A Minimum-Slip Attitude Control Method for the Erebus Walking Robot

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ABSTRACT

In this report, a method for attitude control of statically-stable walking robots is developed such that slip is minimized when the body is leveled by using only the actuators which cause vertical link motions. If all legs of a walker have three independent translational degrees of freedom, then an attitude control method that uses all actuators may be used to level the body with no foot slippage. However, there are some advantages in reducing the number of controllable degrees-of- freedom, as with the Erebus walker. The task is then to determine the best method to adjust the attitude of the walker body such that foot slippage and the build-up of internal link forces is minimized. The method that does this -- the minimum-slip z-axes attitude control method -- is developed in this technical report for that purpose.

1.0 INTRODUCTION

This document outlines research that was undertaken for achieving attitude control (leveling motions) on the Erebus walking robot, Dante. Design considerations for the Erebus walker dictated that full three-dimensional leg motions were not appropriate. To reduce the mass of the walker, a reduced number of degrees-of-freedom was deemed necessary. Therefore, the all-axes leveling method may not be used to level Dante; foot slippage and/or build-up of internal link forces (resulting in flexion) is inevitable for body attitude maneuvers. The prospect of having to face these effects on the very rugged terrain of Mount Erebus (an active volcano in Antarctica) motivated the present research on how to level the walker with only the vertical actuators of the legs, in such a manner which minimizes foot slippage and leg flexion.

The report first considers conventional leveling with only the vertical actuations of all legs -- the *simple-z-axes attitude control method*. The simple-z-axes attitude control method was supplanted by the *iso-altitude attitude control method* in work performed for the AMBLER program [1]. The iso-altitude attitude control method had some (small) advantages over the simple-z-axes attitude control method and was seen as the best z-axes leveling method at that time [2]. However, an insufficient amount of analysis was undertaken with regard to foot slippage. The purpose of the work presented here is to develop a new method for body attitude control with only the vertical actuations which minimizes foot slippage. The new method is called the *minimum-slip z-axes attitude control method*. For traversal of rugged, natural terrain, the minimum-slip z-axes attitude control method is the most appropriate when only the vertical actuators of a walker may be used for attitude control.

2.0 SIMPLE-Z-AXES LEVELING

The *Simple Z-Axes attitude control method* (SZA) is based on the same approximations used by Klein in his implementation of control for the OSU hexapod [3]. In his work, Klein generates velocity commands for the hexapod using the active compliance control scheme. The SZA attitude control method uses the same small angle approximations used by Klein, and also uses only vertical actuators for attitude control of the body. These approximations are widely used for attitude control on other walking machines [4] - [6]. In order to explain the SZA method, consider leg i of a walker whose x- and y-axes are in the horizontal plane (for a level walker). Referring to Figure 1, θ is the change in tilt about the x-axis, and γ is the change in tilt about the y-axis.

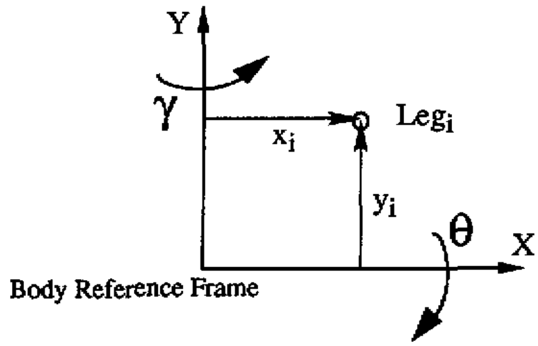

Figure 1. Rotations Required for Attitude Control of the Body.

To rotate the body by a small angle, γ, about the y-axis, the change in the vertical extension of leg *i* is:

$$\Delta z_i = -x_i \sin\gamma. \tag{1}$$

Similarly, the change of length of the vertical axis of leg *i* to rotate about the x-axis by a small angle θ is given by:

$$\Delta z_i = y_i \sin\theta. \tag{2}$$

To effect a small rotation about both axes, these length changes are superimposed, i.e.:

$$\Delta z_i = y_i \sin\theta - x_i \sin\gamma. \tag{3}$$

The above method for attitude control succeeds in bringing the body close to the desired inclination. However, this is at the cost of body repositioning and possible foot slippage. Figure 2 depicts the motion of an Ambler-equivalent leg [2] leveling from a tilt of -20 degrees using the SZA method, assuming that the foot of this leg does not slip. In leveling from a tilted configuration by the SZA method, the body coordinate frame (the control point) height at the end of the trajectory is higher than its starting position. Furthermore, the body frame shifts by a greater distance in the horizontal plane. When the AMBLER levels from a tilt of about 5 degrees, the body frame translates about 30 cm in the horizontal plane and rises about 1.9 cm in the vertical plane. However, the resulting motion is kinematically inadmissable if more than one leg's motion is considered. Therefore, for the entire walker, there will be a combination of a body frame motion similar to that shown in Figure 4, as well as some foot slippage and/or flexion of the walker's legs.

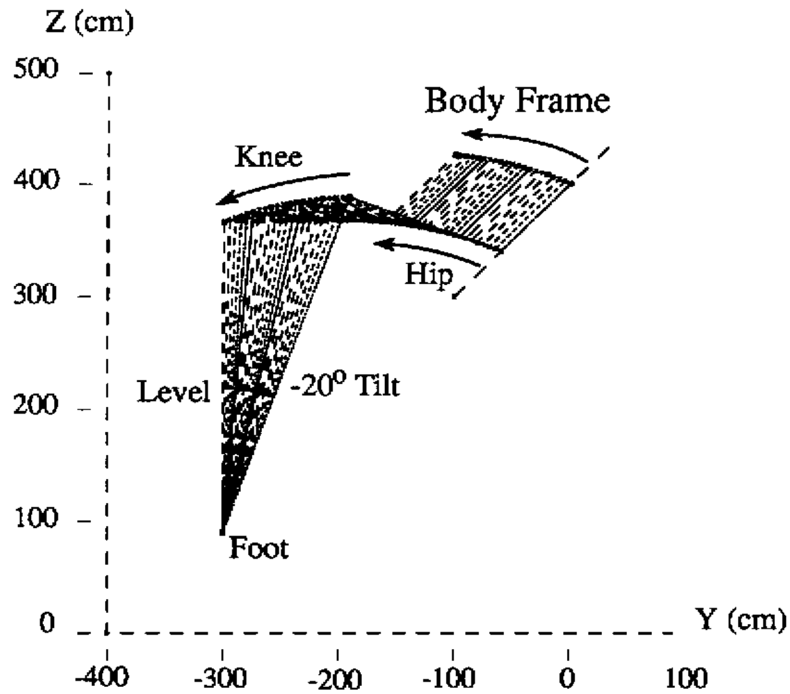

Figure 2. Displacement of the Body Frame, Knee, and Hip when Using the SZA Method.

3.0 ISO-ALTITUDE LEVELING

Since the body-reference coordinate frame changed elevation, as well as swayed in the horizontal direction, the SZA attitude control method had some undesirable body motions in the vertical direction. In this section, a method is shown which utilizes only the z-axes but still keeps the body frame from making excursions in the vertical direction during tilting or leveling maneuvers. This method is called the *Iso-Altitude Z-Axes attitude control method* (IAZA), and was developed by Gonzalez de Santos [1, 2]. Consider a leveled leg that rotates by θ about the x-axis. Figure 3 depicts a leg which is moving from a tilted to a level configuration. The horizontal link length is the same for both leg positions. The body frame translates in the horizontal plane maintaining a constant height a_i, which is the same as the z-coordinate of the leveled leg, z_{Li}.

The length a_i for leg i at rotation θ about the x-axis is given by:

$$a_i = \begin{bmatrix} \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} y_i \\ z_{Ti} \end{bmatrix}, \tag{4}$$

which is a constant parameter. Consider a walker body rotation about the x-axis from θ_1 to θ_2. Corresponding to this case one may write for leg i:

$$\begin{bmatrix}\sin\theta_1 & \cos\theta_1\end{bmatrix}\begin{bmatrix}y_i \\ z_{1i}\end{bmatrix} = \begin{bmatrix}\sin\theta_2 & \cos\theta_2\end{bmatrix}\begin{bmatrix}y_i \\ z_{2i}\end{bmatrix}, \tag{5}$$

and the incremental change in the vertical link length of leg i, $\Delta z_{\theta i} = z_{2i} - z_{1i}$, to achieve tilt θ_2 is:

$$\Delta z_{\theta i} = \frac{z_{1i}(\cos\theta_1 - \cos\theta_2) + y_i(\sin\theta_1 - \sin\theta_2)}{\cos\theta_2}. \tag{6}$$

Similarly, the incremental change in the vertical link length may be found, $\Delta z_{\gamma i} = z_{2i} - z_{1i}$, due to a rotation, γ, about the y-axis as:

$$\Delta z_{\gamma i} = \frac{z_{1i}(\cos\gamma_1 - \cos\gamma_2) - x_i(\sin\gamma_1 - \sin\gamma_2)}{\cos\gamma_2}. \tag{7}$$

Finally, the contribution of a small rotation about both the x- and y- axes may be determined through superposition:

$$\Delta z_i = \Delta z_{\theta i} + \Delta z_{\gamma i}$$

$$= \begin{bmatrix}\dfrac{\sin\gamma_2 - \sin\gamma_1}{\cos\gamma_2} & \dfrac{\sin\theta_1 - \sin\theta_2}{\cos\theta_2} & \left(\dfrac{\cos\theta_1}{\cos\theta_2} + \dfrac{\cos\gamma_1}{\cos\gamma_2} - 2\right)\end{bmatrix}\begin{bmatrix}x_i \\ y_i \\ z_{1i}\end{bmatrix}. \tag{8}$$

The above equation can be used to bring the body to level, or to tilt it such that the body frame does not move vertically. Figure 4 shows a simulation of a leg when the body of the walker is leveled from an initial tilt of -20 degrees with the usual assumption that there is no slippage of the foot of that leg. Note that the body frame has no motion in the vertical plane in this example. With the iso-altitude method, it is not necessary to do any work against gravity if the body frame is chosen to be the walker c.g. location. However, all of the z-axes attitude control methods translate the center of the body frame horizontally by about 30 cm for an initial 5-degree tilt under the assumptions that the foot of the simulated leg does not translate and that the links and joints are rigid. This implies that for all of these leveling methods the feet of the walker will slip.

