Three Dimensional Migration and Forward Modelling of Ground Penetrating Radar Data

Robert Beck and Jim Osborn

CMU-RI-TR-91-12

The Robotics Institute
Carnegie Mellon University
Pittsburgh, Pennsylvania 15213

June 1991

Sponsored by the U.S. Environmental Protection Agency Risk Reduction Laboratory under Cooperative Agreement #CR-815233 and The Ben Franklin Technology Center of Western Pennsylvania and in Cooperation with RedZone Robotics, Pittsburgh, Pennsylvania.

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Abstract

Carnegie Mellon University is automating the use of Ground Penetrating Radar (GPR) for the cleanup of hazardous waste sites. The current focus is on the development of an automated subsurface mapping system to locate buried objects and geological structures so that sources and migratory pathways of contaminants can be identified and catalogued. The subsurface maps are produced using the non-invasive sensing capabilities of GPR. GPR operates similar to conventional radar, but the data acquired is more difficult to process due to the nonhomogeneous nature of the subsurface medium. Though sometimes used in waste site characterization, GPR deployment, data acquisition, and interpretation are human driven processes. The potential of GPR to generate accurate three dimensional subsurface maps has not been fully realized previously. The Site Investigation Robot uses robots to position a GPR transducer to exploit the accurate, repeatable positioning available from automated equipment. By combining the use of a position cognizant, all terrain mobile robot and a linear scanning mechanism, GPR records are acquired in a two dimensional grid on the ground surface. This method of collection simplifies processing and the positional location of features of interest in the data. To achieve accurate positional accuracy of the located subsurface objects, correct modelling of the subsurface medium and the antenna is crucial. The subsurface medium can be modelled as a composition of multiple subvolumes, each one being characterized by its own electrical parameters. The radar wave's energy can be calculated at any point in the subsurface using these parameters and the distance the wave travelled. Analysis of the wave's propagation in the subsurface is simplified by the use of a technique known as ray tracing. Two criteria are used to determine if the use of ray tracing is valid: the wavelength of the radar waves must be short in comparison to the resolution of the external perturbations of any expected objects and the medium must be an isolator. Modelling of the GPR antenna is best introduced by the analogy of a laser and a flash light. A laser is a highly focused beam of light, but the intensity of the light transmitted by a flash light decreases rapidly with distance from the center of the light. The GPR antenna operates in a similar fashion to the flash light. Two models of the antenna are used: the energy of the antenna's beam is assumed constant at all points equidistant from the antenna and the empirical measurements of the beam's attenuation pattern. Because of the lack of focus in the antenna's beam, the data collected at one surface location represents reflections from all objects and interfaces within the beam. Migration is the signal processing operation used to reverse a spatial integration effect in the collected data. Homogeneous migration assumes the electrical characteristics of the subsurface are constant throughout the entire volume processed; heterogeneous migration, a more realistic operation, allows these parameters to vary. Forward Modelling simulates, on a computer, the collection of radar data under user specified conditions. There are two purposes for forward modelling: test the migration operation and as a learning tool for understanding the propagation of GPR waves in the subsurface. Because of the design of the software, the user can easily change parameters that control the simulation (e.g. - antenna height). The accurate tracing of radar waves requires a model which includes the reflection and refraction of rays at subsurface interfaces and objects. This model is a necessity for heterogeneous migration and forward modelling (not currently implemented).

1.0 Executive Summary

1.1 Statement of Purpose

Carnegie Mellon University is automating the use of Ground Penetrating Radar (GPR) for the cleanup of hazardous waste sites. The current focus is on the development of an automated subsurface mapping system to locate buried objects and geological structures so that sources and migratory pathways of contaminants can be identified and cataloged. The subsurface maps produced use the non-invasive sensing capabilities of GPR. GPR operates on principles similar to conventional radar, but the data acquired is more difficult to process due to the heterogeneous nature of the subsurface medium. Though sometimes used in waste site characterization, GPR deployment, data acquisition and interpretation are human driven processes. The potential of GPR to generate accurate three dimensional subsurface maps is thus not fully realized in practice. The Site Investigation Robot project uses robots to position GPR transducers to exploit the accurate, repeatable positioning available from automated equipment. By combining the use of a position cognizant, all-terrain mobile robot and a linear scanning mechanism, it is possible to acquire GPR records in a two dimensional grid on the ground surface.

Data collected using the GPR antenna does not accurately reflect the structure of the subsurface for two reasons. The volumetric nature of the antenna beam causes spatial integration of the subsurface structure before the data is collected. The second reason for inaccuracy in the reflected data is distortion. During propagation, the radar wave is convolved with the linear system which represents the structure of the subsurface. Also, the antenna pulse is not a perfect impulse function; this causes the collected data to be convolved with imperfections in the antenna pulse. Both of these problems are addressed by the deconvolution operation (not explained in this paper).

Two GPR antennas are used: one for transmission and one for reception of radar waves. High frequency radar waves transmitted by the send antenna are reflected back to the receive antenna by interfaces between subsurface materials with different electromagnetic characteristics. Reflections received from the same point on the surface are stored in a one dimensional data array, and tagged with the surface position. All the GPR data collected over the entire two dimensional grid form a volume. The operation of the GPR antenna can be illustrated using a laser and a flashlight. A laser is a highly focused beam of light, but the light intensity of a flashlight decreases rapidly with distance from the center of the beam. The GPR antenna operates in an similar fashion to the flashlight. Because of this lack of focus in the antenna beam, the data collected at one grid location represents reflections from all objects and interfaces within the antenna's beam. Migration is the signal processing operation used to reverse this distortion in the collected GPR data. Due to the heterogeneous nature of the subsurface, the exact shape of buried objects can never be fully recovered by the migration operation. Conventional image processing operations are used on the post migrated data to decrease the needed for a highly skilled interpreter. Forward Modelling simulates, on a computer, the collection of radar data under user specified conditions. There are two purposes for modelling GPR: test the migration operation and aid in the understanding of how GPR waves propagate in the subsurface.

1.2 Summary of Results

Since the GPR antenna's beam is three dimensional, three dimensional migration is the most appropriate technique to reverse the spatial integration phenomena. Testing of the algorithm was done using real data from a test pit (Figure 1) filled with dry sand and a metallic drum and data generated from forward modelling.

The migration operation causes positive reinforcement in the area where reflections due to subsurface objects or interfaces occurred. However, the migration operation causes negative reinforcement in the areas where there are no reflections due to subsurface objects or interfaces. For areas where there are only weak reflections the interference caused by the migration operation will produce noise. Real GPR data (Figures 2, 3) contains values with positive and negative signs, thus allowing migration's constructive and destructive interference to work. Because the Forward Modelling was done in the time domain, the generated radar data only contains positive signs. Therefore, only the constructive interference works when migration is applied to data generated by forward modelling (Figures 4,5).

