A Normal Form Augmentation Approach to Adaptive Control of Space Robot Systems

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Abstract

In this paper, we model a free-floating space robot system as an *extended robot* which is composed of a pseudo-arm representing the base motion resulting from six hyperthetic passive joints, and a real robot arm. The model allows us to categorize the space robot as an under-actuated system, and reveal fundamental properties of the system. Through input-output linearization of the model, we demonstrate a non-trivial internal dynamics, and propose an adaptive control scheme based on a normal form augmentation approach. This approach overcomes two fundamental difficulties in adaptive control design of space robot systems, i.e., nonlinear parameterization of the dynamic equation, and uncertainty of kinematic mapping from Cartesian space to joint space.

1 Introduction

The use of robotic technology has been proposed for future space exploration, and considerable research efforts have been directed to the conceptual development and experimental study. Recently, dynamics and control problems of a space robot system, where the mass and inertia of robot manipulators are significant in comparison to those of the spacecraft, or space station, or satellite which we refer to as *the base*, have been investigated. Longman [1] presented the kinematic relationship between joint and inertia spaces, and a method to compute the workspace of a space robot. Vafa and Dubowsky [2], and Papadojoulous and Dubowsky [3] introduced the concept of virtual manipulator to represent the dynamics of a space robot and made it possible to reproduce the kinematic behavior of a space robot by the kinematics of a modified fixed-base robot.

Because of the high dynamic coupling between the robot arm and the base, the control of a space robot system becomes difficult. Xu et al. in [4] and [5] investigated the dynamic property of a space robot system and found that there are two fundamental problems in controlling space robot systems. First, the dynamic equation of the system cannot be linearly parameterized. This results in infeasibility of most adaptive control and nonlinear control schemes that are currently used in robot control, because the linear parameterization is a prerequisite of these schemes. Second, when the base is free-floating, the kinematic mapping from inertia space (or Cartesian space), in which the robot tasks are usually specified, to joint space, where the control is executed, becomes non-unique due to non-integrable angular momentum conservation. This may cause unavailability of the reference trajectory in joint space. On the other hand, because of dynamic interaction, the kinematic relationship is no longer only kinematically dependent; it is a function of dynamics. Therefore, the kinematic relationship is subject to the dynamic parameter uncertainty in most space robot applications, and the reference trajectory in joint space is also required to be available for both cases of a space robot with free-floating base and attitude-controlled base.

In order to overcome the above problems, we present in this paper a **normal form augmentation approach** to adaptive control of a space robot system when the base is free-floating. Based on this approach, we first model the entire system as an **extended robot** which is composed of a pseudo-arm representing the base motion with respect to the orbit and a real robot arm to facilitate derivations of dynamics. Then, we discuss major properties of the inertial and Jacobian matrices of the system. An input-output exact linearization procedure is performed for the derived model, and an internal dynamic property is revealed. The concept of *internal dynamics* is of significance to gain more insight into the dynamic interaction of the entire space robot system, and demonstrate the nonlinear parameterization of the system as well. The proposed method is also useful to model any under-actuated dynamic system, where the number of actuated joints is less than the total number of joints, i.e., there are passive joints associated no matter whether they are real or virtual.

With the extended robot model and the normal form augmentation approach, we then develop an adaptive control scheme that can avoid the aforementioned two problems, i.e., the nonlinear parameterization and the unavailability of joint reference trajectory. To show the feasibility of the proposed approach and adaptive control scheme, a simulation study is presented.

2 Extended Robot Model

In this section, we propose an extended robot model to represent a free-floating space robot system for facilitating the modeling complexity and revealing intrinsic properties of internal dynamics. Consider a space robot system that is composed of a base which could be a space station, a space shuttle or satellite, free-floating in the space and an m-joint robot arm mounted on the base. If the reference orbit of the space station or the shuttle is considered as a *virtual fixed base*, then the base has six degrees of freedom: three for translation of its centroid and three for rotation about the reference frame. Therefore, we can view the base as the end-effector of a *pseudo-arm* which includes 6 passive joints. Combining this 6-joint pseudo-arm with the connected m-joint real robot arm, we constitute an extended robot model having totally $m + 6 = n$ joints. This extended robot model has its *fixed* base on the orbit and its end-effector that performs de-

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sired tasks referred to the fixed base. In this way, we may consider the extended robot model of a space robot system as a regular fixed-based robot, except that the number of actuated joints is less than the number of its total joints. We can therefore treat a space robot system as an **under-actuated dynamic system**.

2.1 The Inertial Matrix

Since the extended robot model has $n = 6 + m$ joints, the n by n inertial matrix W can be partitioned into four blocks,

$$W = \begin{pmatrix} W_{bb} & W_{br} \\ W_{rb} & W_{rr} \end{pmatrix}, \tag{1}$$

where W_{bb} is the 6 by 6 symmetric submatrix attributed by the floating base, W_{rr} is the m by m symmetric block representing the inertial matrix of the real robot arm with respect to the fixed base, and $W_{br} = W_{rb}^T$ is the 6 by m submatrix representing the interaction between the floating base and the robot arm.