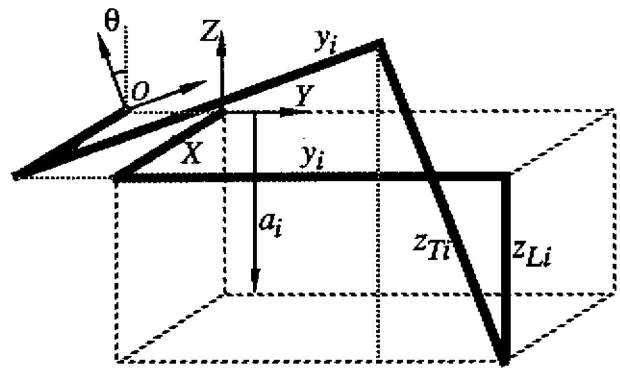

Figure 3. Tilted Leg and Leveled Leg with no Horizontal Link Motions.

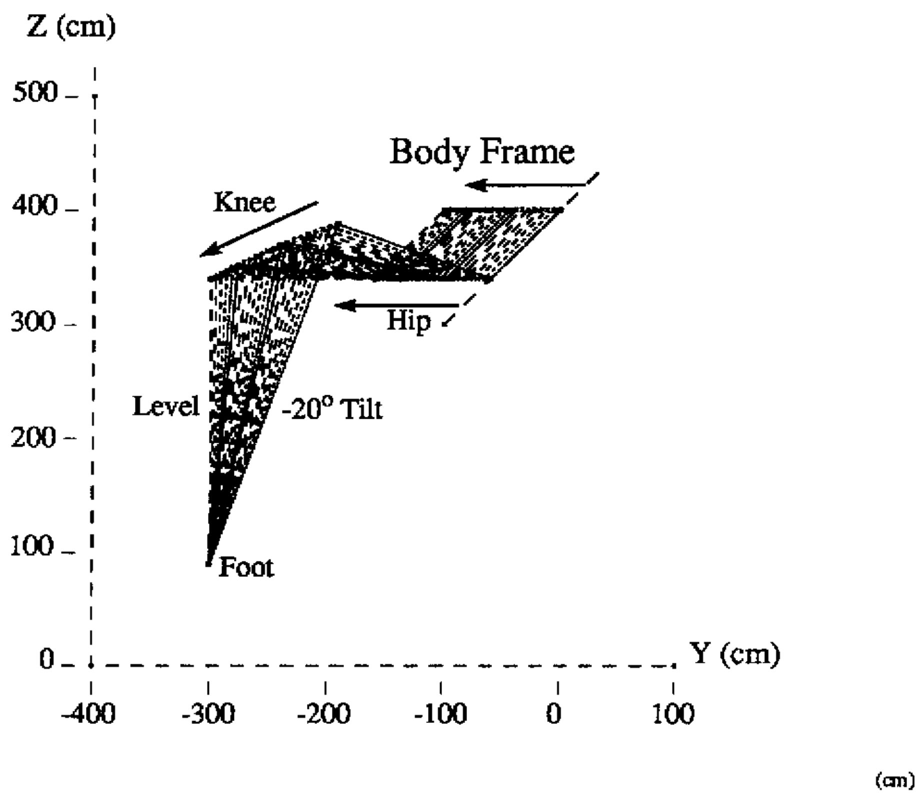

Figure 4. Displacement of the Body Frame, Knee, and Hip when Using the IAZA Method.

4.0 MINIMUM-SLIP LEVELING

The *Minimum-Slip Z-Axes attitude control method*, (MSZA), is developed in this section. To clarify how the MSZA method works, first a discussion of the procedure for calculating the change in link lengths for attitude control of a two-legged planar walker is presented. This is followed by the actual calculations required for such a walker. The method used assumes that one foot does not slip; a three-legged planar case follows to show that the commanded changes in link lengths are the same, irrespective of which leg is assumed to be stationary if the slopes at the feet of all legs are equal. By the principle of superposition, the planar case is expanded to a (real) three-dimensional walker with n legs.

4.1 Procedure

Consider a rigid, two-legged, planar walker depicted in Figure 5 on level and sloped ground. For both terrains the walker is shown in a level and tilted configuration. The heavy lines and dashed lines correspond to the level and tilted configurations, respectively. In this example it is assumed that Foot 1 does not slip when moving from the initial (level) to final configuration. However, Foot 2 still slips, even though the minimum-slip attitude control method is used.

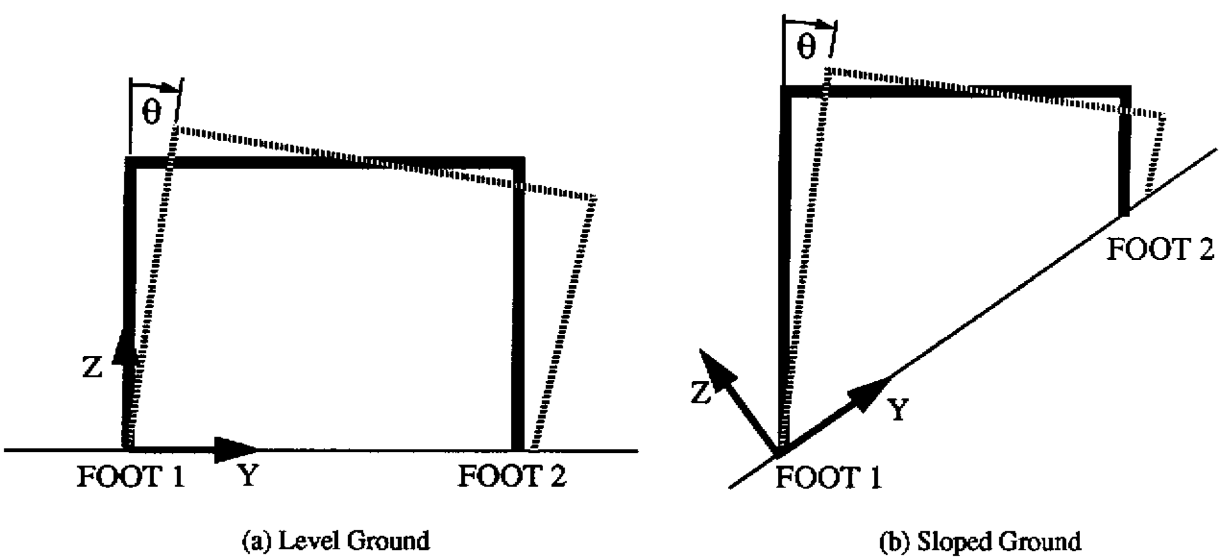

Figure 5. A Two-Legged, Planar Walker on Level Ground and on a Slope.

The relevant kinematic constraint that is used in the minimum-slip analysis for this case is that the feet do not leave the terrain surface (the line connecting the two foot locations). The changes in the vertical link lengths to achieve the desired change in tilt, θ, are determined by enforcing that there is no motion of the second foot in the z-coordinate direction for the cases (and coordinate frames) shown in Figure 5. Having determined the desired change in link lengths, the predicted foot slippage may be obtained in the y-coordinate direction.

The above method is readily expanded to take into account irregular terrain elevations and varying local terrain slopes. For example, consider the three-legged planar walker depicted in Figure 6. Once again, in this example it is assumed that Foot 1 does not slip. The other feet should slip along the lines which represent the local terrain slopes. For the purposes of evaluating the planned leveling motion, a method for predicting which feet are the least likely to slip follows.

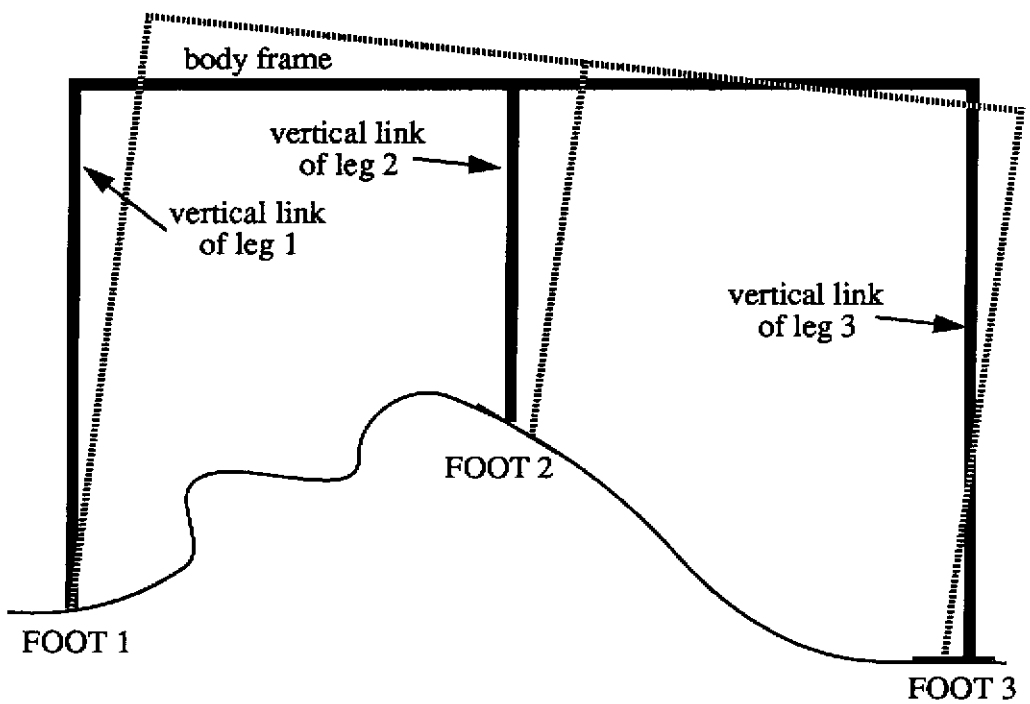

Figure 6. A Three-Legged Planar Walker on General Terrain.

