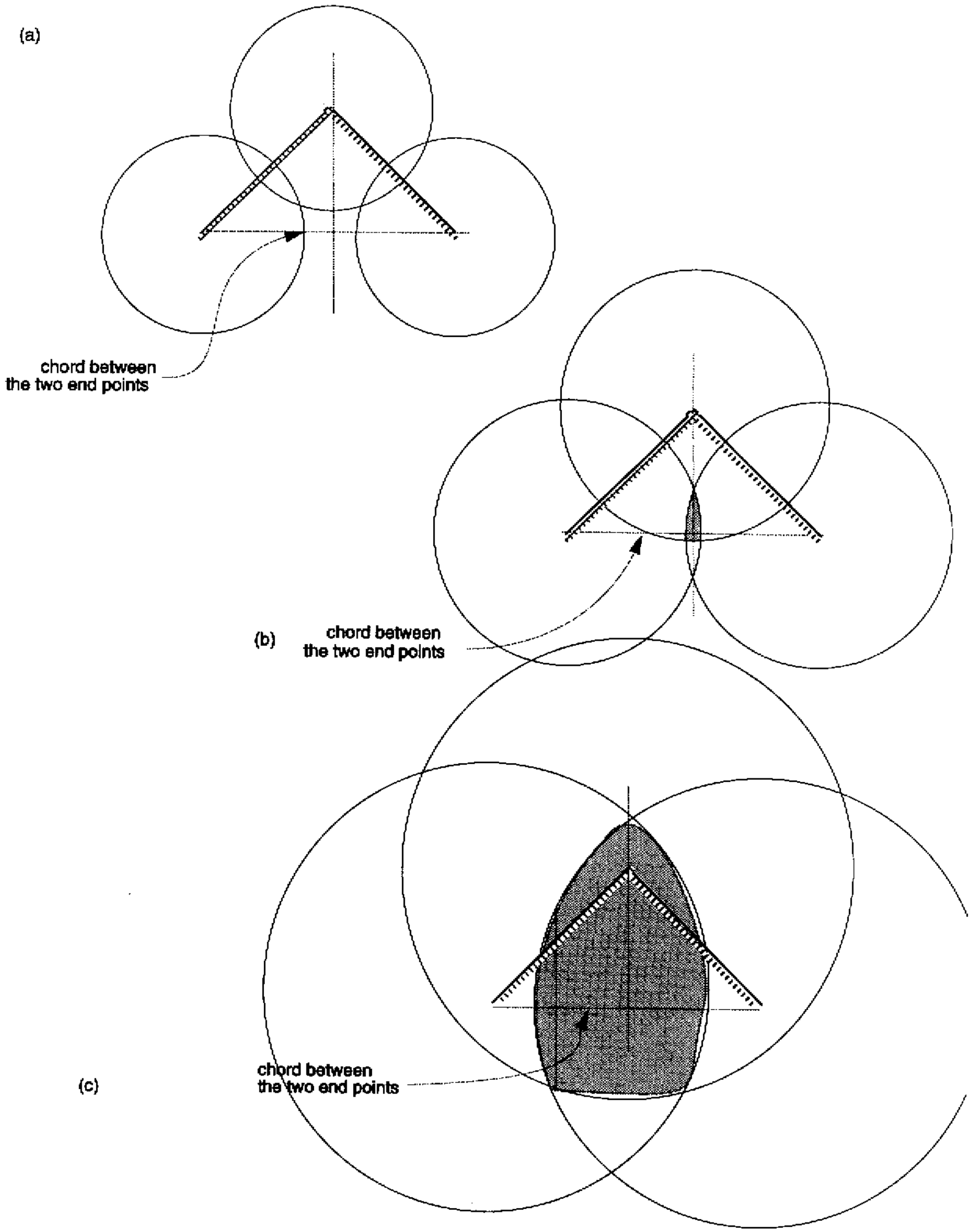


Figure 6.7 The range field of view constraint evaluated on a convex feature.

6.1.3.2 $r_{min} \le |\vec{w}_i|$

A point w is visible if from this point:

$$r_{min} \le |\vec{w}_i| \qquad [6\text{-}5]$$

Equation [6-5] creates a space of invalid viewpoints that separates the sensor from the boundary of the feature. The sensor cannot be in this region. This reduces the set of feasible sensor positions by eliminating those that are closest to the feature.

6.1.3.3 $r_{min} \le |\vec{w}_i| \le r_{max}$

Figure 6.8 is an illustration of the set of feasible viewpoints generated by evaluating the set of points that satisfies each side of the inequality, then subtracting the left hand side from the right hand side. As indicated in the illustration, the space of feasible viewpoints is proportional to r_{max}. Likewise, the space of viewpoints that are invalid is proportional to r_{min}.