Based on this partition, we can write the inverse W^{-1} of the inertial matrix by [6]

$$\begin{pmatrix} W_{bb}^{-1} + W_{bb}^{-1} W_{br} \tilde{W}_{rr}^{-1} W_{br}^T W_{bb}^{-1} & -W_{bb}^{-1} W_{br} \tilde{W}_{rr}^{-1} \\ -\tilde{W}_{rr}^{-1} W_{br}^T W_{bb}^{-1} & \tilde{W}_{rr}^{-1} \end{pmatrix}, \tag{2}$$

where

$$\tilde{W}_{rr} = W_{rr} - W_{br}^T W_{bb}^{-1} W_{br}. \tag{3}$$

It is noted that W is invertible if both W_{bb} and $\tilde{W}_{rr}$ are nonsingular. W_{bb}, as an upper-left block of (1), is positive-definite, symmetric, and thus invertible. Whereas for the matrix $\tilde{W}_{rr}$ in (3), referred to as the **effective inertial matrix** of the robot arm, we can also show that it is a positive-definite symmetric matrix. In fact, let

$$T_{br} = \begin{pmatrix} -W_{bb}^{-1} W_{br} \\ I \end{pmatrix}. \tag{4}$$

The product of W and T_{br} becomes

$$W T_{br} = \begin{pmatrix} O \\ \tilde{W}_{rr} \end{pmatrix}. \tag{5}$$

Thus, the matrix T_{br} defined in (4) is virtually a right-transformation operator mapping W to $\tilde{W}_{rr}$. Then, premultiplying (5) by T_{br}^T, we immediately have

$$T_{br}^T W T_{br} = \tilde{W}_{rr}.$$

Since T_{br} contains an m by m identity matrix located at the bottom position of (4), T_{br} is always full-ranked. Therefore, we can readily show that the $\tilde{W}_{rr}$ is positive-definite and symmetric, and is thus always invertible.

2.2 The Jacobian Matrix

The kinematic relationship of a space robot system can be developed based on the extended robot model. Suppose the m-dimensional Cartesian displacement of the robot end-effector with respect to the fixed base is chosen as an output vector which is a differentiable function of the joint position q, and is denoted by $y = h(q) \in \Re^m$. The Jacobian matrix of y is determined by

$$J = \frac{\partial h}{\partial q} = (J_b \;\; J_r), \tag{6}$$

where $J_b = \partial h / \partial q_b$ is of m by 6 and $J_r = \partial h / \partial q_r$ is of m by m. Likewise, we can also define an **effective Jacobian matrix** by

$$\tilde{J}_r = J T_{br} = J_r - J_b W_{bb}^{-1} W_{br}. \tag{7}$$

The definitions of the effective Jacobian matrix $\tilde{J}_r$ and the effective inertial matrix $\tilde{W}_{rr}$ show that the motion of the space robot arm mounted on the base, unlike the fixed-base robot, is determined by not only the robot motion itself, but also the interaction of the base motion, and this interaction can be characterized by the matrix T_{br} in (4).

Since $\dot{y} = J\dot{q}$ and $\ddot{y} = J\ddot{q} + \dot{J}\dot{q}$, we have

$$J\ddot{q} = (J_b \;\; J_r)\ddot{q} = \ddot{y} - \dot{J}\dot{q}. \tag{8}$$

The above equation cannot be uniquely solved for $\ddot{q}$, because J is now an m by $n = m + 6$ matrix. In the space robot system, $\ddot{q}$ is also restricted by its dynamic equation. Since the dynamic equation for a space robot system can be written by

$$W\ddot{q} + C\dot{q} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}, \tag{9}$$

where C is an n by n matrix representing centrifugal and Coriolis factors and has the following property [7, 8] :

$$z^T C z = \frac{1}{2} z^T \dot{W} z, \tag{10}$$

for an arbitrary $z \in \Re^n$, and $\tau \in \Re^m$ are joint torques of the m-joint robot arm. Thus,

$$(W_{bb} \;\; W_{br})\ddot{q} = -(I \;\; O)C\dot{q}. \tag{11}$$

This equation represents the dynamic constraint for $\ddot{q}$. Combining (11) with the kinematic equation (8), we can solve for $\ddot{q}$. Therefore, we are motivated to define the coefficient matrix of $\ddot{q}$ as an index to monitor control quality of the space robot system, i.e.,

$$Q = \begin{pmatrix} W_{bb} & W_{br} \\ J_b & J_r \end{pmatrix}. \tag{12}$$

By calculating the determinant of Q defined in (12), we obtain [6]

$$\det(Q) = \det(W_{bb}) \det(J_r - J_b W_{bb}^{-1} W_{br}) = \det(W_{bb}) \det(\tilde{J}_r).$$

Since W_{bb} is nonsingular, the invertibility of $\tilde{J}_r$ is equivalent to the invertibility of Q. It will be shown by simulation study that at the kinematic singularity, i.e., at points of $\det(J_r) = 0$, the control may not necessarily break, because $\tilde{J}_r$ can still be nonsingular. However, considering the extreme case, in which the base is so heavy that W_{bb}^{-1} tends to be zero matrix and $\det(\tilde{J}_r) \approx \det(J_r)$, the control breaks at the kinematic singularity. Therefore, the kinematic singularity should be avoided for a *safe* control process.

3 Internal Dynamics

A nonlinear autonomous dynamic system can be modeled by the following state equation and output equation:

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x) u_i = f(x) + G(x)u \\ y &= h(x), \end{aligned} \tag{13}$$

where $G(x) = (g_1(x) \; \cdots \; g_m(x))$. u and y are the system input and output, respectively, with the same dimension, i.e., $u, y \in \Re^m$. The extended robot model defines $n = 6 + m$ joint variables, and the state vector x is thus a $2n$-dimensional vector $x = \begin{pmatrix} q \\ \dot{q} \end{pmatrix}$. While the system input $u = (\tau_1 \; \cdots \; \tau_m)^T$ contains all m joint torques/forces of the robot arm if no reaction wheel is considered in the base control. If the output has the same dimension as the input, based on (7), the effective Jacobian matrix $\tilde{J}_r$ becomes an m by m square matrix, and is invertible if and only if the control quality matrix Q is nonsingular.