4.1.1 Non-Slip Prediction

In this section, the methods that may be used to predict which foot is least likely to slip are discussed. The ability to predict which foot (or feet) do not slip is required in order to obtain the correct minimum-slip results. If the feet of the walker have dissimilar terrain slopes, there is a significant difference between the calculated changes in link lengths for a desired attitude control maneuver. In this case, if the foot that is presumed to be stationary slips, then the resulting leveling control, in general, will not minimize slip. However, the control action will at least still bring the walker close to the desired body attitude.

There are two distinct ways by which determination of a foot which should not slip is possible. One may accomplish this by evaluating the forces acting on the feet of the walker. An alternative method is based on characterizing the configuration of the walker (i.e., its stance). For a more reliable determination, a method that is comprised of both examining foot forces and walker configuration is proposed.

The ability to sustain (horizontal) tractive forces for different terrain geometries and vertical loads was discussed in [7]. For example, consider a foot on a slope of sand, as depicted in Figure 7. Not surprisingly, the higher the vertical load (with respect to the body's coordinate frame), the greater the horizontal force that can be sustained, for any terrain geometry. Therefore, legs with high vertical loads are less likely to slip.

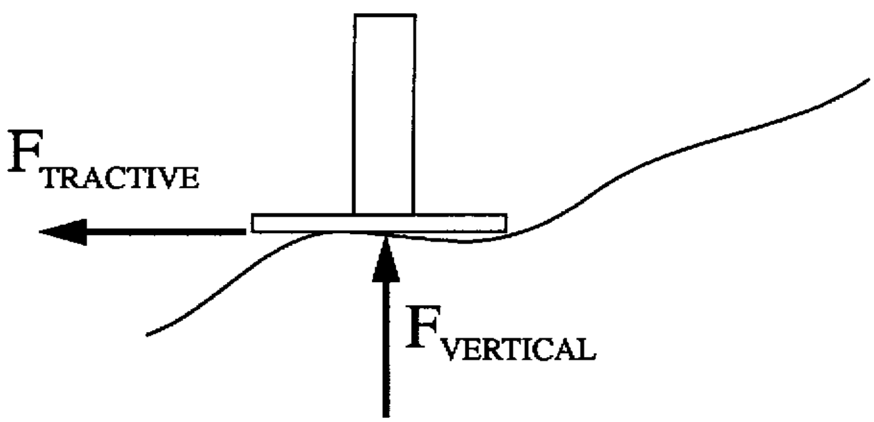

Figure 7. Forces Acting on a Foot.

One could simply select the leg with the highest vertical force (with respect to the walker) as the non-slipping leg. On the other hand, terrain slope effects could also be incorporated in the decision-making process. For example, the maximum horizontal tractive force for a foot of the AMBLER is plotted as a function of the vertical load and terrain slope for loosely-consolidated sand in Figure 8. The combination of vertical load and terrain slope can be used to determine the maximum expected tractive force that may be sustained by a given walker foot on given terrain, if the vertical foot force is known. In this case, the foot which is expected to be able to sustain the highest maximum tractive force is presumed not to slip.

To clarify the meaning of "vertical" load and terrain "slopes" in the context of a rapelling robot (such as Dante), consider the planar, two-legged walker descending down a slope as shown in Figure 9. In this figure, the "vertical load" is the force N_i for foot i, the tractive force is T_i, and the local terrain slope at foot i is θ_i, with respect to the walker reference coordinate frame.

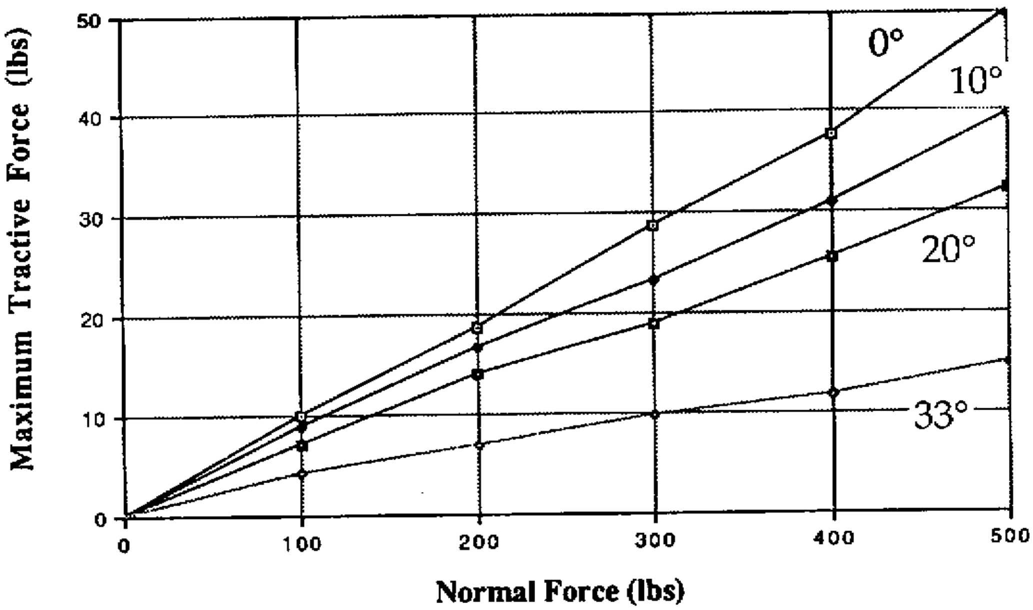

Figure 8. Maximum Available Tractive Force For an AMBLER Foot on Sloped Sand.

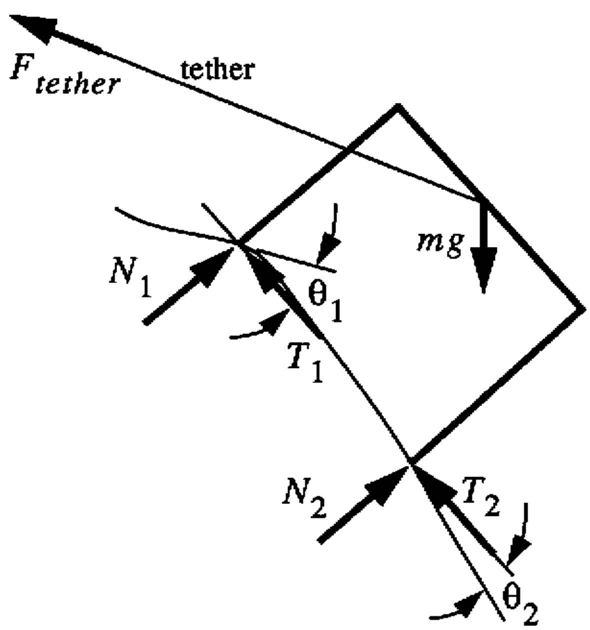

Figure 9. A Rapeller Climbing a Slope.

In experimental work with the AMBLER, it was noted that foot slippage is most likely to occur when the foot/terrain contacts are at dissimilar elevations (with respect to the body coordinate frame) [1, 2]. It was noted that the foot (or feet) which are very high or very low are likely to slip when leveling with only the z-axes. This observation was further corroborated by simulations. Therefore, one of the legs that has an extension length that is closest to the average vertical extension could be chosen as the non-slipping leg. A more accurate method would be to resolve a change in body roll and pitch to a tilt about an axis, the location of this axis being where the body does not rise or fall during the attitude control maneuver. The leg that is closest to this axis is least likely to slip. The above determinations of which foot is least likely to slip are based on mechanism geometry.

Ideally, both the mechanism geometry and the forces exerted on the feet by the terrain contacts should be considered when predicting which foot should not slip. The following method for determining which foot is least likely to slip takes these two factors into account. For a walker with n ground-contacting feet, discard the foot in ground contact which has the longest vertical length and the foot with the shortest vertical length, leaving $n-2$ feet. Of the remaining feet, choose the foot with the highest vertical foot force as the non-slipping foot.

4.2 Planar Case, 2 Legs

In this section, the relevant equations for the minimum-slip z-axes attitude control method for the case of a two-legged planar walker are developed. Consider the two-legged planar walker in Figure 10, where θ_1 and θ_2 correspond to the initial and final tilts of the walker about the x-axis (coming out of the page). When the walker is level, the tilt θ is zero degrees. Therefore, the range in body tilt will be defined as: $-90° < \theta \leq 90°$. The local slope at Foot 2 is θ_{F2}, where a slope of zero degrees corresponds to level ground. The possible range in local slope is: $-90° < \theta_{F2} \leq 90°$. Note that for the two configurations shown in Figure 10, that θ_{F2} is positive, while θ_1 and θ_2 are negative. The body frame has fixed length, W_{12}, which is positive (as in this example) when the $\tilde{y}$-coordinate of the slipping foot is positive (with respect to the $\tilde{Y}$-$\tilde{Z}$ coordinate frame at the non-slipping foot). The original (vertical and positive) leg lengths are l_1 and l_2 for legs one and two, respectively. The goal is to determine the new vertical leg lengths, l_1^n and l_2^n, such that the walker's body attitude (about the x-axis) is changed from θ_1 to θ_2.