FIGURE 1 **Configuration of Test Pit**

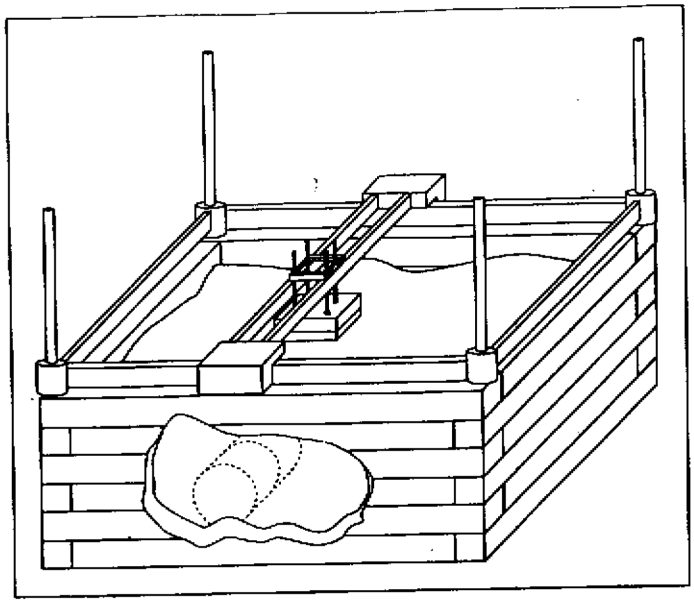


FIGURE 2 **Unprocessed Data from the Test Pit**

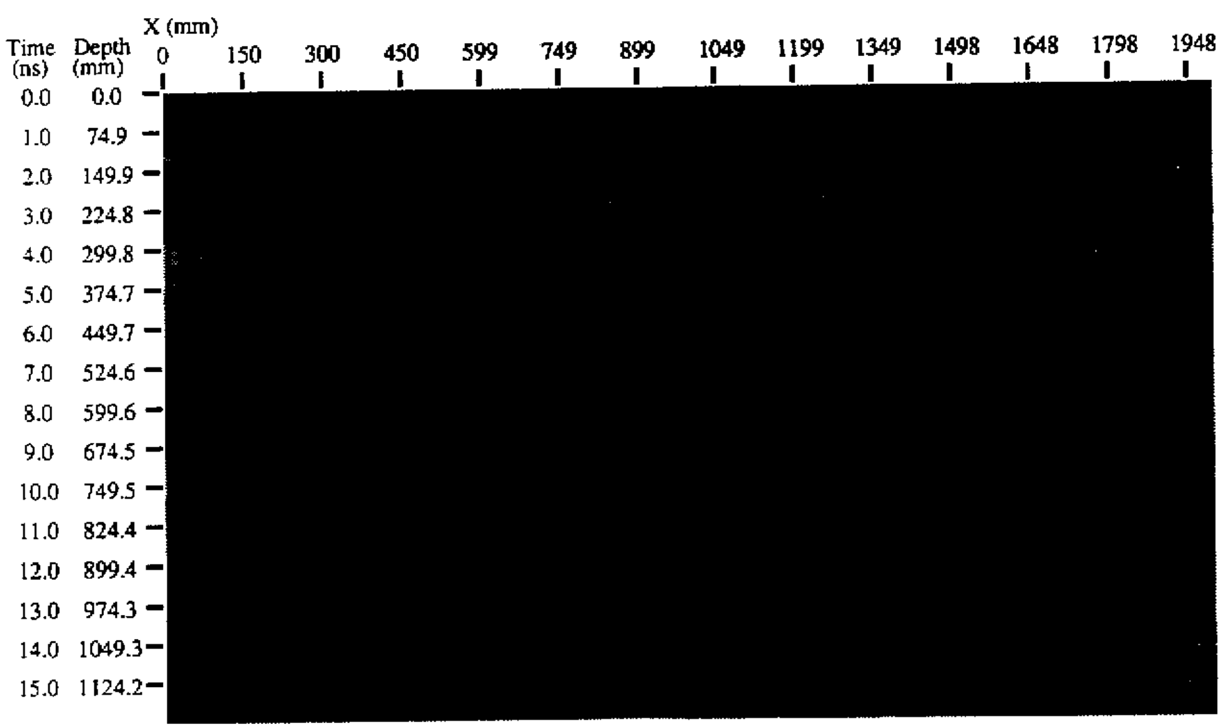


FIGURE 3 **3D Migrated Data from the Test Pit**

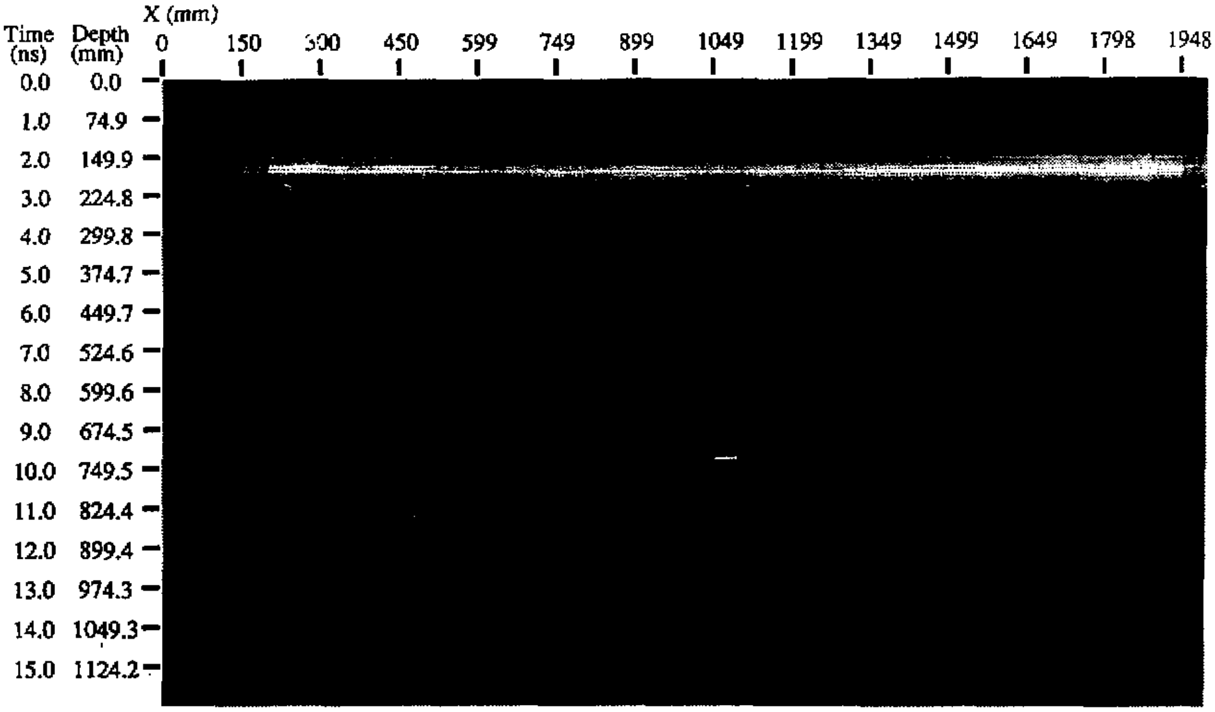


FIGURE 4 **Unprocessed Data Generated by Forward Modelling**

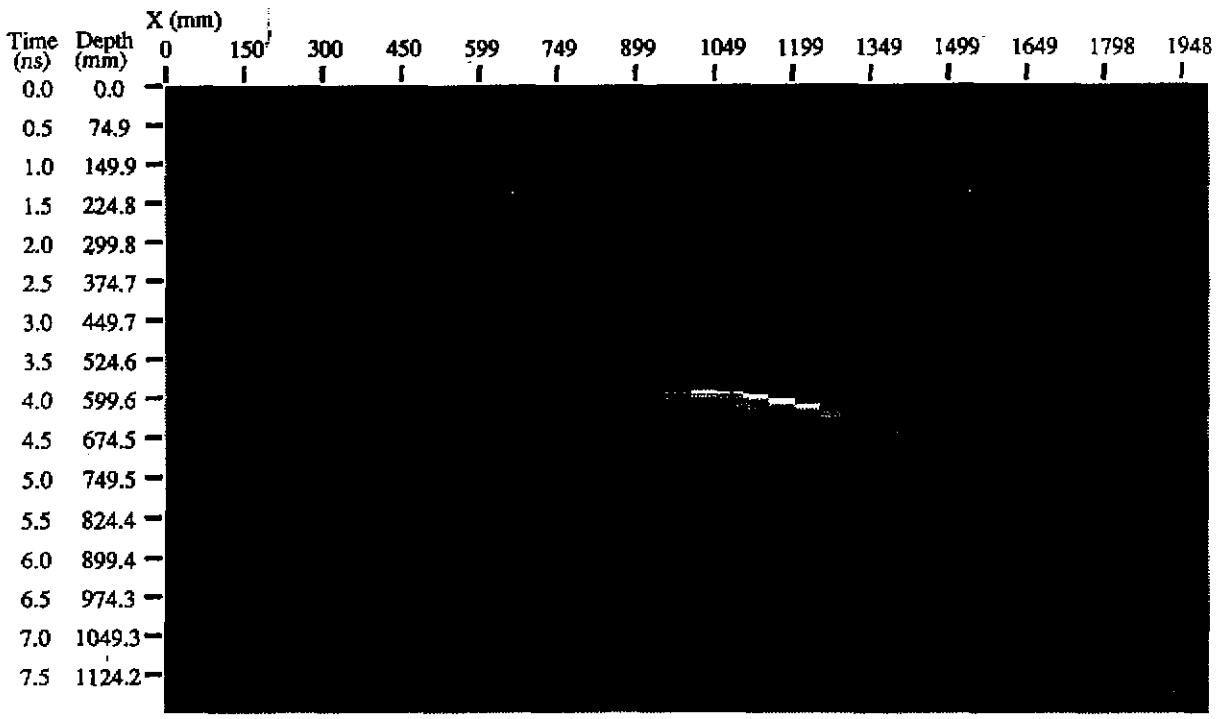
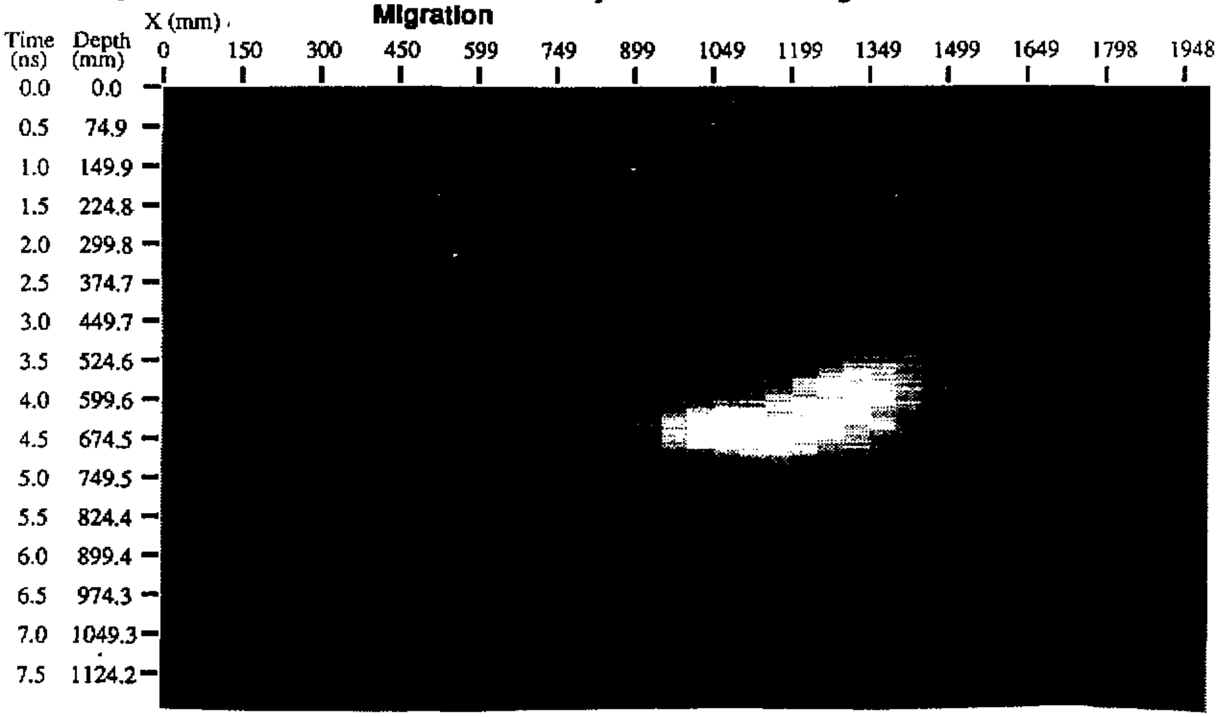


FIGURE 5 **Data Generated by Forward Modelling after Three Dimensional Migration**



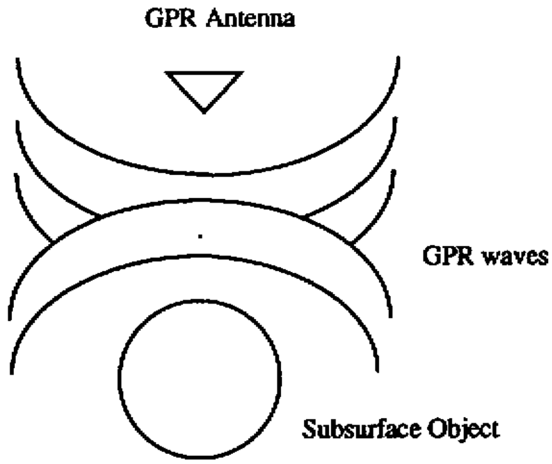
1.3 Summary of Recommendations

Implementation of three dimensional heterogeneous migration and forward modelling is the next important step. Since the velocity of GPR waves varies significantly in different medium, the positional accuracy of the located subsurface objects should significantly improve using a heterogeneous algorithm. Current evaluation of the algorithm's accuracy is done in a semi-quantitative manner; another important contribution is a system to perform automatic quantitative evaluation of the algorithm's accuracy.

2.0 Modelling of the Subsurface Medium

Analysis of GPR wave propagation is done using two tools. Electromagnetic wave theory is the tool which best models the physics involved. The mathematics are simpler if ray tracing, a technique from geometric optics, can be used. (Nayar 1989) Two criteria (Ulriksen 1982) are used to justify the use of ray tracing: the wavelength of the radar waves is much less than the resolution of the external perturbations for any buried object and the medium is an isolator. Since the smallest expected wavelength is 6 centimeters, these two criteria are satisfied.