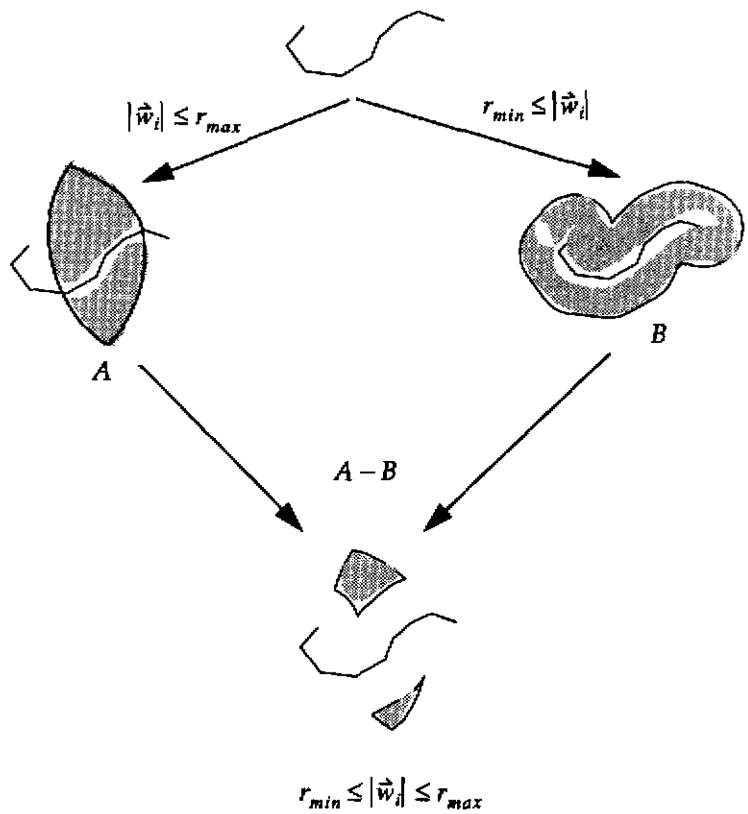


Figure 6.8 The set of feasible viewpoints that satisfy the range field of view constraint.

6.1.4 Accuracy

The range accuracy is a function of the angle of incidence of the beam. A maximum angle of incidence, γ, is selected to ensure that the accuracy of the range image is within acceptable boundaries. The angle is picked from a calibrated chart of range error versus target orientation [48]. The viewpoints are then selected such that the angle of incidence of the beam at every point along the contour of the feature is less than the maximum allowable angle of incidence.

From any position that satisfies the accuracy constraint, the angle of incidence of a beam to every point i on the contour of the feature is less than or equal to γ. Specifically, a viewpoint w is feasible if for every point i on the contour of the feature:

$$\operatorname{acos}\left(\frac{\vec{w}_i \bullet \vec{N}_i}{|\vec{w}_i||\vec{N}_i|}\right) \le \gamma \qquad [6\text{-}6]$$

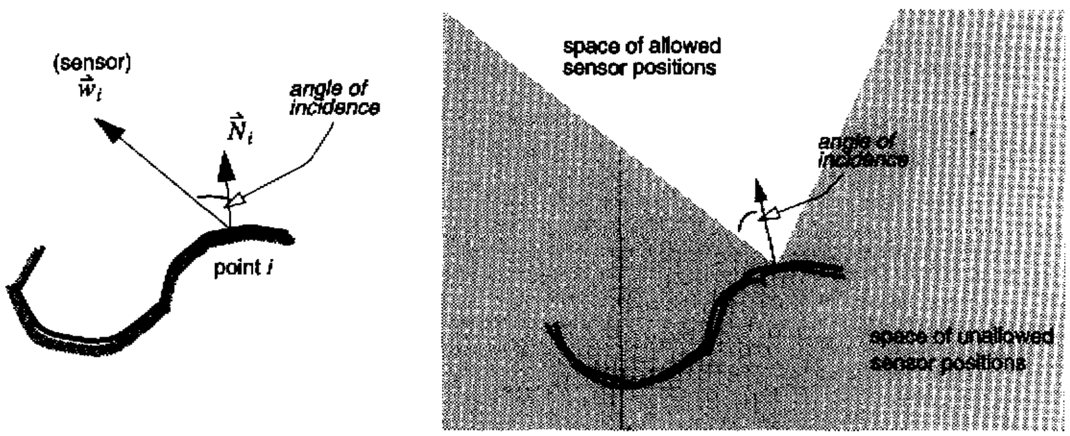


Figure 6.9 A single beam striking a target and the region of allowed sensor positions.

From observation it is shown that the accuracy constraint described by equation [6-6] is a subset of equation [6-1] that describes the visibility constraint. Since $\gamma \le 90^o$, a point that satisfies the accuracy constraint also satisfies the visibility constraint. Therefore, it is only necessary to evaluate one of these constraints. The accuracy constraint is more restrictive, therefore, to ensure that all constraints are satisfied, it is only necessary to evaluate the constraint given by equation [6-6].

The set of feasible viewpoints depends on the geometry of the feature and on the maximum allowable angle of incidence γ. The wider γ, the larger the set of feasible viewpoints, the narrower γ, the smaller the set. Three ways to increase the set of feasible viewpoints are to:

1) increase the maximum allowable angle of incidence;

2) reduce the length of the segments that define the contour of the feature;

3) only sample a portion of the entire feature.