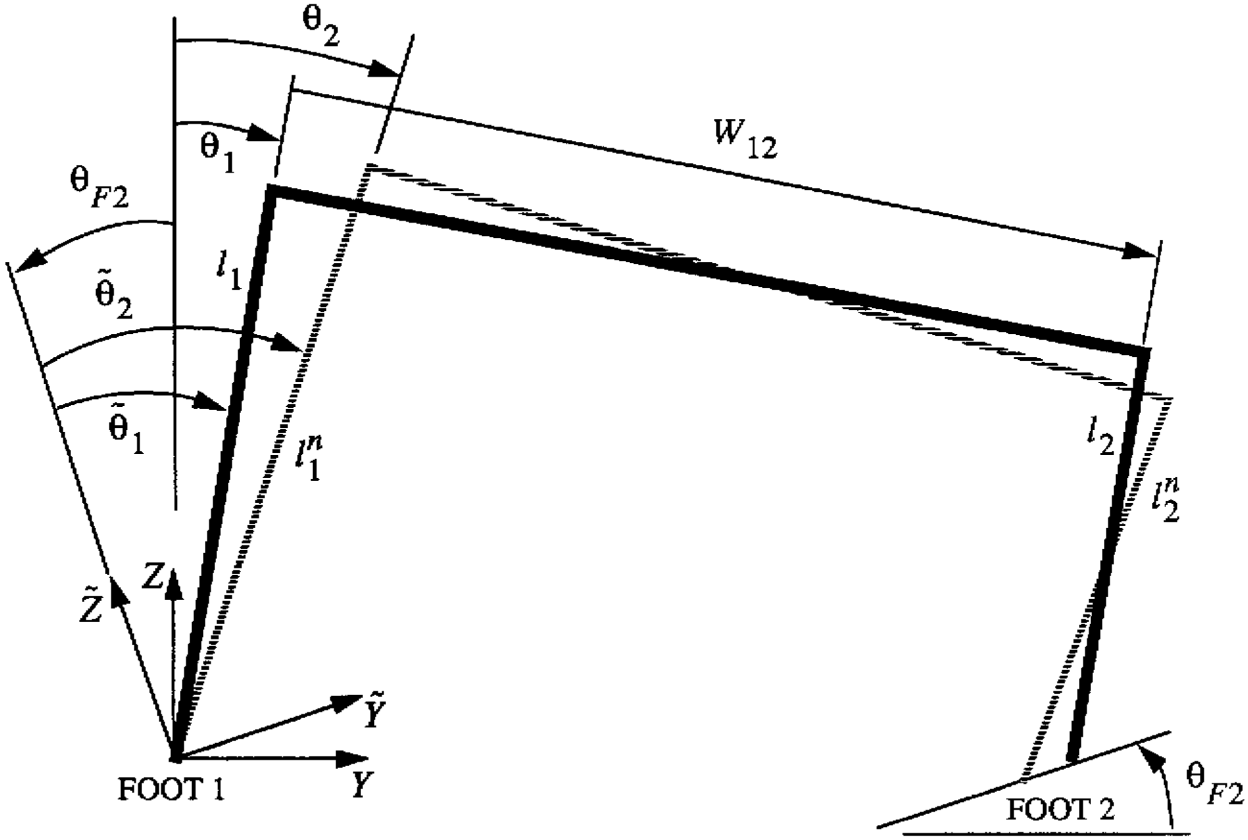

Figure 10. A Planar Two-Legged Walker.

A coordinate frame (with the z-axis oriented vertically) is placed at the non-slipping foot, which is arbitrarily presumed to be Foot 1 in this analysis for the purposes of developing the relevant equations. When calculating the slippage of a foot, this coordinate frame is transformed by the angle of the local slope at the slipping foot. Therefore, the initial and final body tilts are $\tilde{\theta}_1$ and $\tilde{\theta}_2$ with respect to the new coordinate frame, respectively. In the absence of a terrain map, the slope at Foot 2 may be approximated by:

$$\theta_{F2} = \operatorname{atan}\left(\frac{z_2 - z_1}{y_2 - y_1}\right), \tag{9}$$

where the y- and z-coordinates above are the coordinates of the foot locations in a fixed vertical ground reference frame. The transformed initial and final walker tilts are then:

$$\tilde{\theta}_1 = \theta_1 - \theta_{F2} \tag{10}$$

$$\tilde{\theta}_2 = \theta_2 - \theta_{F2}. \tag{11}$$

We will define the change in vertical leg length of leg i, Δl_i as:

$$\Delta l_i = l_i^n - l_i. \tag{12}$$

The "vertical" height of Foot 2 with respect to Foot 1 in the $\tilde{z}$-coordinate direction before and after changing the body height are:

$$\tilde{z}_2 = l_1 \cos\tilde{\theta}_1 + W_{12} \sin\tilde{\theta}_1 - l_2 \cos\tilde{\theta}_1 \tag{13}$$

and

$$\tilde{z}_2^n = l_1^n \cos\tilde{\theta}_2 + W_{12} \sin\tilde{\theta}_2 - l_2^n \cos\tilde{\theta}_2, \tag{14}$$

respectively. Since there is no foot motion in the $\tilde{z}$-coordinate direction, the above two equations may be equated, i.e.:

$$\begin{aligned} & l_1 \cos\tilde{\theta}_1 + W_{12} \sin\tilde{\theta}_1 - l_2 \cos\tilde{\theta}_1 \\ & = (l_1 + \Delta l_1) \cos\tilde{\theta}_2 + W_{12} \sin\tilde{\theta}_2 - (l_2 + \Delta l_2) \cos\tilde{\theta}_2. \end{aligned} \tag{15}$$

The change in leg lengths are thus related:

$$\Delta l_1 - \Delta l_2 = \Lambda, \tag{16}$$

where:

$$\Lambda = \frac{l_1 (\cos\tilde{\theta}_1 - \cos\tilde{\theta}_2) + l_2 (\cos\tilde{\theta}_2 - \cos\tilde{\theta}_1) + W_{12} (\sin\tilde{\theta}_1 - \sin\tilde{\theta}_2)}{\cos\tilde{\theta}_2}. \tag{17}$$

One could arbitrarily set the value of either Δl_1 or Δl_2, and use Equation (16) in order to determine the change in the other leg's vertical length. The value that is "arbitrarily" set will determine the nominal altitude (height) of the machine. The method to determine which combination of Δl_1 and Δl_2 is appropriate is discussed in Section 5.0.

The $\tilde{y}$-coordinate of Foot 2 with respect to Foot 1 before and after changing the body attitude are:

$$\tilde{y}_2 = -l_1 \sin\tilde{\theta}_1 + W_{12} \cos\tilde{\theta}_1 + l_2 \sin\tilde{\theta}_1 \tag{18}$$

and

$$\tilde{y}_2^n = -l_1^n \sin\tilde{\theta}_2 + W_{12}\cos\tilde{\theta}_2 + l_2^n \sin\tilde{\theta}_2, \tag{19}$$

respectively.

The foot slippage may now be found by determining the motion in the $\tilde{y}$-coordinate direction:

$$\begin{aligned} slippage &= \tilde{y}_2^n - \tilde{y}_2 \\ &= -(l_1 + \Delta l_1)\sin\tilde{\theta}_2 + W_{12}\cos\tilde{\theta}_2 + (l_2 + \Delta l_2)\sin\tilde{\theta}_2 \\ &\quad + l_1 \sin\tilde{\theta}_1 - (W_{12}\cos\tilde{\theta}_1 + l_2 \sin\tilde{\theta}_1) \\ &= \xi - \Lambda \sin\tilde{\theta}_2, \end{aligned} \tag{20}$$

where:

$$\xi = l_1(\sin\tilde{\theta}_1 - \sin\tilde{\theta}_2) + W_{12}(\cos\tilde{\theta}_2 - \cos\tilde{\theta}_1) + l_2(\sin\tilde{\theta}_2 - \sin\tilde{\theta}_1), \tag{21}$$

and Λ is determined by Equation (17). Equation (20) may be further simplified to:

$$\tilde{y}_2^n - \tilde{y}_2 = (l_2 - l_1)\sin(\tilde{\theta}_2 - \tilde{\theta}_1) + W_{12}(1 - \cos(\tilde{\theta}_2 - \tilde{\theta}_1)). \tag{22}$$

4.3 Planar Case, 3 Legs

When applying the minimum-slip z-axes attitude control method, it is assumed that one foot does not slip. If the slope at each footfall is equal, then the commanded changes in vertical link lengths to achieve a desired attitude control maneuver is the same no matter which foot is presumed to not slip. In this section, this result will be demonstrated for a three-legged planar walker.