FIGURE 6 **Illustration of Subsurface GPR Wave Propagation**



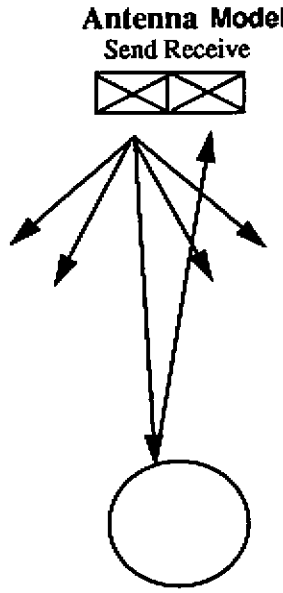
Ray Tracing Justification Criteria

$$\lambda = \frac{c}{(f \times \sqrt{\varepsilon})} \quad where\ f = 500\ Mhz\ and\ c = 3\times10^{8}\ meters\ per\ second$$

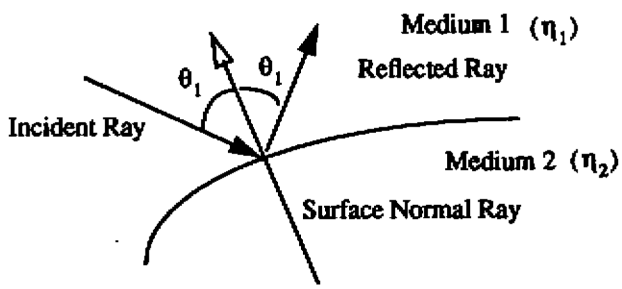
Examples *Dry Sand* $(\varepsilon = 4)\ \lambda = 30\ cm$ *Saturated Sand* $(\varepsilon = 81)\ \lambda = 6\ cm$

It is desired to trace the propagation of radar waves in the subsurface. Since it is valid to use ray tracing, tracing the propagation of waves reduces to the tracing of vectors. Modifications to pure ray tracing are based on the antenna's beam pattern (see section 3). Figure 7 illustrates the principles of ray tracing (Born 1980) at a subsurface interface. The angles of reflection and refraction are a function of the medium's electrical characteristics. The refracted ray lies in the same plane, called the plane of incidence, as the incident ray and the normal to the surface. The reflected ray lies in the plane of incidence.

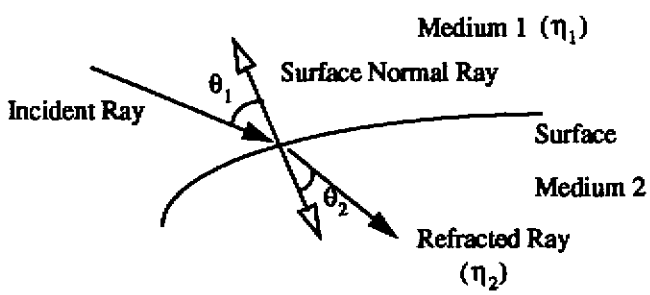
FIGURE 7 **Illustration of Ray Tracing**



η = *index of refraction* $\qquad \eta = \sqrt{\varepsilon \times \mu} \qquad \eta_2 \sin\theta_2 = \eta_1 \sin\theta_1$



Law of Reflection



Law of Refraction

3.0 Antenna Modelling

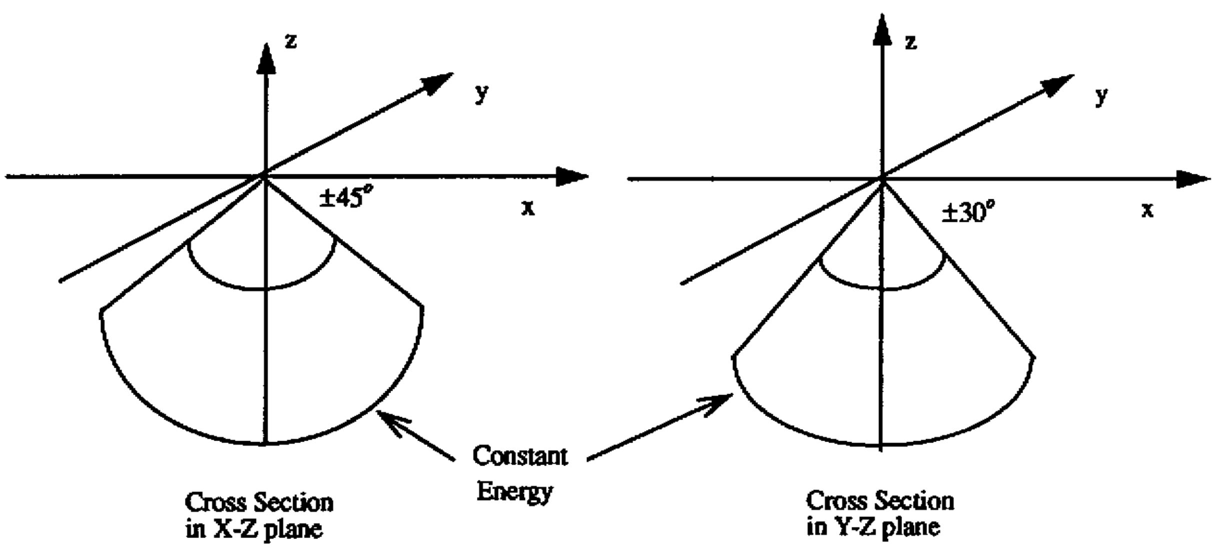
There are three factors that affect the amplitude of the transmitted GPR signal: the bandwidth, the beam pattern, and the radar beam's attenuation in the subsurface medium. The bandwidth describes the change in signal amplitude versus frequency. The 500 Mhz antenna has a bandwidth of $\pm 250 Mhz$. This means that the half power point (3 db) is at 250 and 750 Mhz. Since all the processing in this paper was done in the time domain, the bandwidth of the antenna was not used.

The beam pattern describes the attenuation of the signal which is a result of the unfocused nature of the antenna pulse. In part 1, all energy in the beam pattern is assumed to be constant. A more accurate model (part 2) takes into account the geometrical spreading losses of the radar wave.

3.1 Constant Energy Beam Pattern

The first approximation of the beam pattern assumes the radar wave's energy is constant for all points equidistant from the antenna. The range of the antenna's beam pattern, as obtained from the manufacturer (Operations Manual 1987), is $\pm 45°$ for the x-axis cross section and $\pm 30°$ for the y-axis cross section (Figure 8). The analogy of the arc for the three dimensional case is a sphere bounded by an elliptical cone. One x-y section taken from the intersection of the sphere with the elliptical cone is an ellipse.