Based on the space robot dynamic equation given in (9), we can now deduce that

$$\dot{x} = \begin{pmatrix} \dot{q} \\ \ddot{q} \end{pmatrix} = \begin{pmatrix} \dot{q} \\ -W^{-1}C\dot{q} \end{pmatrix} + \begin{pmatrix} O \\ W^{-1} \end{pmatrix} \begin{pmatrix} 0 \\ u \end{pmatrix}. \qquad (14)$$

This exhibits that $f(x)$ in (13) is just the first term on the right-hand side of (14), and $G(x)$ is a $2n$ by m matrix formed by the last m columns of the matrix $\begin{pmatrix} O \\ W^{-1} \end{pmatrix}$, where O is the n by n zero matrix. Based on (2) and (4), we obtain

$$G(x) = (g_1(x) \ \cdots \ g_m(x)) = \begin{pmatrix} O \\ T_{br}\tilde{W}_{rr}^{-1} \end{pmatrix}.$$

In order to study a MIMO nonlinear system, let us find the *relative degree*, r, of the system at a point x^0. The relative degree indicates how high order of output time-derivatives is at least required for uniquely inverting the system. In general, each output channel has its individual value of the relative degree so that $r = \{r_1, \cdots, r_m\}$ if the system has totally m output channels. The formal definition of the relative degree for a MIMO system of the state equation (13) is as follows [9]:

Definition 1 *$r = \{r_1, \cdots, r_m\}$ is a relative degree (vector) of the system if*

1. *for all $1 \le j \le m$, $1 \le i \le m$ and all $k < r_i - 1$, and for all x in a neighborhood of x^0, Lie derivatives*

$$L_{g_j} L_f^k h_i(x) = 0,$$

2. *and the m by m matrix*

$$D(x) = \begin{pmatrix} L_{g_1} L_f^{r_1 - 1} h_1(x) & \cdots & L_{g_m} L_f^{r_1 - 1} h_1(x) \\ \cdots & \cdots & \cdots \\ L_{g_1} L_f^{r_m - 1} h_m(x) & \cdots & L_{g_m} L_f^{r_m - 1} h_m(x) \end{pmatrix}$$

is nonsingular at $x = x^0$.

In the above definition, *Lie derivative* for a scalar function $h_i(x)$ along a vector field η is defined via a dual product between the gradient of h_i and the vector η, i.e., $L_\eta h_i(x) = \frac{\partial h_i}{\partial x}\eta$. The matrix $D(x)$ is often called the *decoupling matrix* [9, 10].

For a space robot system, if Cartesian displacement of the robot end-effector is chosen as an output vector, we can prove that $r_1 = r_2 = \cdots = r_m = 2$. Let an *extend Lie derivative* for a vector field $h(q) = (h_1(q) \ \cdots \ h_m(q))^T$ be

$$L_\eta h(q) = \begin{pmatrix} L_\eta h_1(q) \\ \vdots \\ L_\eta h_m(q) \end{pmatrix}.$$

It can be seen that each $L_{g_j} h(q) = 0$. Furthermore, since $L_f h(q) = (J \ \ O) f = J\dot{q}$,

$$(L_{g_1} L_f h(q) \ \ \cdots \ \ L_{g_m} L_f h(q)) = \begin{pmatrix} \frac{\partial J\dot{q}}{\partial q} & \frac{\partial J\dot{q}}{\partial \dot{q}} \end{pmatrix} G(x)$$
$$= J T_{br} \tilde{W}_{rr}^{-1} = \tilde{J}_r \tilde{W}_{rr}^{-1}.$$

The resultant m by m matrix is just the decoupling matrix $D(x)$ for the robot system. Clearly, if $\det(Q) \ne 0$, then $D(x) = \tilde{J}_r\tilde{W}_{rr}^{-1}$ is nonsingular so that the relative degree $r_1 = \cdots = r_m = 2$. This also explains why it is always required to know up to the desired output acceleration in trajectory-tracking control design.

In a space robot system, the extended robot model possesses n joint variables, and the state vector is $2n$-dimensional, while the input $u = \tau$ and output $y = h(q)$ are both $m = (n-6)$-dimensional vectors. Because $r_i = 2$ in each output channel, based on nonlinear control system theory [9, 10], there can be a $2m$-dimensional subsystem which is *input-output exactly linearizable*, while the remaining $2n - 2m = 12$-dimensional subsystem is unlinearizable by the input-output procedure. Such an unlinearizable subsystem is referred to as the *internal dynamics.* Unless the base is also driven by attitude thrust jets or gyroscopic devices, the internal dynamics for the space robot system has 12 dimensions. For the $2m$-dimensional linearizable portion, we may define a new state vector $\zeta = \begin{pmatrix} y \\ \dot{y} \end{pmatrix}$, and the new dynamic equation is exactly linearized in *Brunovski canonical form.* This linearized equation is equivalent to $\ddot{y} = v$ with a new input $v \in \Re^m$. Moreover, if knowledge on all system parameters is available, the real robot input u can be resolved in terms of the new input v,

$$u = D^{-1}(x)(v - b(x)) = \alpha(x) + \beta(x)v, \qquad (15)$$

where $b(x) = L_f^2 h(x)$, and $\alpha(x) = -D^{-1}(x)b(x)$ and $\beta(x) = D^{-1}(x)$. Equation (15) is known as *static state-feedback control* [9].