Consider the three-legged planar walker depicted in Figure 11. Note that the slopes at each foot/terrain contact are equal. The choice of which foot is assumed to be stationary (non-slipping) does not matter for the walker in this type of configuration. To prove this, it is first assumed that the foot of leg 1 does not slip, and the relative vertical commanded motions corresponding to Equation (16) are formed:

$$\Delta l_1 - \Delta l_2 = \Lambda_{12} \tag{23}$$

$$\Delta l_1 - \Delta l_3 = \Lambda_{13}. \tag{24}$$

Then it is assumed that the foot of leg 2 does not slip. The change in the extension of leg 3 to the change in the extension of leg 2 is then determined:

$$\Delta l_2 - \Delta l_3 = \Lambda_{23}. \tag{25}$$

It will then be shown that the condition (25) is consistent with the conditions (23) and (24), i.e.:

$$\begin{aligned} \Lambda_{13} - \Lambda_{12} &= \Delta l_1 - \Delta l_3 - \Delta l_1 + \Delta l_2 \\ &= \Delta l_2 - \Delta l_3 \\ &= \Lambda_{23}. \end{aligned} \tag{26}$$

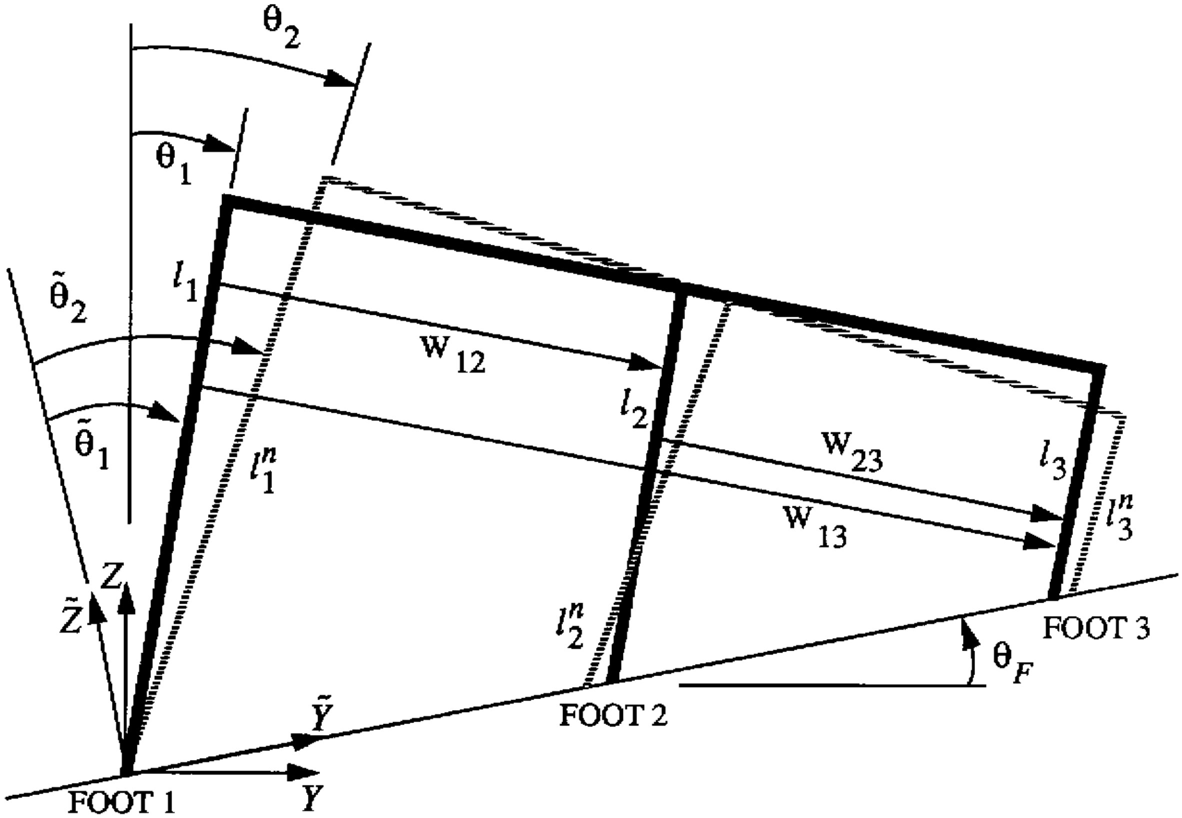

Figure 11. A Three-Legged Planar Walker On a Constant Slope.

First the change in the length of leg 2 relative to the change in length of leg 1 is found from (23) with:

$$\Lambda_{12} = \frac{l_1(\cos\tilde{\theta}_1 - \cos\tilde{\theta}_2) + l_2(\cos\tilde{\theta}_2 - \cos\tilde{\theta}_1) + W_{12}(\sin\tilde{\theta}_1 - \sin\tilde{\theta}_2)}{\cos\tilde{\theta}_2}. \quad (27)$$

Then the change in the length of leg 3 relative to the change in length of leg 1 is found from (24) with:

$$\Lambda_{13} = \frac{l_1(\cos\tilde{\theta}_1 - \cos\tilde{\theta}_2) + l_3(\cos\tilde{\theta}_2 - \cos\tilde{\theta}_1) + W_{13}(\sin\tilde{\theta}_1 - \sin\tilde{\theta}_2)}{\cos\tilde{\theta}_2}. \quad (28)$$

If the foot of leg 2 is presumed not to slip instead of foot 1, then the change in the length of leg 3 relative to the change in length of leg 1 is found from (25) with:

$$\Lambda_{23} = \frac{l_2(\cos\tilde{\theta}_1 - \cos\tilde{\theta}_2) + l_3(\cos\tilde{\theta}_2 - \cos\tilde{\theta}_1) + W_{23}(\sin\tilde{\theta}_1 - \sin\tilde{\theta}_2)}{\cos\tilde{\theta}_2}. \quad (29)$$

Now subtracting (27) from (28) yields:

$$\Lambda_{13} - \Lambda_{12} = \frac{l_3(\cos\tilde{\theta}_2 - \cos\tilde{\theta}_1) + W_{13}(\sin\tilde{\theta}_1 - \sin\tilde{\theta}_2)}{\cos\tilde{\theta}_2}$$

$$\frac{-l_2(\cos\tilde{\theta}_2 - \cos\tilde{\theta}_1) - W_{12}(\sin\tilde{\theta}_1 - \sin\tilde{\theta}_2)}{\cos\tilde{\theta}_2}$$

$$= \frac{l_3(\cos\tilde{\theta}_2 - \cos\tilde{\theta}_1) + (W_{12} + W_{23})(\sin\tilde{\theta}_1 - \sin\tilde{\theta}_2)}{\cos\tilde{\theta}_2}$$

$$\frac{-l_2(\cos\tilde{\theta}_2 - \cos\tilde{\theta}_1) - W_{12}(\sin\tilde{\theta}_1 - \sin\tilde{\theta}_2)}{\cos\tilde{\theta}_2}$$

$$= \frac{l_2(\cos\tilde{\theta}_1 - \cos\tilde{\theta}_2) + l_3(\cos\tilde{\theta}_2 - \cos\tilde{\theta}_1) + W_{23}(\sin\tilde{\theta}_1 - \sin\tilde{\theta}_2)}{\cos\tilde{\theta}_2}$$

$$= \Lambda_{23}, \tag{30}$$

Q.E.D.

4.4 Three-Dimensional Case, n Legs

The leveling equations for tilting about the x-axis were developed in Section 4.2. In this section the same method will be applied to tilting about the y-axis. The changes in leg lengths are then combined for tilting maneuvers about both axes of the three-dimensional walker.

4.4.1 Tilting in the X-Z Plane

Consider the two-legged planar walker in Figure 12, where γ_1 and γ_2 correspond to the initial and final tilts of the walker about the y-axis (coming out of the page). When the walker is level, the tilt γ is zero degrees. Therefore, the range in body tilt will be defined as: $-90° < \gamma \leq 90°$. The local slope at Foot 2 is γ_{F2}, where a slope of zero degrees corresponds to level ground. The possible range in local slope about the y-axis is: $-90° < \gamma_{F2} \leq 90°$. Note that for the two configurations shown in Figure 12, that γ_{F2} is positive, while γ_1 and γ_2 are negative. The body frame has fixed length, W_{12}, which is negative in this case, as the $\tilde{x}$-coordinate of the slipping leg is negative. The original (vertical and positive) leg lengths are l_1 and l_2 for legs one and two, respectively. The goal is to determine the new vertical link lengths, l_1^n and l_2^n, such that the walker's body attitude about the y-axis is changed from γ_1 to γ_2.

A coordinate frame is placed at the non-slipping foot, which is presumed to be Foot 1 in this derivation. When calculating the slippage of a foot, this coordinate frame is transformed by the angle of the local slope at the slipping foot. Therefore, the initial and final body tilts about the y-axis are $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$, respectively. In the absence of a terrain map, the slope at Foot 2 may be approximated by:

$$\gamma_{F2} = \operatorname{atan}-\left(\frac{z_2 - z_1}{x_2 - x_1}\right), \tag{31}$$

where the x- and z-coordinates above are the coordinates of the foot locations in a vertical ground reference frame.