Figure 6.11 is an illustration of the accuracy constraint evaluated for a line segment. The maximum allowable angle of incidence is constant in all three sub-figures. What changes is the length of the segment. The maximum angle of incidence is preserved in the set of feasible viewpoints, what does change however is the distance between the apex of the feasible region and the line segment. The longer the line, the larger the distance. This distance is also a function of the maximum allowable angle of incidence. The wider the angle, the closer the apex will be to the feature. It is also important to note that in this example, the set of feasible viewpoints are concentrated along a line that bisects the feature.

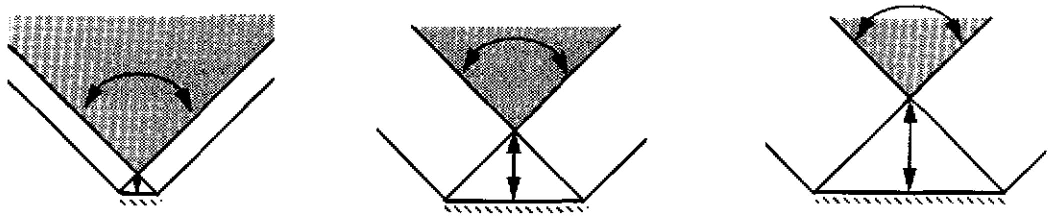
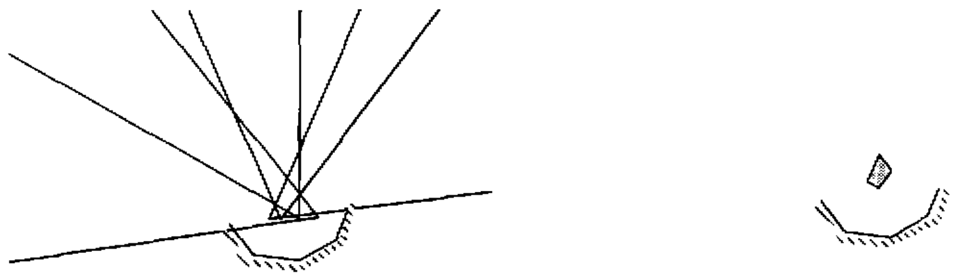


Figure 6.11 The accuracy constraint for a line segment of different lengths.

If the feature is concave, the set of feasible viewpoints is the region that results from the intersection of the cone at every point along the contour of the feature. The region that results tends to concentrate along a point that is near the intersection of the normal vectors at every point along the contour of the feature. Note that there may be no solution if the distance between the apex of the cone and the line feature for which it is defined is too large thus causing the cone to not intersect with other cones near the point of concentration.

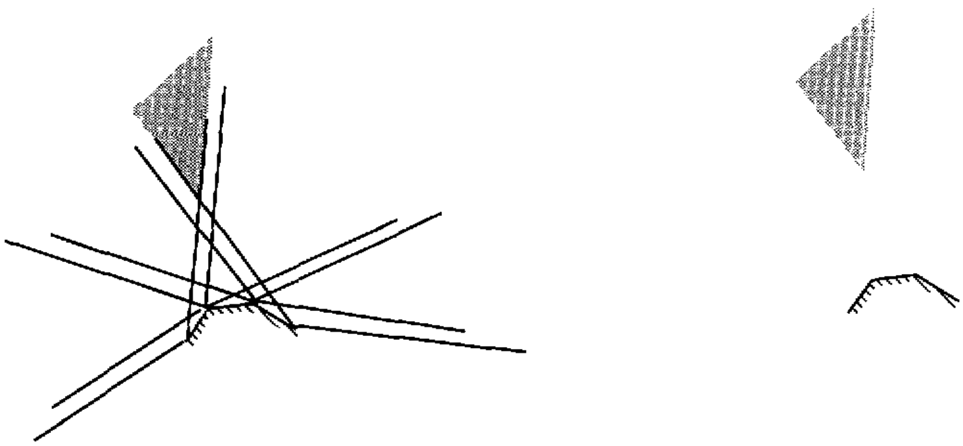
If the feature is convex, the set of feasible viewpoints tends to concentrate along a line that bisects the chord that connects the two end points of the feature (Figure 6.13). There is no solution if the angle spanned by the orientation angles of the normal vectors exceeds 360^0 - γ.



a) The accuracy cones for different segments along the contour of a concave feature.

b) The set intersection of the accuracy cones.

Figure 6.12 The accuracy constraint for a concave feature.



a) The accuracy cones for different segments along the contour of a convex feature

b) The set intersection of the accuracy cones.

Figure 6.13 The accuracy constraint for a convex feature.

6.1.5 Spatial resolution

A point w is a feasible viewpoint if the distance between any two consecutively sampled points sampled from w is less than a maximum allowable distance Y. The set of feasible viewpoints can be increased by increasing the angular resolution of the sensor (making $\Delta\theta$ smaller) or by increasing Y.