According to (14), we have

$$J\ddot{q} + JW^{-1}C\dot{q} = JW^{-1}\begin{pmatrix} 0 \\ u \end{pmatrix} = \tilde{J}_r\tilde{W}_{rr}^{-1}u = D(x)u. \qquad (16)$$

Using $v = \ddot{y} = J\ddot{q} + \dot{J}\dot{q}$ and substituting $J\ddot{q} = v - \dot{J}\dot{q}$ into (16) result in

$$\begin{aligned} \alpha(x) &= \tilde{W}_{rr}\tilde{J}_r^{-1}(JW^{-1}C\dot{q} - \dot{J}\dot{q}) \\ \beta(x) &= \tilde{W}_{rr}\tilde{J}_r^{-1}. \end{aligned} \qquad (17)$$

Their counterparts, in a full-actuated fixed-base robot system that is exactly linearizable, can be determined by

$$\begin{aligned} \alpha_o(x) &= C\dot{q} - WJ^{-1}\dot{J}\dot{q} \\ \beta_o(x) &= WJ^{-1}. \end{aligned} \qquad (18)$$

Comparing (17) to (18), we find that for a space robot system, due to the existence of internal dynamics, the property of linear parameterization in $\alpha(x)$ and $\beta(x)$ is no longer valid. The fact of the above nonlinearized parameterization has been revealed by authors in [5] and here we demonstrate this fact in a different angle. To overcome this problem, we will develop a **normal form augmentation approach** in the next section.

4 Normal Form Augmentation Approach

Since the robot task is usually specified in terms of Cartesian displacement of an end-effector, we choose the Cartesian displacement as a system output $y = h(q) \in \Re^m$. Furthermore, since $2n - 2m$ unobservable variables constitute the states of the internal dynamics, the six joint positions in q_b of the base and their time-derivatives may be the best choice of states to represent the internal dynamics. Therefore, we now define an augmented output vector $y_a = \begin{pmatrix} y \\ q_b \end{pmatrix} \in \Re^n$, and its time-derivative

$$\dot{y}_a = \begin{pmatrix} \dot{y} \\ \dot{q}_b \end{pmatrix} = \begin{pmatrix} J_b & J_r \\ I & O \end{pmatrix} \begin{pmatrix} \dot{q}_b \\ \dot{q}_r \end{pmatrix} = J_{sq}\dot{q}, \qquad (19)$$

where I is the 6 by 6 identity matrix and O is the 6 by m zero matrix. The n by n square Jacobian matrix J_{sq} defined in (19) can be inverted to

$$J_{sq}^{-1} = \begin{pmatrix} O & I \\ J_r^{-1} & -J_r^{-1}J_b \end{pmatrix} \qquad (20)$$

if J_r in $J = (J_b \ \ J_r)$ is nonsingular. Using $\ddot{y}_a = J_{sq}\ddot{q} + \dot{J}_{sq}\dot{q}$ and substituting $\ddot{q} = J_{sq}^{-1}(\ddot{y}_a - \dot{J}_{sq}\dot{q})$ into (9), we obtain

$$WJ_{sq}^{-1}\ddot{y}_a - WJ_{sq}^{-1}\dot{J}_{sq}\dot{q} + C\dot{q} = \begin{pmatrix} 0 \\ u \end{pmatrix}. \tag{21}$$

Premultiplying (21) by $J_{sq}W^{-1}$ yields

$$\ddot{y}_a - \dot{J}_{sq}\dot{q} + J_{sq}W^{-1}C\dot{q} = J_{sq}W^{-1}\begin{pmatrix} 0 \\ u \end{pmatrix}. \tag{22}$$

The above equation can be decomposed into two parts

$$\ddot{y} - \dot{J}\dot{q} + JW^{-1}C\dot{q} = JW^{-1}\begin{pmatrix} 0 \\ u \end{pmatrix} = \tilde{J}_r\tilde{W}_{rr}^{-1}u, \tag{23}$$

and

$$\ddot{q}_b + (I \ \ O)W^{-1}C\dot{q} = (I \ \ O)W^{-1}\begin{pmatrix} 0 \\ u \end{pmatrix}. \tag{24}$$

Clearly, (23) represents the linearizable subsystem of the system, while (24) describes the internal dynamics. If the static state-feedback control law (15) with (17) is applied to the subsystem (23), it can be immediately obtained that $\ddot{y} = v$, provided that link parameters are known. Therefore, if we define $e(t) = y_d(t) - y(t)$, and

$$v = \ddot{y}_d + k_v\dot{e} + k_p e, \tag{25}$$

the dynamics of the linearizable subsystem of the space robot is equivalent to

$$\ddot{e} + k_v\dot{e} + k_p e = 0, \tag{26}$$

where k_v and k_p are constant control gains. Obviously, k_v and k_p should be chosen such that the linear error equation (26) is Hurwitz, i.e., all roots of the corresponding characteristic equation have negative real parts.

Since the complete set of equations including the linearized portion and the internal dynamics portion is called the *normal form* [9, 10], we refer to the definitions of y_a and J_{sq} and the derivation of equation (21) as a **normal form augmentation approach**.