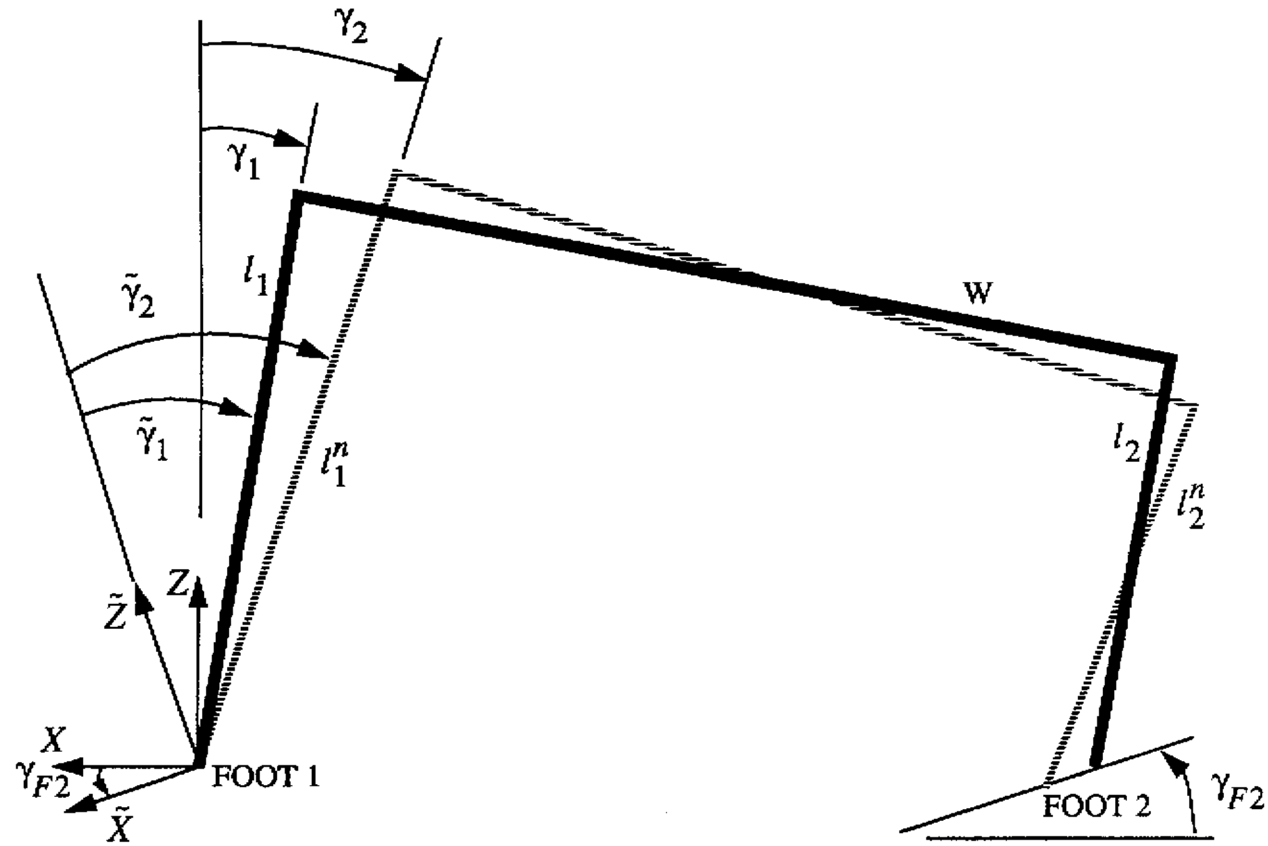

Figure 12. Rotation in the X-Z plane.

The transformed initial and final walker tilts are then:

$$\tilde{\gamma}_1 = \gamma_1 - \gamma_{F2} \tag{32}$$

$$\tilde{\gamma}_2 = \gamma_2 - \gamma_{F2}. \tag{33}$$

The change in vertical leg length of leg i, Δl_i will be defined as:

$$\Delta l_i = l_i^n - l_i. \tag{34}$$

The "vertical" height of Foot 2 with respect to Foot 1 in the $\tilde{z}$-coordinate direction before and after changing the body height are:

$$\tilde{z}_2 = l_1 \cos\tilde{\gamma}_1 - W_{12} \sin\tilde{\gamma}_1 - l_2 \cos\tilde{\gamma}_1 \tag{35}$$

and

$$\tilde{z}_2^n = l_1^n \cos\tilde{\gamma}_2 - W_{12}\sin\tilde{\gamma}_2 - l_2^n \cos\tilde{\gamma}_2, \tag{36}$$

respectively. Since there is no foot motion in the $\tilde{z}$-coordinate direction, then the above two equations are equal, i.e.:

$$\begin{aligned} & l_1 \cos\tilde{\gamma}_1 - W_{12}\sin\tilde{\gamma}_1 - l_2 \cos\tilde{\gamma}_1 \\ &= (l_1 + \Delta l_1)\cos\tilde{\gamma}_2 - W_{12}\sin\tilde{\gamma}_2 - (l_2 + \Delta l_2)\cos\tilde{\gamma}_2. \end{aligned} \tag{37}$$

The change in leg lengths are thus related:

$$\Delta l_1 - \Delta l_2 = \Lambda, \tag{38}$$

where:

$$\Lambda = \frac{l_1(\cos\tilde{\gamma}_1 - \cos\tilde{\gamma}_2) + l_2(\cos\tilde{\gamma}_2 - \cos\tilde{\gamma}_1) - W_{12}(\sin\tilde{\gamma}_1 - \sin\tilde{\gamma}_2)}{\cos\tilde{\gamma}_2}. \tag{39}$$

The $\tilde{x}$-coordinate of Foot 2 with respect to Foot 1 before and after changing the body attitude are:

$$x_2 = l_1 \sin\tilde{\gamma}_1 + W_{12}\cos\tilde{\gamma}_1 - l_2 \sin\tilde{\gamma}_1 \tag{40}$$

and

$$x_2^n = l_1^n \sin\tilde{\gamma}_2 + W_{12}\cos\tilde{\gamma}_2 - l_2^n \sin\tilde{\gamma}_2, \tag{41}$$

respectively.

The foot slippage may now be found by determining the motion in the $\tilde{x}$-coordinate direction:

$$\begin{aligned} slippage &= \tilde{x}_2^n - \tilde{x}_2 \\ &= (l_1 - l_2)\sin(\tilde{\gamma}_2 - \tilde{\gamma}_1) - W_{12}(1 - \cos(\tilde{\gamma}_2 - \tilde{\gamma}_1)). \end{aligned} \tag{42}$$

4.4.2 Procedure to Level a Walker

The procedure for leveling two-legged, planar walkers in the Y-Z and X-Z planes was developed in Sections 4.2 and 4.4.1, respectively. These will now be combined into a procedure to implement minimum-slip z-axes attitude control for (three-dimensional) walkers which have n legs. Consider a statically-stable walker with n legs (and feet) which is commanded to change body attitude by a pitch θ about its x-axis, and a roll γ about its y-axis. Not all feet are necessarily in ground contact; however, the feet in the air are moved in like manner as with the feet in ground contact, in order to maintain their distance from the terrain surface during the leveling maneuver. The procedure for determining the link length changes to achieve the desired body attitude is depicted in Figure 13, and described below.

The first step that needs to be taken is to determine the slope of each terrain contact in both the y- and x-coordinate directions. If the slopes are not known (from terrain maps), the average slope may be inferred from the relative terrain elevations of the different footholds as in Equations (9) and (31). These elevations may be estimated from leg lengths, and body attitude readings. For feet that are in the air, we may arbitrarily choose the "slope," but to ensure no unexpected collision with the terrain, it is perhaps best to presume that they are "on" slopes equal to the local terrain slopes near them for the purposes of calculating the change in their respective link lengths.

The next step is to use the method developed in Section 4.1.1 in order to predict which foot is the least likely to slip. In the absence of foot force sensing, this will be the foot of the leg whose vertical length is the closest to the average vertical length of all legs. If force sensing is available, it will be the foot of the leg that has the greatest force exerted on its foot in the direction normal to the ground surface, excluding the highest and lowest feet.

Having predicted which foot is the least likely to slip, the foot deflection relationship for all feet relative to the non-slipping foot is determined in the same manner as that shown in the planar mechanisms. Let the foot that is presumed to be stationary be denoted as Foot j. For each of the other feet i, $(i = 1, 2, \ldots n, i \neq j)$, then the change in the vertical leg extensions for a change in pitch, θ, about the x-axis are determined by:

$$\Delta l^{\theta}_{j} - \Delta l_i^{\theta} = \Lambda_i^{\theta} \tag{43}$$

as described in Section 4.2, in the context of Foot 1 and Foot 2. Similarly, to compensate for the change in roll, γ, the following changes in vertical link lengths:

$$\Delta l^{\gamma}_{j} - \Delta l_i^{\gamma} = \Lambda_i^{\gamma}, \tag{44}$$

are applied, as described in Section 5.4.1. Equations (43) and (44) are then combined to yield:

$$\Delta l_i = \Delta l_j - \Lambda_i^{\theta} - \Lambda_i^{\gamma}. \qquad (45)$$