Figure 6.14 is an illustration of the scanning pattern of a laser range sensor. The beams are separated an angular distance of $\Delta\theta$. This causes the sensor to sample points that are separated by a distance of

ΔY from each other. The distance between any two consecutively sampled points y_{i+1} and y_i is given by

$$\Delta y = \frac{r_i \, Sin(\Delta\theta)}{Cos(\theta + \Delta\theta)} \qquad [6\text{-}7]$$

where

$$\Delta y = y_{i+1} - y_i = r_{i+1} \, Sin(\theta + \Delta\theta) - r_i \, Sin(\theta) \qquad [6\text{-}8]$$

and

$$r_i = \frac{h}{Cos\theta} \quad \text{and} \quad r_{i+1} = \frac{h}{Cos(\theta + \Delta\theta)} \qquad [6\text{-}9]$$

and θ is the angle of incidence of the beam given by:

$$\theta = \operatorname{acos}\left(\frac{\vec{w}_i \bullet \vec{N}_i}{|\vec{w}_i||\vec{N}_i|}\right) \qquad [6\text{-}10]$$

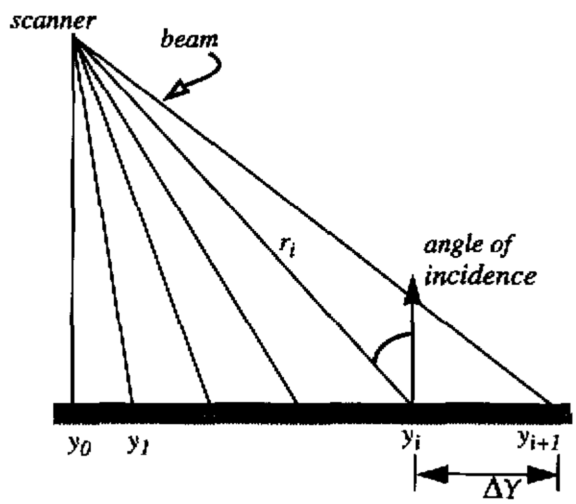


Figure 6.14 An illustration of the scanning pattern of a laser range scanner.

Given a maximum separation between two consecutive points labeled Y and the angular resolution of the sensor, the set of viewpoints from where two consecutive beams will strike a target a maximum distance of Y apart is constrained by a circle with radius R and center x_c, y_c given by:

$$R = \frac{Y}{2 \, Sin(\Delta\theta)} \qquad [6\text{-}11]$$

and

$$(x_c, y_c) = (\frac{Y}{2} + y_i, R\ Cos(\Delta\theta)) \qquad [6\text{-}12]$$

The region is a circle labeled S_i.

The set of feasible viewpoints is obtained by evaluating

$$S = \bigcap_i S_i \qquad [6\text{-}13]$$

over every point on the contour of the feature. For a straight line segment, the set of feasible viewpoint is simply the set intersection of the regions evaluated at the two end points. Figure 6.15 is an illustration of the spatial resolution constraint applied to a single straight line segment. As illustrated in the diagram there is no feasible viewpoint if the distance between the two extreme points (d) is greater than twice the radius of the circle. Contrary to the range field of view constraint, there is a solution for $d = 2\ r$.

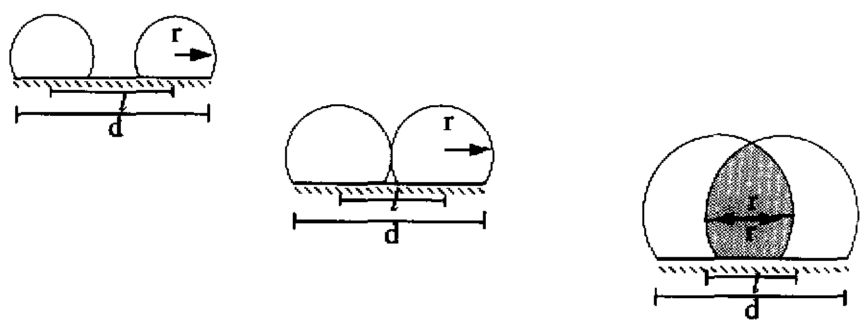


Figure 6.15 Spatial Resolution Constraint.

Figure 6.16 is an illustration of the set of feasible viewpoints for a concave feature. As with the range field of view constraint, the set of feasible viewpoints tends to be concentrated in a region that is near the intersection of the bisectors of each segment that composes the contour of the feature. This region of concentration also appears to be crossed by the bisector of the chord that connects the two end points.

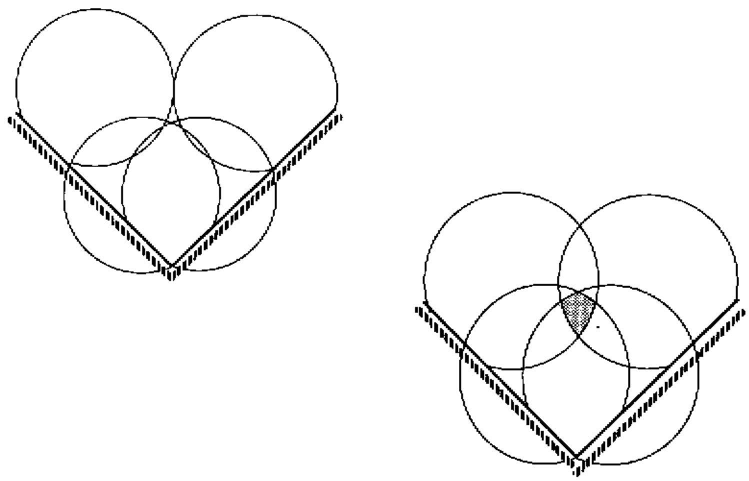


Figure 6.16 Spatial Resolution Constraint for a concave feature.