Based on the concept of the augmented square Jacobian matrix J_{sq}, we further define a **Cartesian inertial matrix** as follows,

$$M = J_{sq}^{-T}WJ_{sq}^{-1}. \tag{27}$$

Thus, if J_{sq} is nonsingular, the dynamic equation (21) of the space robot system can be rewritten as

$$M\ddot{y}_a - M\dot{J}_{sq}\dot{q} + J_{sq}^{-T}C\dot{q} = J_{sq}^{-T}\begin{pmatrix} 0 \\ u \end{pmatrix}. \tag{28}$$

Substituting the inverse kinematics $\dot{q} = J_{sq}^{-1}\dot{y}_a$ into (28), we obtain a dynamic equation in terms of the augmented velocity $\dot{y}_a$ and acceleration $\ddot{y}_a$,

$$M\ddot{y}_a + G\dot{y}_a = J_{sq}^{-T}\begin{pmatrix} 0 \\ u \end{pmatrix}, \tag{29}$$

where

$$G = J_{sq}^{-T}CJ_{sq}^{-1} - M\dot{J}_{sq}J_{sq}^{-1}. \tag{30}$$

Based on the dynamic equation (29) represented in Cartesian space, it is noted that if one defines a new augmented input

$$v_a = \ddot{y}_a = \begin{pmatrix} \ddot{y} \\ \ddot{q}_b \end{pmatrix}, \tag{31}$$

then the static state-feedback control can be

$$\begin{pmatrix} 0 \\ u \end{pmatrix} = J_{sq}^T(Mv_a + G\dot{y}_a). \tag{32}$$

Obviously, to realize the above **augmented state-feedback control model** associated with the linearized augmented system given by (31) for the space robot system, in addition to the knowledge of y_d and the invertibility of J_{sq}, the following two conditions must be satisfied:

1. Either desired q_b, $\dot{q}_b$ and $\ddot{q}_b$ can be known, or their actual values are measurable;
2. All the top six components of the resultant vector on the right-hand side of (32) are equal to zero.

In summary, when we *extend* the input vector from $u \in \Re^m$ to $\begin{pmatrix} 0 \\ u \end{pmatrix} \in \Re^n$, and the output vector from $y \in \Re^m$ to $y_a = \begin{pmatrix} y \\ q_b \end{pmatrix} \in \Re^n$, the entire space robot system becomes a full-actuated robot system as if the internal dynamics disappears, provided that the above two conditions are satisfied.

5 Adaptive Control

As discussed in the previous sections, the state-feedback coefficients $\alpha(x)$ and $\beta(x)$ can be determined by equation (17) for a space robot system. Due to the nature of the under-actuated system, both $\alpha(x)$ and $\beta(x)$ are nonlinear functions of physical parameters of robot links. This results in difficulty for adaptive control realization against parameter uncertainty. In this section, we will demonstrate that the normal form augmentation approach expressed in (19) and (21) and the augmented state-feedback control (32) can overcome the nonlinear parameterization problem. Let us start with the assumption that q_b, $\dot{q}_b$ and $\ddot{q}_b$ are measurable. Under this assumption, the augmented output error function between the desired $(y_a)_d = \begin{pmatrix} y_d \\ q_b \end{pmatrix}$ and the actual $y_a = \begin{pmatrix} y \\ q_b \end{pmatrix}$ can be written as $e_a = (y_a)_d - y_a = \begin{pmatrix} e \\ 0 \end{pmatrix}$. Let an *extended augmented error* be defined by

$$s = \dot{e}_a + k_v e_a = \begin{pmatrix} \dot{e} + k_v e \\ 0 \end{pmatrix} \in \Re^n, \tag{33}$$

where $e = y_d - y \in \Re^m$ is the output error function, and $k_v > 0$ is the constant gain. Then, we define a reference output velocity η and a reference output acceleration $\dot{\eta}$ as follows,

$$\eta = \begin{pmatrix} \dot{y}_d + k_v e \\ \dot{q}_b \end{pmatrix} \quad \text{and} \quad \dot{\eta} = \begin{pmatrix} \ddot{y}_d + k_v\dot{e} \\ \ddot{q}_b \end{pmatrix}. \tag{34}$$

Comparing (34) with (33), we have

$$s = \eta - \dot{y}_a, \quad \text{and} \quad \dot{s} = \begin{pmatrix} \ddot{e} + k_v\dot{e} \\ 0 \end{pmatrix} = \dot{\eta} - \ddot{y}_a. \tag{35}$$

Let us now define $E = \frac{1}{2}s^T M s$ to represent an extended error energy, and then,

$$\dot{E} = s^T M\dot{s} + \frac{1}{2}s^T\dot{M}s = s^T M\dot{\eta} - s^T M\ddot{y}_a + \frac{1}{2}s^T\dot{M}s. \tag{36}$$

The second term on the right-hand side of (36) can be determined by (29),

$$-s^T M\ddot{y}_a = s^T G\dot{y}_a - s^T J_{sq}^{-T}\begin{pmatrix} 0 \\ u \end{pmatrix}. \tag{37}$$

By recalling (10), we can derive the third term

$$\begin{aligned} \frac{1}{2}s^T\dot{M}s &= -s^T M\dot{J}_{sq}J_{sq}^{-1}s + \frac{1}{2}s^T J_{sq}^{-T}\dot{W}J_{sq}^{-1}s \\ &= -s^T M\dot{J}_{sq}J_{sq}^{-1}s + s^T J_{sq}^{-T}CJ_{sq}^{-1}s \\ &= s^T Gs = s^T G\eta - s^T G\dot{y}_a. \end{aligned} \tag{38}$$

Finally, (36) can be rewritten as

$$\dot{E} = s^T M\dot{\eta} + s^T G\eta - s^T J_{sq}^{-T}\begin{pmatrix} 0 \\ u \end{pmatrix}. \tag{39}$$