FIGURE 8 **Beam Patterns Assuming Constant Energy**



The equation for the aforementioned cone is shown below (spherical coordinate system).

$$\theta = \operatorname{atan}\left(\sqrt{\frac{1}{(1 + 2 \times (\sin\phi)^2)}}\right)$$

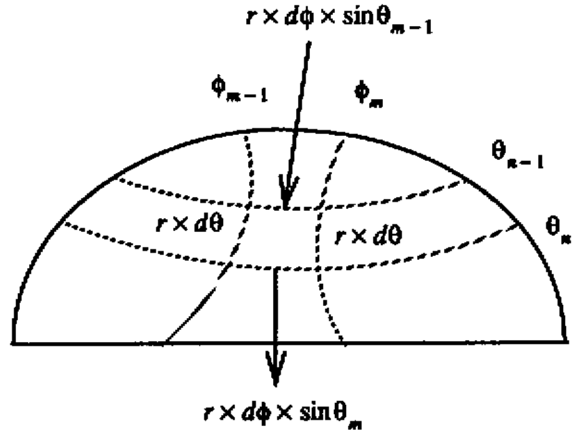
Since the radar energy flux is uniformly distributed across the surface of the sphere, accurate three dimensional processing requires equal weighting of each ray in the radar beam. In order to accomplish this, the sphere is divided into trapezoidal patches of equal area. Figure 9 shows a hemi-sphere with the lengths of the edges labelled for a sample trapezoid. The algorithm divides the sphere's surface into a user specified number of longitudinal strips. Each strip is divided into as many trapezoidal patches of the specified area as possible. The algorithm's performance was measured by calculating the ratio of the area covered by the trapezoids to the total area possible in the sphere (96.48%).

FIGURE 9 **Area of a Trapezoidal Patch**

r = *radius*

$d\phi = \phi_m - \phi_{m-1}$

$d\theta = \theta_n - \theta_{n-1}$



$$Area = \frac{(d\phi)}{4}\ x\ (\sin\theta_{n-1} + \sin\theta_n) \qquad (\sqrt{(4 \times d\theta - d\phi^2 \times (\sin\theta_{n-1} - \sin\theta_n)^2)}\,)$$

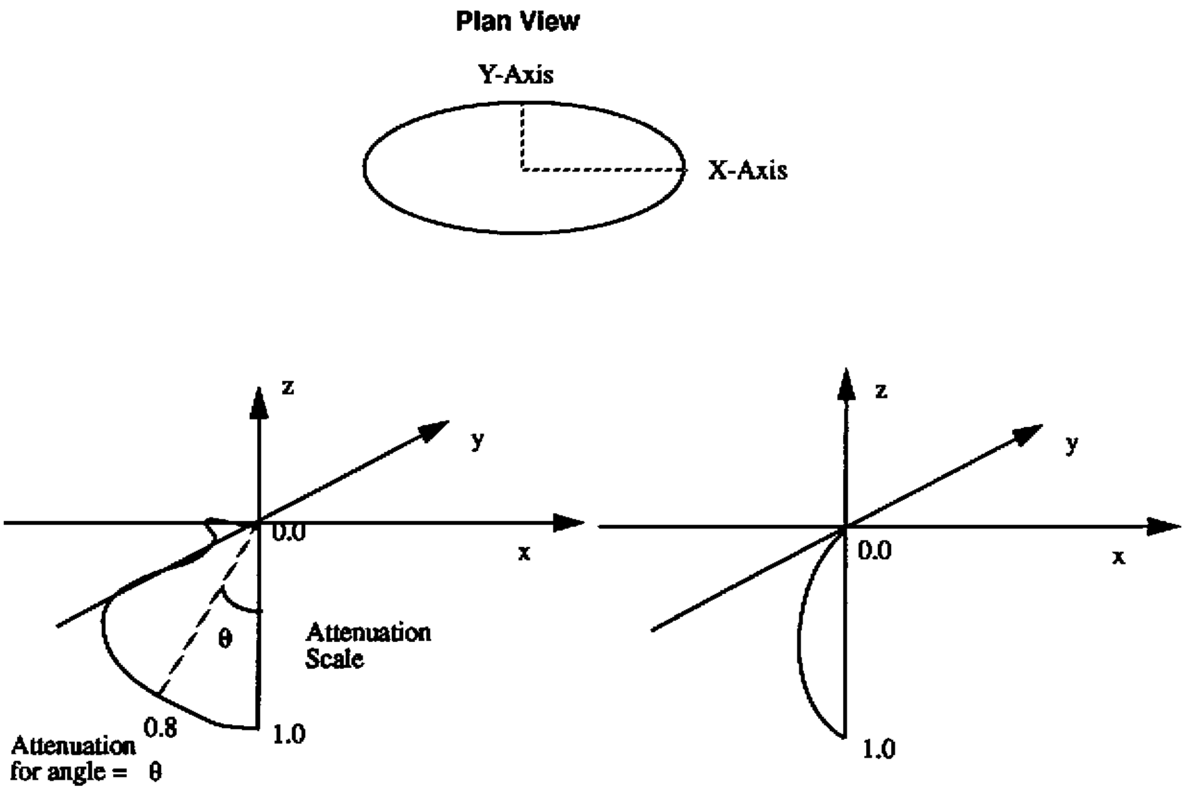
3.2 Beam Attenuation Pattern

Four sets of beam pattern measurements were obtained: one from Geophysical Survey Systems Incorporated (Shutz 1990) and three from Steven Duke's thesis (Duke 1990). The measurements obtained from GSSI characterize all the GPR antennas (medium of air) they manufacture. The data from Duke's thesis describe the 500 Mhz antenna in three different medium.

The beam's attenuation pattern is characterized by two orthogonal cross sections (Figure 10). The contribution of the beam pattern to the attenuation of the wave is a function only of the angle from the z axis and is independent of the distance from the antenna. The effect due to the distance of the wave from the antenna is described by the attenuation equation. Since the migration algorithm is three dimensional, a volumetric beam pattern was needed. This need was supplied by using a volumetric interpolation algorithm. The algorithm uses two steps: a standard cubic spline interpolation algorithm and a custom elliptical interpolation algorithm. The resolution of the beam pattern cross sections provided was too low; therefore, the cubic spline algorithm was used to increase the number of (angle, attenuation) pairs.

The crucial insight used to develop the custom algorithm is now described. A set of concentric ellipses with centers at different values of attenuation (z axis) can be described using the x and y axis cross sections (Figure 11). The z axis is divided into points of equal resolution.