Figure 6.16 is an example that has been generated for a convex feature. In order for the regions to

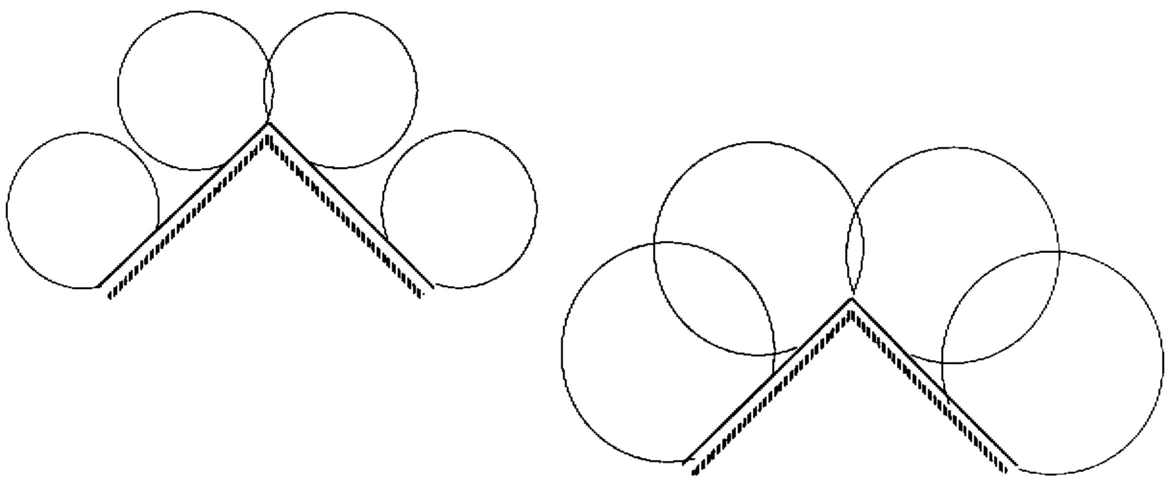


Figure 6.17 Spatial Resolution Constraint for a convex feature.

intersect, thereby generating a set of feasible viewpoints, the radius of the circle must be much larger than for a concave feature. The dimensions of these circles is proportional to the angle between the vectors which are normal to the feature. Thus, to increase the set of feasible viewpoints one could partition the feature such that the angle spanned by the orientation of the normal vectors is as small as possible.

6.1.5.1 Corners

The spatial resolution constraint is evaluated at consecutively sampled points. At corners, one of the points is located in one of the segments, the other point is located in the other segment. This creates a chord between the two points which does not lie entirely on either one of the sub-features. The spatial resolution constraint is evaluated at every chord and intersected with the set of possible sensor positions. The regions that are evaluated at each of these chords reduce the space of feasible sensor positions.

Figure 6.18-a is a diagram that illustrates the chord for different points on the contour of the feature. Figure 6.18-b is a diagram that illustrates part of the circles (defined by equations and) generated for each chord.

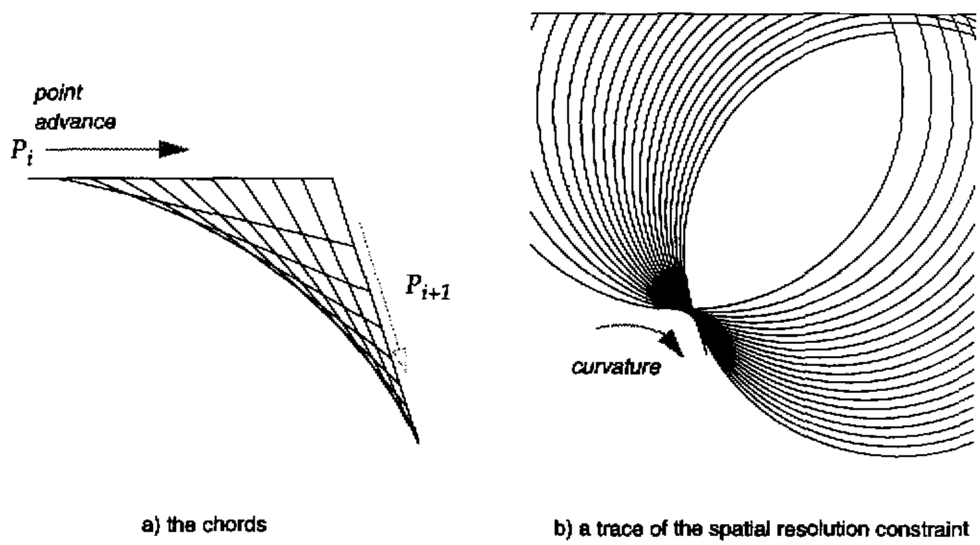


Figure 6.18 A trace of the chord for a corner angle equal to 45°.