We now define a following control law

$$\begin{pmatrix} 0 \\ u \end{pmatrix} = J_{sq}^T[M_m\dot{\eta} + G_m\eta + \begin{pmatrix} H(\dot{e} + k_v e) \\ \delta \end{pmatrix}], \tag{40}$$

where W_m and C_m, respectively, represent the inertial matrix W and the matrix C in the *model plant*, and accordingly, $M_m = J_{sq}^{-T} W_m J_{sq}^{-1}$ and $G_m = J_{sq}^{-T} C_m J_{sq}^{-1} - M_m \dot{J}_{sq} J_{sq}^{-1}$. In (40), H is an m by m positive-definite, symmetric constant weighting matrix. The vector $\delta \in \Re^6$ in the control law (40) plays a key important role in realizing the second condition for the augmented state-feedback control proposed in the last section. In fact, since

$$J_{sq}^T = \begin{pmatrix} J_b^T & I \\ J_r^T & O \end{pmatrix},$$

the control law (40) can be splitted into two portions,

$$0 = (W_{bb} \;\; W_{br})_m J_{sq}^{-1} \dot{\eta} + (J_b^T \;\; I) G_m \eta + J_b^T H(\dot{e} + k_v e) + \delta \quad (41)$$

$$u = (W_{br}^T \;\; W_{rr})_m J_{sq}^{-1} \dot{\eta} + (J_r^T \;\; O) G_m \eta + J_r^T H(\dot{e} + k_v e). \quad (42)$$

It is clear that since δ *only* appears in (41), δ can simply be evaluated to ensure that (41) vanishs.

Let ξ be the parameter column vector that lists all real physical objective parameters to be identified. Let ξ_m be the corresponding parameter vector for the model plant of the space robot system. Now, substituting the control law (40) into (39), we further obtain

$$\dot{E} = s^T Y \phi - s^T \begin{pmatrix} H(\dot{e} + k_v e) \\ \delta \end{pmatrix}, \quad (43)$$

where $Y\phi = (M - M_m)\dot{\eta} + (G - G_m)\eta$, and Y is a matrix function of q, $\dot{q}$, $\ddot{q}_b$, and y_d, $\dot{y}_d$ and $\ddot{y}_d$, and is independent of the objective physical parameters, while $\phi = \xi - \xi_m$ is the parameter deviation vector between the real plant and the model plant.

We now define an adaptation law for the system,

$$\dot{\phi} = -\Gamma Y^T s, \quad (44)$$

where Γ is a constant *adaptation gain matrix* and is also positive-definite and symmetric. Then, a following Lyapunov function can be adopted to justify the stability of the space robot system with the control law (40) and the adaptation law (44),

$$V_L = E + \frac{1}{2}\phi^T \Gamma^{-1} \phi = \frac{1}{2} s^T M s + \frac{1}{2}\phi^T \Gamma^{-1} \phi. \quad (45)$$

Clearly, $V_L > 0$, and $V_L = 0$ only at the equilibrium point of this adaptive system, i.e., $\begin{pmatrix} e \\ \dot{e} \end{pmatrix} = 0$ and $\phi = 0$. Taking time-derivative for V_L, we have

$$\begin{aligned} \dot{V}_L &= \dot{E} + \dot{\phi}^T \Gamma^{-1} \phi \\ &= s^T Y \phi - s^T \begin{pmatrix} H(\dot{e} + k_v e) \\ \delta \end{pmatrix} - s^T Y \phi \\ &= -(\dot{e} + k_v e)^T H (\dot{e} + k_v e) \end{aligned} \quad (46)$$

which is negative-definite and is zero only at the equilibrium point.

Therefore, the control law (42) and the adaptation law (44) *asymptotically stabilize* the entire space robot system to track a desired trajectory described in terms of y_d, $\dot{y}_d$ and $\ddot{y}_d$. Since J_{sq}^{-1} is heavily involved in the control law and adaptation law, the stability also requires that J_r be nonsingular. Whereas the feasibility of the above adaptive control law depends on whether the desired joint trajectory of the pseudo-arm representing the floating base is available, or q_b, $\dot{q}_b$ and $\ddot{q}_b$ are measurable. In general, it is not easy to determine the desired q_b and its time-derivatives *kinematically* based on a given desired Cartesian trajectory, because all joint variables are also constrained by the dynamic equation (11). An alternative is to measure the base position, velocity and acceleration with respect to the orbit. This is the *cost* we have to pay in achieving the linear parameterization to control the system adaptively against parameter uncertainty.

6 Simulation Study

We now discuss a simulation study to verify the augmented state-feedback control model and adaptive control scheme developed in the previous sections. The space robot system to be simulated is a robot arm with two revolute joints mounted on a free-motion base moving on 2D plane. Since a rigid body on 2D plane possesses 3 d.o.f.: two for its centroid position and one for the orientation about the axis normal to the plane, the base can be viewed as the *end-effector* of a three-joint pseudo-arm. We consider that the first two joints of the pseudo robot are prismatic along two sliding axes that are perpendicular to each other, while the third joint rotates about the axis normal to the 2D plane. In the pseudo-arm model, the total mass and inertia moments of the floating base are only given to the third link, and the first two have zero masses. Combining with the two-joint planar robot arm, the extended robot model has totally five joints, i.e., $n = 5$ and $m = 2$.

The system input $u \in \Re^2$ contains two torques of joint 4 and joint 5. Whereas the output $y = h(q) \in \Re^2$ represents translational motion of the end-effector with respect to the orbit. A schematic diagram of the space robot system for the case study is shown in Figure 1. The Denavit-Hartenberg (D-H) table including joint variables and link length parameters of the extended robot model is also given in Figure 1.