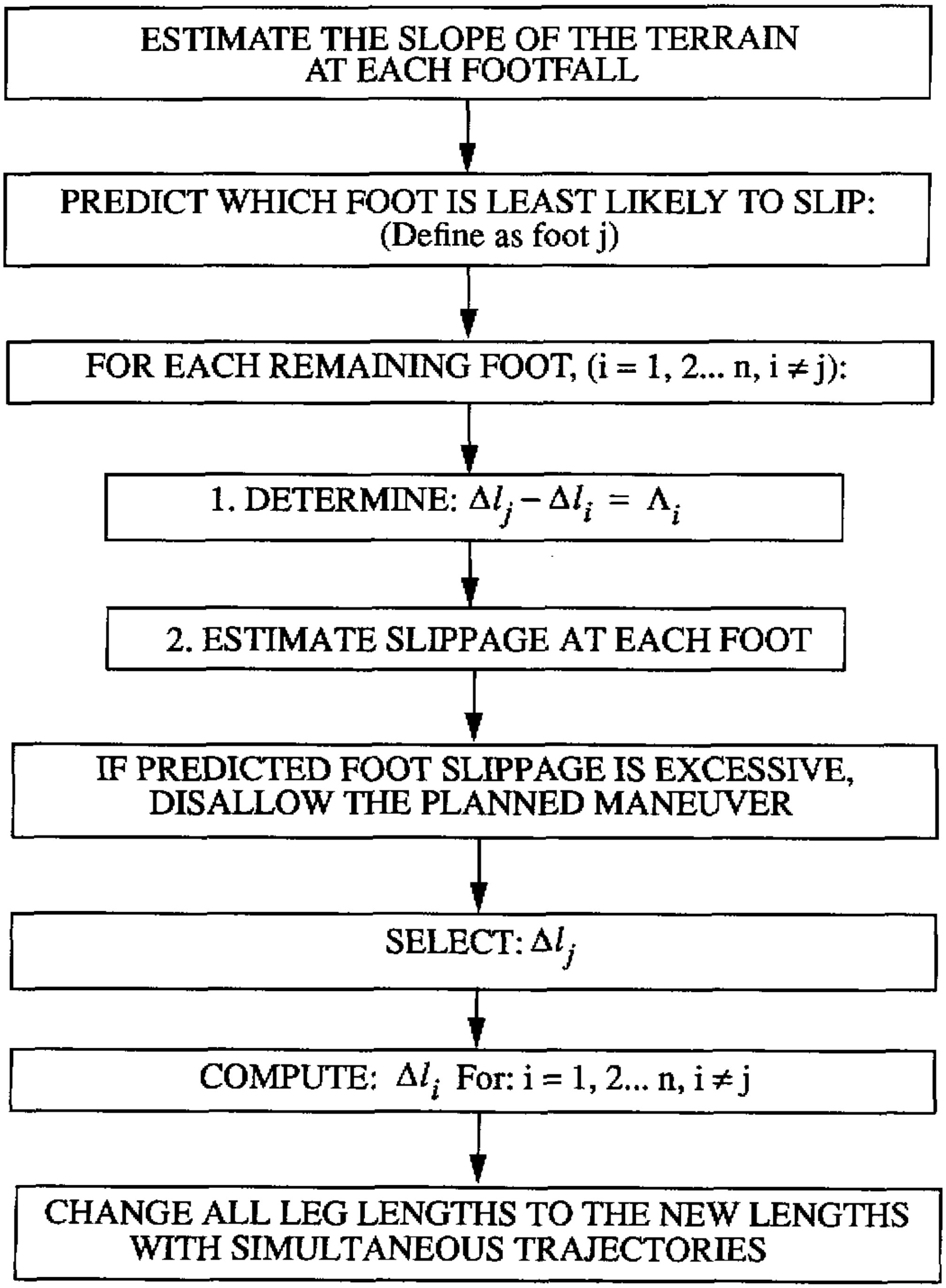

Figure 13. Procedure for Implementing Attitude Control on a Walker.

The next step in the solution procedure is to specify the change in the vertical extension of leg j, namely, Δl_j. This is done by using the methods outlined in Section 5.0. The proper selection of the parameter Δl_j will yield an appropriate body altitude (height with respect to the walker body coordinate reference frame). Having chosen, Δl_j, then the change in the vertical extensions for all other legs may be calculated from Equation (45).

In order to achieve a smooth leveling motion, the commanded leg length changes should be executed simultaneously; i.e., all joint motions should start and stop at the same time, as well as have a constant speed trajectory from the initial to the final leg lengths.

5.0 COMBINING BODY ATTITUDE AND ALTITUDE

In the preceding section, the determination of the relative changes in pairs of leg lengths in order to achieve attitude control was formulated. In this section, a method for specifying the leg length changes, given the commanded relative changes, is described. The factor that decides the specific leg length changes is the specified nominal altitude of the body with respect to the ground (in the body-referenced coordinate frame). The method also determines if the proposed change in body attitude is within possible joint limits.

For the minimum slip method, it is assumed that one leg does not slip, and the others will slip by various amounts, depending on the mechanism geometry. The minimum slip method equations then form a set of relative changes in leg extensions. Consider a walker with n feet (not all of which are necessarily in ground contact). In the minimum-slip analysis assume that leg j does not slip. The minimum-slip method generates $n-1$ equations of the following form:

$$\Delta l_j - \Delta l_i = \Lambda_{ji}, (i = 1, 2, \dots n, i \neq j). \tag{46}$$

Recall that the new absolute length of leg i, $(i = 1, 2, \dots n)$, is determined by:

$$l_i^n = l_i + \Delta l_i \tag{47}$$

where: l_i is the original vertical length of leg i,

l_i^n is the new commanded vertical length of leg i, and

Δl_i is the commanded change in the length of the vertical link of leg i.

The first step in determining the desired link length changes is to first set:

$$\Delta l_j = 0, \tag{48}$$

even though the final commanded change in the vertical length of leg j may be non-zero. Having done this, the other final commanded vertical leg lengths (for no change in the length of leg j) of all other legs may be found by applying (23) and (47) to all legs. This results in a set of feasible new link lengths that will achieve the desired levelling (assuming that no joint motion limits are violated). The next step of the process is to adjust all of these final leg lengths by the same amount in order to achieve the desired nominal height (in the body z-frame) relative to the footfall locations, or to avoid joint motion limit constraints.

For the sake of convenience, we will denote the final vertical leg lengths determined above (with no change in the length of leg j) as l_i^{ξ}, $(i = 1, 2, \ldots n)$. Assuming that all legs have the same vertical stroke, find the legs with the shortest and longest vertical extension, (l_i^{ξ}), whether or not these lengths are physically realizable due to joint motion limits. The values of the shortest and longest extensions will be denoted l_{low} and l_{high}, respectively. If the difference between these two values exceeds the vertical stroke of the legs, then the planned attitude control maneuver cannot be executed for any given walker altitude. This condition should be checked by the gait planner.

Next, the nominal height of the body, l_{nom}, is selected. For example, if l_{min} and l_{max} represent the shortest and longest vertical leg extension available, respectively, then setting l_{nom} at a value equal to:

$$l_{nom} = \frac{l_{min} + l_{max}}{2}, \tag{49}$$

selects a body height that is in the middle of the available range for the given tilt. Choosing the nominal leg length as such generally allows for larger changes of tilt during any subsequent attitude control maneuver. l_{nom} could be set to a larger (corresponding to higher) value if terrain obstacle clearance is an issue. Alternatively, l_{nom} could be set to a lower value in order to gain stability.

The next step is to determine the increment of change in the lengths of the "low" and "high" leg, l_{inc}, such that the two legs are equally close to the length corresponding to the nominal height, l_{nom}. Then all leg commanded leg heights previously determined by assuming no change in the length of leg j are changed by the chosen increment, i.e.:

$$l_i^n = l_i^{\xi} + l_{inc}. \tag{50}$$

<u>EXAMPLE:</u>

Let: $l_{low} = 0.2$,

$l_{high} = 0.5$, and

$$l_{nom} = 0.7.$$

The problem is to determine l_1^n and l_2^n, (corresponding to the feet l_{low} and l_{high}, respectively). The incremental change in the vertical link length is found as follows:

$$l_{inc} = \frac{2l_{nom} - l_{low} - l_{high}}{2}$$

$$= \frac{2(0.7) - 0.2 - 0.5}{2}$$

$$= 0.35 \tag{51}$$

Therefore, the new commanded lengths of leg 1 and leg 2 are:

$$l_1^n = 0.2 + 0.35 = 0.55, \text{ and} \tag{52}$$

$$l_2^n = 0.5 + 0.35 = 0.85, \tag{53}$$

respectively.

6.0 LIMITING FOOT SLIPPAGE

It is desirable to avoid walker motions that result in unfavorable interaction with the terrain. One such interaction is foot slippage due to execution of z-axes attitude control. While the actual foot slippage, in general, will be less than that predicted (due to compliances), one may avoid excessive foot slippage by limiting the allowable predicted foot slippage. In this section, the determination of how much the walker attitude may change given a specified maximum allowable predicted foot slip is developed.

Given a user-specified maximum allowable predicted foot slippage in the $\tilde{x}$- and $\tilde{y}$-coordinate directions, the maximum change in the rotation of the walker about the $\tilde{y}$- and $\tilde{x}$-axes is to be determined, respectively. The procedure to determine the maximum allowable change in tilt about these axes is exactly the same. The following analysis is in the context of tilting about the $\tilde{x}$-axis (corresponding to Section 4.2). Recall from (22) that the slippage between a pair of feet (foot 1 and foot 2) is determined by:

$$\tilde{y}_2^n - \tilde{y}_2 = (l_2 - l_1)\sin(\tilde{\theta}_2 - \tilde{\theta}_1) + W_{12}(1 - \cos(\tilde{\theta}_2 - \tilde{\theta}_1)). \tag{54}$$

For the purposes of clarity, let:

$$a\sin(\tilde{\theta}_2 - \tilde{\theta}_1) + b(1 - \cos(\tilde{\theta}_2 - \tilde{\theta}_1)) = c_y \tag{55}$$

where: $a = l_2 - l_1$,

$b = W_{12}$, and

c_y is the specified maximum allowable slip in the $\tilde{y}$-coordinate direction.

First rearrange (55) to obtain:

$$a\sin(\tilde{\theta}_2 - \tilde{\theta}_1) - b\cos(\tilde{\theta}_2 - \tilde{\theta}_1) = d \tag{56}$$

where $d = c_y - b$. For small changes in tilt (less than 15 degrees), (56) may be simplified to:

$$a(\tilde{\theta}_2 - \tilde{\theta}_1) = c_y, \tag{57}$$

by using small angle approximations. For a 15-degree change in tilt, the use of (57) results in approximately a 3.5% error in the determination of the allowable change in tilt $(\tilde{\theta}_2 - \tilde{\theta}_1)$ for the specified maximum slip. Therefore,

$$|\tilde{\theta}_2 - \tilde{\theta}_1| = \frac{c_y}{a} = \frac{c_y}{l_2 - l_1} \tag{58}$$

is the maximum change in tilt about the $\tilde{x}$-axis for the specified maximum slip, c, in the $\tilde{y}$-axis direction.