6.2 Dependencies

A constraint that is a subset of another constraint can be eliminated without affecting the result.

1. From equations [6-1] and [6-6], it is clear that the accuracy constraint is a subset of the visibility constraint. The implicit equation that describes a feasible point is the same with a change only in the constant that establishes the limit. Since the accuracy constraint is more restrictive. It dominates over the visibility constraint. That is, a feature is visible if it satisfies the accuracy constraint.

2. The angular field of view constraint is composed of three regions: F the set of viewpoints from where the entire feature is visible, V the viewpoints that are on the half-plane defined by a line crossing the two end points, and C a circle defined on the line segment connecting the two end points and the angular field of view of the sensor.

The constraint is valid only in the space where the viewpoints are visible. Therefore one can evaluate only one or the other. If the viewpoint is not in $V \cap C$ (if $\alpha < 180^o$), V (if $\alpha = 180^o$) or $V \cup C$ (if $\alpha > 180^o$), then the feature is not visible. Checking if a feature is in one of these regions takes two evaluations.

6.3 Discussion

Accuracy is more restrictive than visibility. But if both are requirements, only one of these is needed.

Constraints such as the range field of view, accuracy, and spatial resolution pull the solution to a triangle that is oriented in the middle of the chord when the feature is convex and to a point that is near the intersection of the vectors that are perpendicular to each segment when the feature is concave. The accuracy constraint pulls the solution a distance away from the line segment by a distance that is proportional to the length of the feature. Similarly, when the feature has a concave corner, the spatial resolution constraint pulls the solution a distance away from the line segment. On the other hand, it pulls the solution towards the feature when the feature is convex.

The maximum range field of view of the range accuracy constraint brings the solution close to the feature whereas the minimum range field of view eliminates points that are close to the feature.

Chapter 7

Conclusions

This research presents a method to synthesize the viewpoints which satisfy five constraints that characterize the quality and completeness of an image. The organization of this section is as follows: the method is summarized in section 7.1, the results are reported in section 7.2, and future work is projected in 7.5.

7.1 Summary

A method has been developed to position a sensor to acquire an accurate and complete description of a feature. The method synthesizes all of the possible locations from where the feature is visible and within the sensor's field of view. It also synthesizes all of the locations from where the sensor can acquire dense and accurate data. The viewpoints that are in both sets are chosen as candidate sensor positions.

Finding the viewpoint is done in two parts. First, the set of candidate sensor positions is generated for each constraint using the previously described method. One of these points that is common to all constraints is selected to position the sensor. Second, the sensor is oriented so that from the selected position, the feature is in the sensor's field of view. Dividing the procedure into two steps enabled the use of simple geometric algorithms to synthesize the set of candidate viewpoints.

The method to synthesize the set of all possible viewpoints is analogous to the *constructive solid geometry* (CSG) method employed to build geometric solid models[38]. In CSG modeling, a set of primitives (cubes, spheres, cylinders, etc.) are combined using boolean operators (union, intersection, difference) to produce a solid model. In this method, two primitives (a block[1] and a circle) and four operators (complement, union, intersection, difference, and body sweep) are used to generate the space of all candidate viewpoints[2].

1. A block is the implementation of an infinite *line* which divides the space of all viewpoints into two sets: the set that is in front of the line and the set that is in back.

The orientation of the sensor is resolved from the *minimum spanning sector* that completely encloses the feature. The minimum spanning sector is the sector that is obtained after projecting the feature onto the contour of a unit circle centered about the sensor. The spanning angle of this cone is the minimum angular field of view that is required to sense the feature from that sensor position. The vector that bisects the sector is the senor's orientation. An advantage of this method over other approaches[13][52] is that it is able to determine the orientation of the sensor even when the sensor is located inside the cavity of a concave feature.

A divide and conquer approach is used to find multiple viewpoints when a single image is not sufficient to obtain an accurate, dense and complete description of the feature. A feature is recursively subdivided until for each sub-feature there is at least one viewpoint from where the sensor can acquire an image that satisfies all of the visibility, detectability, density and accuracy constraints.

The method has been tested, in simulation, on a complex feature for different sensor configurations (angular field of view, range field of view, angular resolution) and task parameters (maximum allowable distance between two points and angle of incidence of the beam).

7.2 Results

Methods were generated to position a sensor to acquire accurate and complete information.

The methods were demonstrated on a number of examples. The examples were generated using different features, sensors with different range field of view, angular field of view and angular resolution characteristics, and different image accuracy and density specifications. The last of these examples generated the viewpoints that from where a sensor could acquire a complete and accurate image of the interior of a cavity.

The algorithms have been implemented for two dimensional polygonal objects using ACIS, a geometric solid modeler. The results validated the method.

2. *Union*, *intersection* and *difference* are standard boolean set operations. The *complement* operation produces a region which is equal to the set difference between a region that encloses the set of all possible viewpoints and a primitive. The *body sweep* operation produces a region that is equivalent to the union of many instances of one primitive which are closely spaced together along a contour of a given path.

Two algorithms were compared to determine the location at which to partition a feature to generate multiple viewpoints from where the sensor was to acquire multiple images. The first one recursively divided the feature in half after each iteration. The second strategy divided the feature at points of inflection.