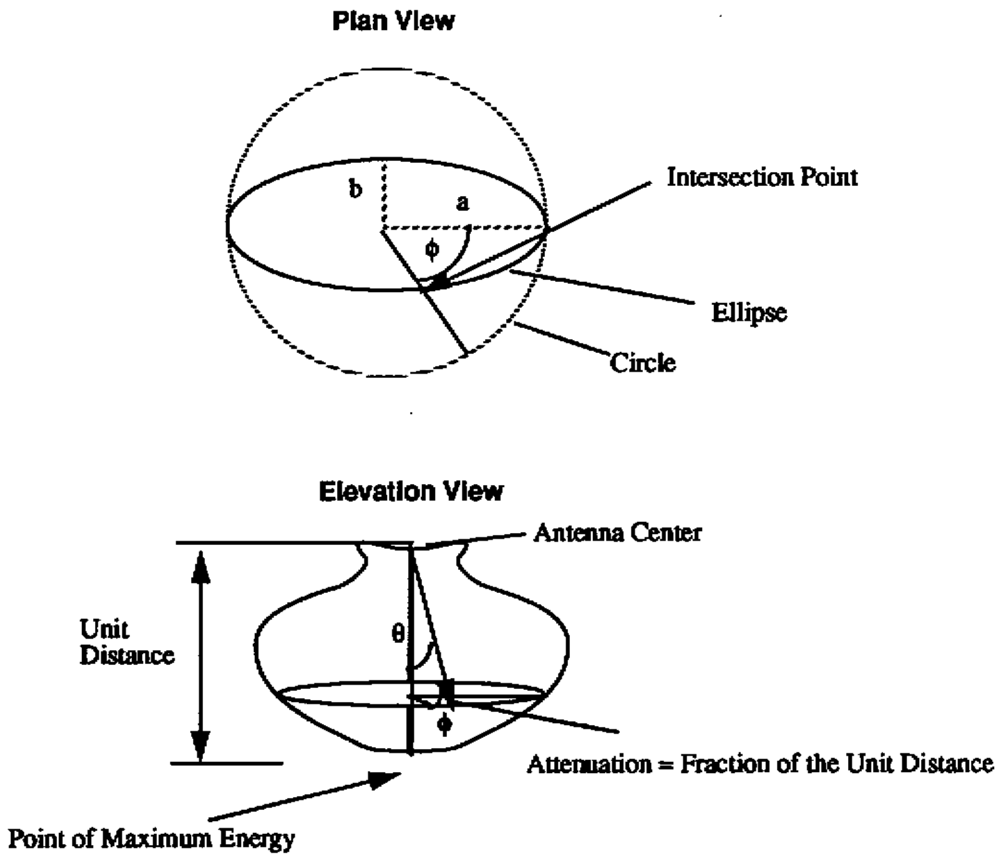
FIGURE 10 **Format of Beam Pattern Measurements**



Cross Section in
X-Z Plane

Cross Section in
Y-Z Plane

FIGURE 11 **Step Two in Volumetric Interpolation**

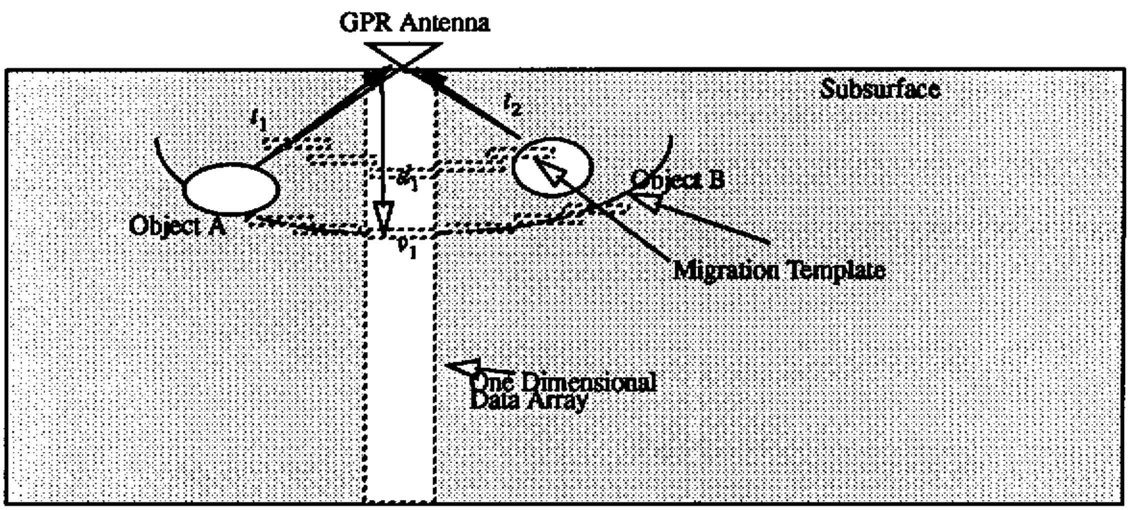


4.0 Migration

4.1 Motivation

Migration is a processing operation that reverses the effect of the unfocused nature of the antenna beam in the collected data. In section 2 it was shown that it is valid to use the ray tracing technique from geometric optics in analyzing GPR wave propagation. Figure 12 is used to show the need for the migration processing operation (Operations Manual 1987); the example is two dimensional for simplicity. The dotted line rectangle represents the one dimensional data array which contains GPR data collected from one position on the surface. The data array is tagged with the surface position, but all the data in the array is not a result of reflections from that position. Each entry in the data array represents all objects and interfaces equidistant from the antenna. Two rays will be traced in this example. Ray 1 strikes object A, and is reflected back to the antenna with a time delay of t_1. Ray 2 strikes object B, and is reflected back to the antenna with a time delay of t_2. For homogeneous medium, the distance the ray travelled is calculated by multiplying the time delay by the velocity of light in the subsurface medium (Operations Manual 1987). This distance is converted to an index into the data array. The top data array entry contains data representing objects and interfaces 50 picoseconds away from the antenna. Every other data array entry contains data collected at an integer multiple of 50 picoseconds away from the antenna. As Figure 12 shows, reflections of GPR waves that are stored in this data array can be from any object within the antenna beam. The problem just described is also true for the three dimensional case.

FIGURE 12 **Motivation for Migration**



$$Distance\ Step = 0.5 \times Time\ Step\ x\ \frac{3\times10^{11}}{\sqrt{\varepsilon}}\ mm$$

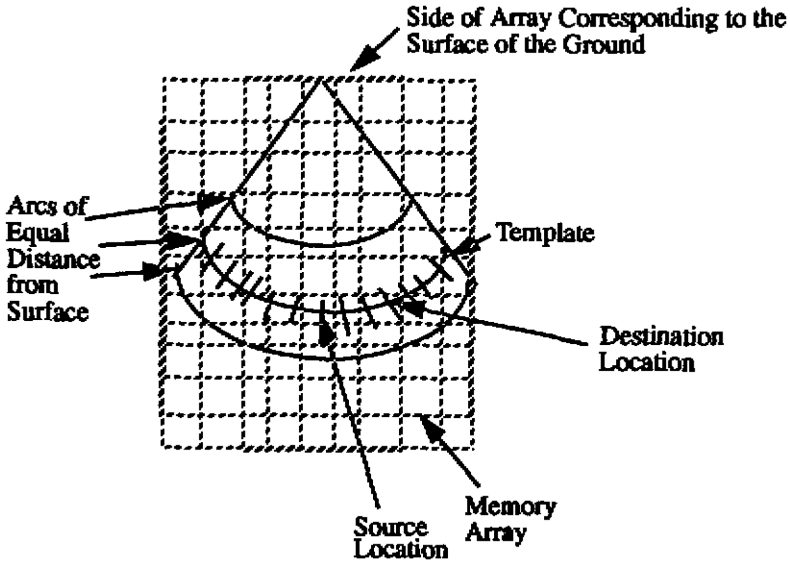
d_1 = *distance index corresponding to* t_1