We can derive the dynamic equation for the system via (9). As we have seen, the adaptive control scheme developed in the preceding section assumes that only the dynamic parameters appearing in the inertial matrix W are to be identified, and those geometric parameters, such as the robot link lengths a_4 and a_5 and the base centroid x-coordinate a_3 are not included for adaptation. Moreover, we assume that the centroid of each robot link has only one non-zero coordinate which is along x-axis, denoted by $-\bar{x}_i$ for $i = 4, 5$, and define the remaining length of each link as $l_i = a_i - \bar{x}_i$. Under such conditions, each inertia tensor Φ_i of the base and two links is reduced to be diagonal and only the inertia moment about z-axis will be used, i.e., $m_i k_{zi}^2$, where k_{zi} is the gyration radius about z-axis and m_i is the mass of link i.

Therefore, totally eight parameters involved in W are to be identified. The parameter vector ξ can thus be defined as

$$\xi = (m_3 \;\; m_4 \;\; m_5 \;\; m_3 k_{z3}^2 \;\; m_4 k_{z4}^2 \;\; m_5 k_{z5}^2 \;\; m_4 l_4 \;\; m_5 l_5)^T. \quad (47)$$

With the definition of ξ, the inertial matrix W and C in (9) can be decomposed into

$$W = \sum_{i=1}^{8} \xi_i W^i, \quad \text{and} \quad C = \sum_{i=1}^{8} \xi_i C^i, \quad (48)$$

where each W^i and C^i are independent of ξ and are the coefficient matrices of each ξ_i.

The robot arm tip position with respect to the fixed base is an output,

$$y = h(q) = \begin{pmatrix} d_2 - a_3 s_3 - a_4 s_{34} - a_5 s_{345} \\ d_1 + a_3 c_3 + a_4 c_{34} + a_5 c_{345} \end{pmatrix}, \quad (49)$$

where s_i and c_i are $\sin\theta_i$ and $\cos\theta_i$, s_{ij} and c_{ij} are $\sin(\theta_i + \theta_j)$ and $\cos(\theta_i + \theta_j)$, and s_{ijk} and c_{ijk} are $\sin(\theta_i + \theta_j + \theta_k)$ and $\cos(\theta_i + \theta_j + \theta_k)$, respectively, for $i, j, k = 3, 4, 5$. Taking partial derivative of $y = h(q)$ with respect to q, we can obtain the Jacobian matrix $J = (J_b \;\; J_r)$. Once J is computed, we can form J_{sq} and J_{sq}^{-1} by (19) and (20), and further determine the 5 by 8 matrix Y in the adaptation law (44) through (27), (30) and (43).

The desired trajectory of the space robot tip point is defined to be a circle with radius $R = 1.2$ and a constant speed $\omega = 1.5$ rad./sec. in clockwise direction. We set a large initial tracking error for both y and $\dot{y}$ to simulate how the space robot control system can catch up to the desired trajectory after interruption by some disturbance. The real plant parameters in vector (47) are fixed to be

$$\xi = (10 \quad 2 \quad 1 \quad 37.5 \quad 6 \quad 3 \quad 3 \quad 1.5)^T.$$

All the model plant parameters in ξ_m used for adaptive control simulation are simply defined to be 1. Moreover, the adaptation gain is defined by

$$\Gamma = \mathrm{diag}\{2,\ 0.1,\ 0.1,\ 4,\ 0.1,\ 0.1,\ 0.2,\ 0.2\},$$

and the control gains are set to be $k_v = 16$ and $H = 10$.

As the simulation results, Figure 2(a) shows the desired and actual trajectories of the robot tip point **A**, and the traces of the base centroid **C** and the fourth joint center (the top point of the base) **B** in the case without parameter deviation. Figure 2(b) plots the same trajectories in the case with parameter deviation but without adaptive control. In Figure 3, part (a) shows the resultant trajectories with parameter adaptation, and part (b) gives the input (two joint torques) plot versus time in the same condition. The tracking errors versus time in the cases with and without adaptive control are shown in Figure 4. Also, the parameter adaptation processes for m_4 and $m_5 k_{z5}^2$ versus time are shown in Figure 5. All these resultant plots verify that the proposed model and adaptive control for the space robot system are feasible and effective.

7 Conclusion

We presented an *extended robot model* to describe the dynamics of a space robot system when the base is free-floating. The extended robot model consists of a pseudo-arm representing the base motion and a real robot arm. Based on the presented model, the system is categorized as an under-actuated mechanism, and the property of the inertial matrix and Jacobian matrix are discussed. The model is of significance in analysis of any under-actuated dynamic system no matter whether the passive joints are real or hyperthetic. The concept may be applicable, for instance, to manipulation tasks where robots are operating a tool with passive joints.

Through input-output linearization of the model, we demonstrated the non-trivial internal dynamics which results in fundamental difference between the space robot and fixed-base robot, and reveals the nonlinear parameterization property. Then, we developed an adaptive control scheme based on the *normal form augmentation approach.* This approach makes it possible to solve two fundamental problems in controlling space robot systems, i.e., the unavailability of joint trajectory and the nonlinear parameterization. By simulation study, we demonstrated that the developed control scheme represented in Cartesian space can asymptotically stabilize the space robot system to track a given trajectory. The computational procedure of the adaptive controller and the tracking results with and without parameter adaptation are also provided through the case study.