Subdividing the feature led to an important observation: the spatial resolution constraint is a locally global constraint. The spatial resolution constraint evaluates a constraint on two points. When splitting the feature, the two end points may lie on different sub-features. These same points must be used when evaluating the spatial resolution constraint for each sub-feature.

Dividing the feature at points of inflection gave some advantages that reduced the number of sub-features that were generated. Concave features draw the solution towards the center of the cavity. Convex features on the other hand draw the solutions towards a point that is on a line that is normal to the chord of the feature. This last point is distant from the solution generated by any of the neighboring concave sub-features.

A simulator has been developed to test different sensing and sensor alternatives.

7.3 Conclusions

The viewpoints from where a sensor can acquire accurate and complete information can be synthesized from a description of the sensor, the feature and the image density and accuracy requirements.

Geometric principles can be used to generate efficient algorithms that work for both curved and polygonal segments. The analogy of geometry and set theory enable the explanation, proof and implementation of these algorithms.

Dividing a feature to obtain multiple viewpoints is a simple and direct approach to generate the views that are required to sense a feature.

7.4 Contributions

The contributions of this dissertation are in the areas of range and angular field of view constraints, orienting the sensor and partitioning the feature.

7.4.1 Angular field of view

Cowen was the first to introduce the angular field constraint as a circle defined by the length of a line segment and the angular field of view of the sensor[13]. The difficulty was to apply this idea to an arbitrary feature or collection of features. Cowan opted to generate a conservative solution by enclosing the features by a minimum circumscribing circle. This generates a conservative constraint that limits the location of the sensor which precludes locations that are located inside the cavity of a concave feature.

One important observation generated by this work is that from any position that is in the region of viewpoints from where a feature is visible, the angular field of view required to see an entire feature is equal to the field of view required to see a line segment connecting the two end points of the feature (provided that the sensor is facing the feature). This observation is significant as it enables the synthesis of the exact collection of viewpoints that will satisfy the angular field of view constraint. Extending Cowan's definition of the angular field of view made it possible to describe viewpoints that are inside the cavity of a concave feature. This is particularly useful when scanning a concave feature with a sensor that has a wide field of view which exceeds 180°.

7.4.2 Range field of view constraint

A constraint that was not previously defined in the literature is the range field of view constraint. The range field of view of a laser range scanner is bounded by its ability to detect a target outside of a maximum and minimum range of detection. This limits the set of candidate sensor positions. In this paper the set of viewpoints that satisfy the range field of view constraint was derived. The viewpoints were generated by sweeping two regions derived from the maximum and minimum range of detection across the contour of the feature.

7.4.3 Orienting the sensor

Previous approaches to determine the orientation of the sensor were sensitive to the position and orientation of the sensor [13][53]. These were unable to orient the sensor from inside the cavity of a concave feature. The method implemented in this research is invariant to the position and orientation of the sensor. It is able to determine the orientation of the sensor even from locations that are inside the cavity of a concave feature. The method has been developed only for two dimensional features. However, some preliminary work has been done to develop the approach further to determine the orientation of the sensor in a three dimensional environment. If this method can be generalized it will provide a solution that is much simpler than current alternatives and is linear in the number of vertices of the feature.

It has already been mentioned that the spatial resolution constraint is locally global. The constraint has the characteristic that it evaluation depends on two points along the contour of the feature and not just any individual point. Therefore when dividing the feature to evaluate multiple viewpoints, it is important to keep a set of points from the adjoining feature. These points will be used to evaluate the constraint at the extreme points of the sub-feature.

7.4.4 Partitioning the feature

Previous approaches to automatic sensor planning found a single viewpoint from where the image that was acquired satisfied a set of image constraints. Abrahams et. al. take a dynamic problem of moving obstacles and converts it into a static problem that can be solved by static automatic sensor placement techniques[1]. Tarabanis added a degree of freedom to the problem by determining the configuration of the sensor that will maximize some properties of the image[53]. This research extends the research in sensor placement along another axis, that of partitioning the visual task when a single viewpoint is not sufficient to acquire accurate and complete image.

Finding the minimum number of views is a minimax problem which cannot be solved in a linear search. Although the divide-and-conquer mechanism implemented in this research generates the views that are required to sense the feature, it is easy to show that this number is not in general the smallest number of views that are required. The number of views that are generated depends on the selection of the point at which each feature is divided. The first algorithm that was tested divided the feature at the mid point evaluated along the perimeter of the feature. It did generate a set of views that in totem gave a complete and accurate description of the feature. The second algorithm that was tested which also generated the views that gave a complete and accurate description of the feature divided the feature at the points of inflection.

Further research in this problem might yield an algorithm that will select the split points that are guaranteed to generate the minimum number of views.

7.5 Future Work

7.5.1 Extensions to 3D

Further investigation is required to determine the extensions to three dimensional features. The visibility constraint exploits the characteristics of concavity and convexity to generate the space of candidate sensor positions. Although it appears in principle that similar simplifications should apply to three dimensional features, saddle points that may occur in three dimensional features preclude the direct use of the algorithm. Further investigation is required to determine the points at which a sensor can be partitioned and the effects of a saddle point on the set of valid sensor positions.