d_2 = *distance index corresponding to* t_2

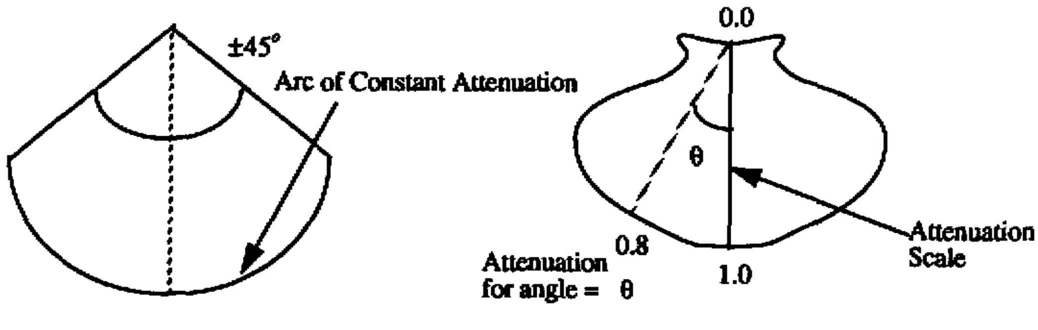
4.2 Explanation

For ease of explanation, the following description of migration is done for the two dimensional case. Changes needed for the three dimensional case are explained later. Migration is done in three steps: calculate a template, use the template to process the section of data, and normalization (Christian 1990). One side of the array corresponds to GPR data collected near the surface of the ground. The template (Figures 13) consists of a sequence of concentric circular arcs of different radii; the center is the data entry corresponding to the surface of the ground and the radius is the distance from the antenna. Each arc is subdivided into a sequence of points; the distance between each point is the same. The point directly below the data array element is called the source element; the rest of the elements are called destination elements.

FIGURE 13 **Explanation of Migration**



a - Migration Template



Constant Attenuation **Beam Pattern Attenuation**

b - Elevation View
Review of Antenna Beam Pattern Models

The first approximation to the antenna's beam pattern assumes the radar wave's energy is constant for all points along each arc's edge. This assumption is the reason for the addition of 100% of each source element to all of its destination elements. A more accurate model of the antenna's beam pattern is shown in Figure 13b. In this model, the energy of the antenna's beam is at its maximum value directly below the antenna. Along an arc of constant depth, the beam's energy decreases as the distance from the source element increases.The attenuation fractions can be used in the migration algorithm for weighting the source element before adding it to a destination element.The extension of migration to three dimensions can now be stated. The three phases of the algorithm (calculate the template, use the template in processing, normalization) are the same. The template used is calculated using one of the algorithms described in section 2: sphere tessellation or volumetric interpolation of the antenna beam's attenuation pattern. The attenuation fraction is used as a scale factor (same method as two dimensional migration).

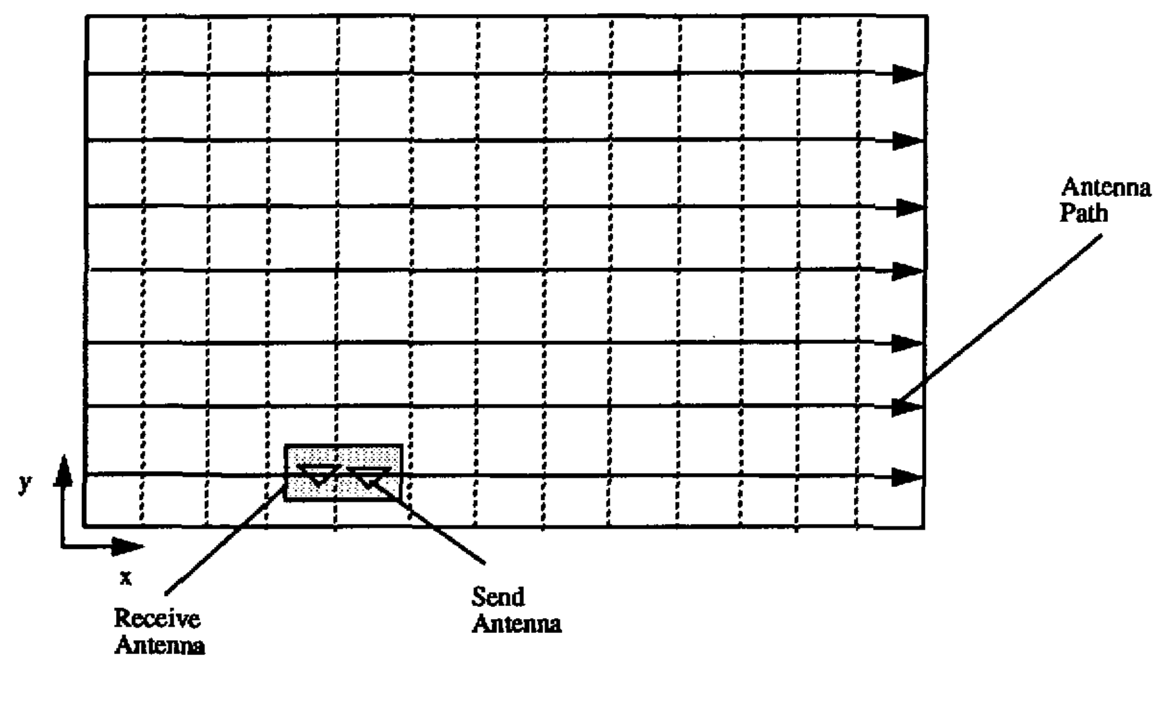
5.0 Forward Modelling

Forward Modelling simulates the generation of GPR data on a computer (Duke 1990). Parameters characterizing the antenna, the subsurface, and the geometry of buried objects are easily changed. This feature makes the forward modelling software a powerful scientific tool. Figure 12 was previously used to explain the motive for migration. It is also an example of how the three dimensional simulator works. For flexibility of implementation, the simulation used a public domain ray tracer to perform the ray-object intersections.

5.1 Simulation of Antenna Movement

Forward modelling of GPR data was done for a rectangular site. Both antennas are moved as a unit (constant antenna separation distance - Figure 14). The site is divided into a rectangular grid. Simulation of the GPR antenna's operation is done or each cell in the grid. The next section explains the simulation of the antenna.

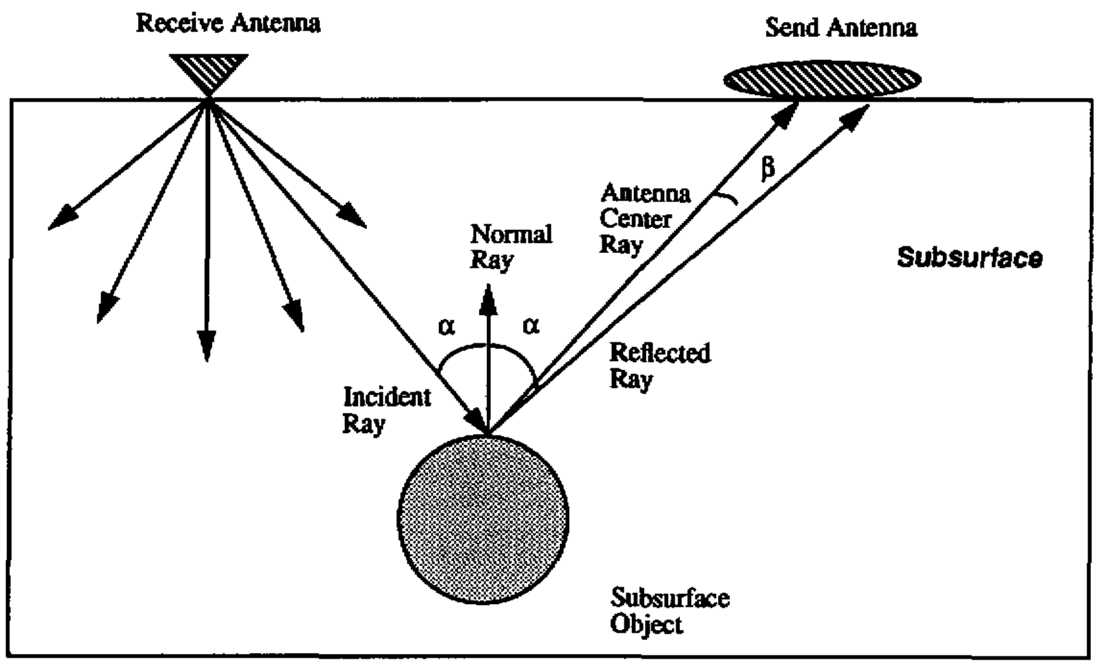
FIGURE 14 **Antenna Movement In Forward Modelling**