Similarly, the maximum change in tilt about the $\tilde{y}$-axis is:

$$|\tilde{\gamma}_2 - \tilde{\gamma}_1| = \frac{c_x}{-a} = \frac{c_x}{l_1 - l_2} \tag{59}$$

where c_x is the maximum allowable slip in the $\tilde{x}$-coordinate direction.

7.0 USE OF Z-AXES-ONLY LEVELING

In this section, the ability to achieve a desired change in body attitude, and the associated foot slippage, is discussed for the simple z-axes, iso-altitude, and minimum-slip attitude control methods. It is shown that the various methods all produce "minimum slip;" however, the minimum-slip method gives the most accurate of leveling responses.

Consider the planar walker depicted in Figure 14. Let the original leg lengths equal 20 and 120 cm, with each foot on flat, level ground. The foot slippage for the Iso-Altitude (IAZA) and Simple-Z-Axes (SZA) attitude control methods is shown in Figure 15 for a 15-degree change in tilt for walker widths from 0 to 250 cm wide. Note that the Simple-Z-Axes method had less foot slippage than the Iso-Altitude method for the same commanded change in walker tilt.

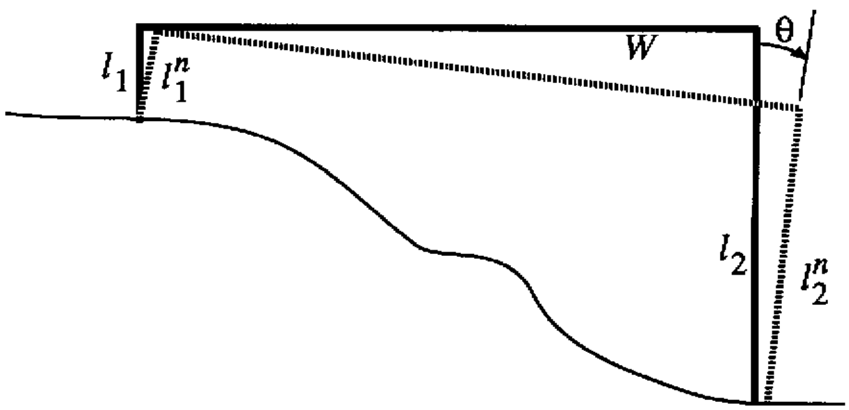

Figure 14. Planar Walker Tilting.

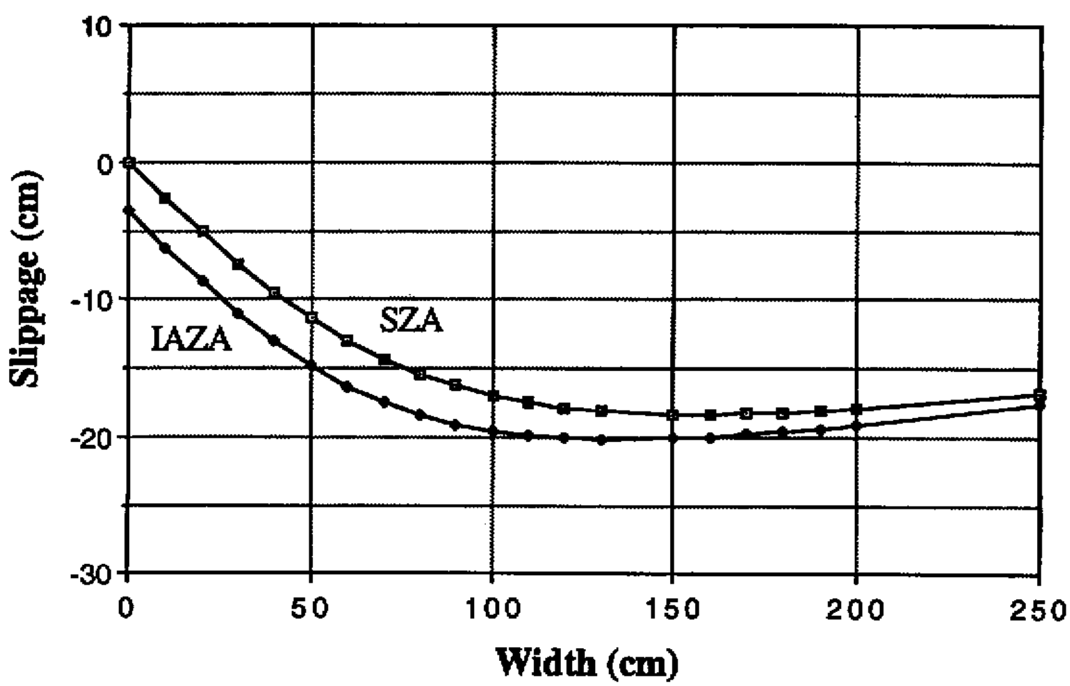

Figure 15. Foot Slippage for a 15-Degree Attitude Maneuver for Varying Walker Widths.

However, the positional performance of the Iso-Altitude attitude control method is much better than that of the Simple-Z-Axes attitude control method. A comparison of the actual resulting tilt and foot slippage is presented in Table 1 for these two methods, as well as the minimum-slip method, for various commanded tilts of a two-legged planar walker. The initial configuration of the walker is level, on flat ground, with initial vertical leg lengths of 50 cm, and the horizontal distance between these legs is also 50 cm. The resulting tilt is calculated from:

$$\theta_{tilt} = \operatorname{atan}\left(\frac{l_2^n - l_1^n}{W_{12}}\right) \tag{60}$$

where l_1^n and l_2^n are the new leg lengths that are obtained from the leveling control equations.

Table 1. A Comparison of the Use of the Different Leveling Methods.

Commanded tilt	SZA method tilt	SZA method slip	IA method tilt	IA method slip	Min-Slip method tilt	Min-Slip method slip
15.00	14.51	1.65	15.00	1.76	15.00	1.76
15.50	14.96	1.75	15.50	1.89	15.50	1.89
15.51	14.97	1.76	15.51	1.89	15.51	1.89
15.52	14.98	1.76	15.52	1.89	15.52	1.89
15.53	14.99	1.76	15.53	1.89	15.53	1.89
15.54	15.00	1.76	15.54	1.90	15.54	1.90
15.55	15.01	1.77	15.55	1.90	15.55	1.90

Interestingly, the commanded changes in link lengths for the iso-altitude method were exactly the same as with the minimum-slip method, even though they were derived from different principles. Additionally, the iso-altitude method uses measurements relative to the body coordinate frame, while the minimum-slip method defines locations with respect to a foot coordinate frame. Examination of the leveling equations shows that the minimum-slip and iso-altitude methods are one and the same for a walker on flat ground. However, the iso-altitude attitude control method does not take terrain slopes into account.

Even though the Simple-Z-Axes method had a smaller slippage for a given commanded tilt than the minimum-slip method, it also failed to tilt by as much as the desired amount. For a commanded tilt of 15 degrees, the SZA method produces a tilt of only 14.51 degrees. When using the SZA attitude control method, a commanded tilt of 15.54 degrees is required in order to obtain a resulting tilt of 15 degrees. At this tilt the foot slippage is 1.76 cm, exactly the same as when tilting a commanded 15 degrees by the minimum slip method. In other words, the mechanism is constrained to minimize slip no matter which leveling method is used. The important point is that the accuracy of the leveling is contingent upon using the appropriate leveling method. Simulations showed that the minimum-slip leveling method yields the desired leveling.

8.0 CONCLUSIONS

A new method for implementing walker body attitude (and altitude) control has been developed which is designed to minimize foot slippage -- ***the minimum-slip attitude control method.*** A very interesting result is that the new method produced equivalent results as the iso-altitude attitude control method when the walker is on flat ground. However, if any of the foot contacts are on sloped ground, these two methods will yield different results. By taking into account the change in walker mechanism geometry, the resulting leg flexion due to using only the z-axes to level is also minimized. Suggested future work will be to implement both the conventional and the newly-proposed forms of attitude control on the Erebus walking robot, Dante, for a variety of terrain types and terrain geometries. The proposed series of experiments should ultimately prove the value of the minimum-slip z-axes attitude control method and ultimately lead to more reliable locomotion when the Erebus robots are deployed to Antarctica.

9.0 REFERENCES

[1] Gonzalez de Santos P., Nagy P.V., and Whittaker W.L., "Attitude Control Methods for Walking Machines," accepted for publication in the Proceedings of the 1992 ASME Mechanisms Conference, Phoenix, AZ, Sept. 13-16, 1992.

[2] Gonzalez de Santos P., Nagy P.V., and W.L. Whittaker W.L., "Leveling of the AMBLER Walking Machine: A Comparison of Methods," CMU Robotics Institute Technical Report CMU-RI-TR-91-13, July 1991.

[3] Klein C.A. and Briggs R.L., "Use of Active Compliance in the Control of Legged Vehicles," IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-10, No. 7, July 1980, pp. 393-400.

[4] Kaneko M., Tanie K., and Than M., "A Control Algorithm for Hexapod Walking Machine Over Soft Ground," IEEE Journal of Robotics and Automation, Vol. RA-4, No. 3, June 1988, pp. 294-302.

[5] Gorinevsky D.M. and Shneider A.Y., "Force Control of Legged Vehicles over Rigid and Soft Surfaces," International Journal of Robotics Research, Vol. 9, No. 2, April 1990, pp. 4-23.

[6] Hirose S., et. al., "TITAN III: A Quadruped Walking Vehicle," Proceedings of Robotics Research, 2nd International Symposium, MIT Press series in artificial intelligence, 1985, pp. 325-331.

[7] Nagy P.V. and Whittaker W.L., "Tractive Force Evaluation for Walking Robots," Proceedings of the Fourth SME World Conference on Robotics Research, Pittsburgh, PA, Sept. 17-19, 1991, pp. 5:45-5:54.