In two dimensions, the viewpoints that satisfy an angular field of view constraint have been determined by the union or intersection of the angular visibility arc (a circle) and the equivalent feature of projection - a line segment whose projection onto the unit circle is equal to the projection of the entire feature.

In a three dimensional environment, the region constrained by a single segment on a three dimensional space is a donut formed by sweeping a planar two dimensional angular visibility arc around the line segment. The angle subtended between the sensor at each point on the donut is constant.

In an orthogonal environment, there is a second donut oriented along the segment. It is possible that these two sets, which are orthogonal to each other, may be sufficient to determine the viewpoints from where the sensor can see the object. Similar to the two dimensional counterpart, if the angular field of view (horizontal and vertical) is less than 180°, the union of these regions swept across lines located across the contour of the feature determine the viewpoints from where the sensor would not be able to detect the feature. If the angular field of view is less than 180^0, the intersection of these regions determines the viewpoints from where the sensor is able to detect the feature.

The orientation of the sensor is obtained from the spanning angle of the minimum spanning sector of the projection of a feature onto the unit circle. In three dimensions the projection can also be obtained by projecting the features onto a three dimensional sphere. The difficulty lies in parameterizing the projection to obtain a vector that represents the sensor's orientation.

Finding the orientation of the sensor when the field of view that is required is less than 180^0 has been addressed in part by Anderson.[3] by enclosing the feature in a unit sphere. This technique does not work when the sensor is located inside the cavity of the feature. Methods based on the projection of the feature onto a spherical coordinate system may not be able to determine the sensor's orientation.

The author has been exploring an alternative which is to project three dimensional feature onto multiple cylinders of unit radius. The axis of each cylinder is along the orthogonal axes of the cartesian plane. The z component of each cylindrical projection is eliminated; only the angular component is preserved. The idea, which has not been further developed, is that these angles are sufficient to orient a sensor such as the ERIM[16] whose field of view can be represented by two orthogonal sweeping planes can be oriented using this technique.

7.5.2 Generalization to other sensors

A light stripe sensor is a natural extension to the class of sensors that are handled by the methods described in this dissertation. The Toyota sensor typifies a two dimensional line stripe scanning device. A plane of light is projected onto the environment. A camera is used to detect the intersection of the light and a feature.

A more complicated extension is the application of this work to stereo perception systems. A stereo system has two cameras which are separated by a baseline (Figure 7.11). The range error is proportional to the baseline distance and to the relative orientation of each camera. It is not sufficient for each of these cameras to be in the space of candidate sensor positions. Instead, an additional constraint which has to do with the depth accuracy has to be evaluated. This constraint depends on the baseline distance and the orientation of each individual camera.

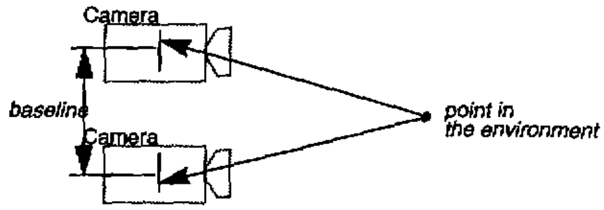


Figure 7.11 A binocular stereo range sensor. It uses two cameras to triangulate the range distance to a point in the environment.

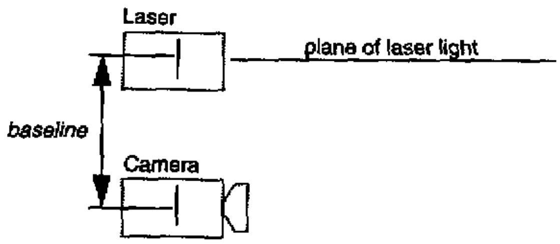


Figure 7.12 A binocular stereo range sensor. It uses two cameras to triangulate the range distance to a point in the environment.

7.5.3 Robustness

The assumption in this work is that models of the feature are provided and that these are fairly good representations of real life features. The sensitivity of a valid viewpoint to errors in the model has yet to be explored.

When a single image does not provide sufficient information, the means to subdivide the feature has been selected quite arbitrarily. This approach does not guarantee a minimum number of views. More formal approaches to select the point at which to partition the feature to obtain the minimum number of views have yet to be developed.

A candidate viewpoint is feasible if the sensor can reach the point from its current location [47]. Sensor placement must be combined with research in safe navigation and obstacle avoidance to generate a sequence of viewpoints that satisfy the image constraints and that can be reached by the sensor.

7.5.4 Exploratory active perception

A perception system may not be able to know the details of what it has not seen. But it may be able to derive and represent the areas that contain missing information. An exploratory active perception system can seek to explore these areas to obtain the missing information.

Areas that are occluded or that do not contain accurate and complete information can be derived from the current display of sensed information [31][35][36][58][59]. Rather than spanning the complete environment, an active perception system can direct the sensor to obtain the best information subject to the characteristics of the sensor, the image constraints, and the characteristics of the object and environment.

This generates an opportunity for an active robotic exploration system that is able to seek missing and accurate information.

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