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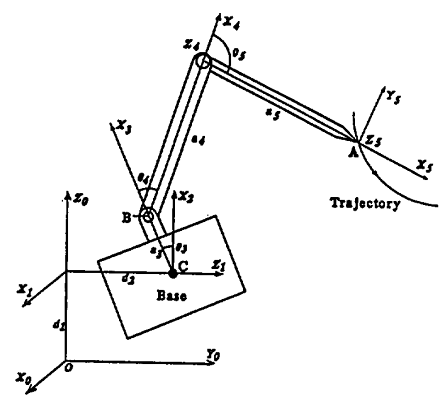
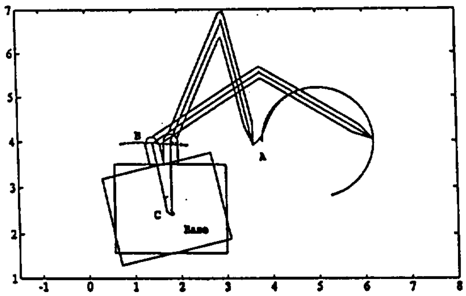


(a) A Schematic Diagram for a Space Robot System with Coordinates Assignment

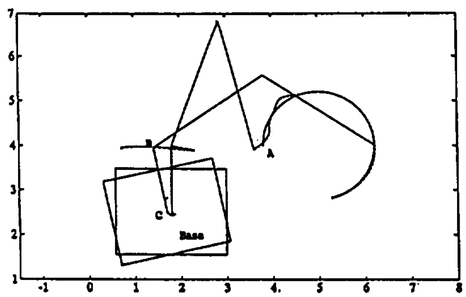
Variable	θ_i	d_i	α_i	a_i	q
d_1	0	d_1	-90^0	0	
d_2	-90^0	d_2	-90^0	0	q_b
θ_3	θ_3	0	0	a_3	
θ_4	θ_4	0	0	a_4	
θ_5	θ_5	0	0	a_5	q_r

(b) Denavit-Hartenberg (D-H) Table for the Extended Robot Model

Figure 1: A Planar Space Robot System for Simulation Study

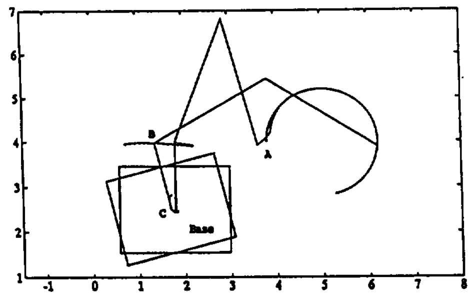


(a) Trajectories without Parameter Deviation

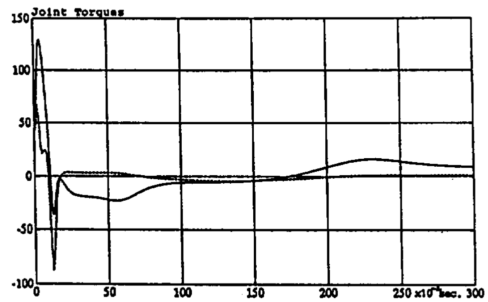


(b) Trajectories with Parameter Deviation but without Adaptation

Figure 2: Comparison of the Cases with and without Parameter Deviation

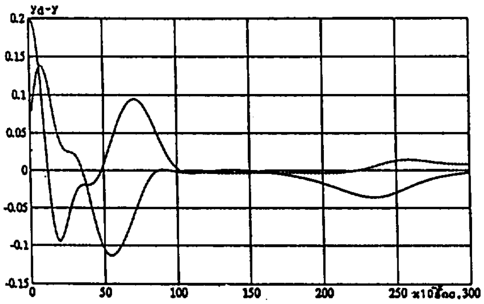


(a) Output Trajectories with Adaptive Control

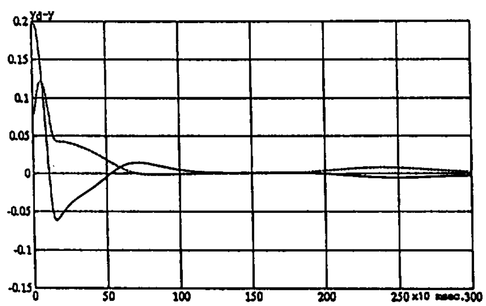


(b) Two Joint Torques vs. Time in the Adaptive Control System

Figure 3: Output and Input of the Space Robot Adaptive Control System

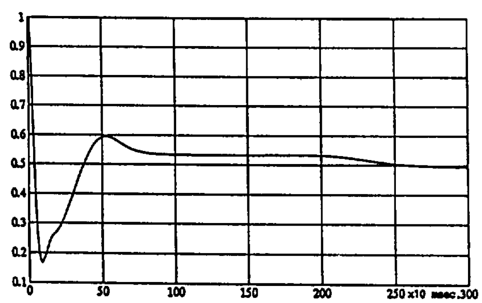


(a) Tracking Error in the Case without Parameter Adaptation

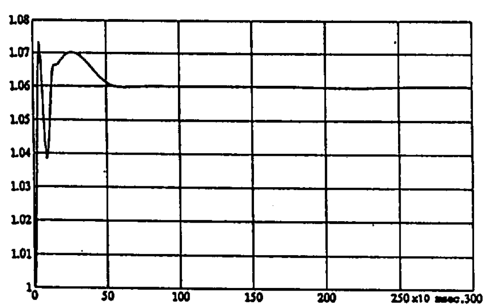


(b) Tracking Error in the Case with Parameter Adaptation

Figure 4: Comparison of Tracking Errors with and without Adaptation



(a) Adaptation Process of the Parameter m_4



(b) Adaptation Process of the Parameter $m_4 k_{zb}^2$

Figure 5: Parameter Adaptation Processes