5.2 Reflection Model

All rays originate at the receive antenna, which is modelled as a point. For simplicity of implementation, the send antenna is modelled as a circular aperture (Pilant 1990) The rays are traced using a constant angular resolution for θ and ϕ. Rays originate at the receive antenna and terminate at the send antenna; consistency with the original ray tracer implementation is obtained by using this method. When a ray intersects an object, the reflected ray is calculated assuming a specular reflection model. Next, the antenna center ray is calculated. Its origin is the object's intersection point and its end point is the center of the send antenna's aperture. The maximum size of the angle between the reflected ray and the antenna center ray (β) is determined by the size of the send antenna's aperture (Shutz 1990).

FIGURE 15 **Reflection Model for Forward Modelling**



5.3 User Created Simulation Scenario

Before making a simulation run, the configuration file must be initialized (Table 1). All parameters characterizing the antenna, the subsurface medium, and the buried objects are defined in this file. The value of these parameters are easily changed without altering any of the simulation code. The definitions of each parameter follow.

Table 1 **Configuration File Parameters**

Site Characterization Parameters (See Figure 14)	
Parameter	Meaning
xbounds	Bounds in the x direction - low limit, high limit, distance step
ybounds	Bounds in the y direction - low limit, high limit, distance step
zbounds	Bounds in the z direction - low limit, high limit, distance step
Antenna Characterization Parameters	
Parameter	Meaning
aheight	antenna height (mm)
timestep	time step (picoseconds)
distbwant	distance between antennas (mm)
anglestep	angle step (radians)
beamx	beam angle - x cross section (radians)
beamy	beam angle - y cross section (radians)
Reflection Model Characterization Parameter	
Parameter	Meaning
speclobeangle	specular lobe angle (radians)
Subsurface Object Characterization Parameters	
Parameter	Subparameters
sphere	center, radius
cone	center of end 1, radius of end1, center of end 2, radius of end 2, length
polygon	number of points (N) point 1 through point N
patch	number of points (N) point 1, normal to point 1 ... point N, normal to point N
Subsurface Characterization Parameter	
Parameter	Meaning
baseperm	base permittivity

6.0 Extensions for Heterogeneous medium

When a volume of GPR data contains subvolumes with different electrical characteristics (heterogeneous media), accuracy of the processing operations can be greatly increased by utilizing a more complete model of the subsurface. The electrical characteristics of the subsurface are used to calculate the velocity of the radar wave in the medium. These characteristics are used in the heterogeneous migration algorithm (Fenner 1985, Ulriksen 1982).

$$\mu = magnetic \;\; susceptibility$$

$$\sigma = conductivity$$

$$\beta = imaginary \;\; part \;\; of \;\; the \;\; wave \;\; number$$

$$\beta = 2\pi f\left(\sqrt{\mu\frac{\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{2\pi f\varepsilon}\right)^{2}}+1\right)}\right)$$

$$Velocity = 2\pi\frac{f}{\beta}$$

A subsurface interface is a division between two subsurface volumes with different electrical characteristics. When a radar wave is incident on a subsurface interface, the amount of radar energy reflected and refracted at this interface is calculated using the impedance of both mediums as shown below.

$$Z_1 = Impedance \;\; for \;\; Layer \;\; 1 \qquad Z = \sqrt{\left(j\times\omega\times\frac{\mu}{(\sigma+j\times\omega\times\mu)}\right)}$$

$$Z_2 = Impedance \;\; for \;\; Layer \;\; 2$$

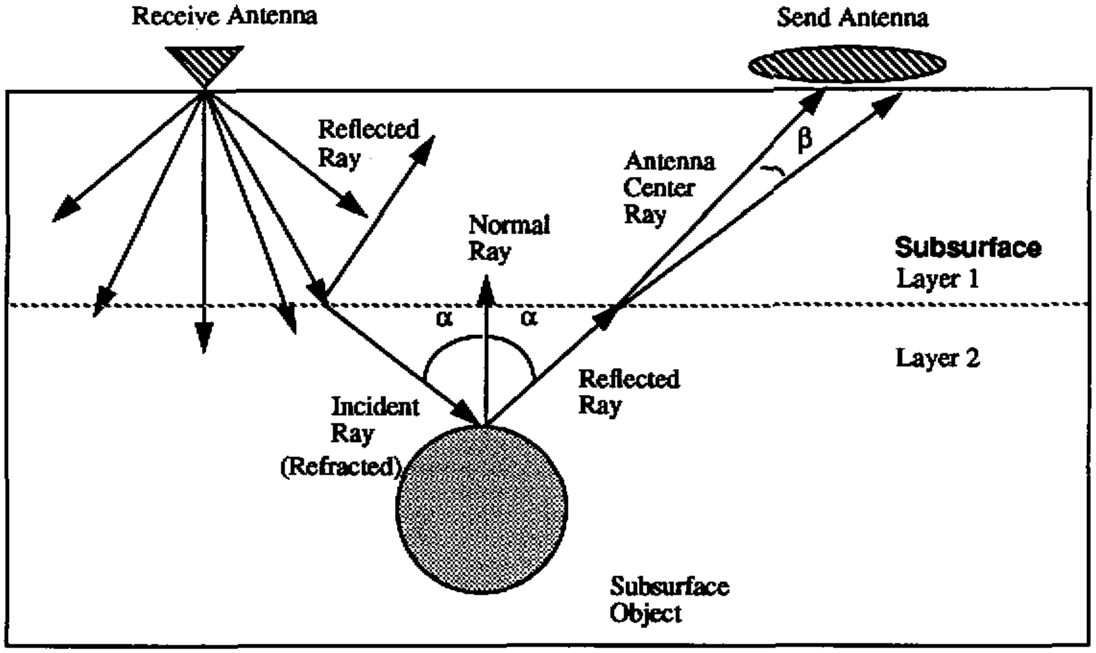
$$Reflected \;\; Energy = \frac{(Z_2-Z_1)}{(Z_2+Z_1)} \;\; Incident \;\; Energy$$

$$Refracted \;\; energy = Total \;\; Energy - Reflected \;\; Energy$$

6.1 Forward Modelling

For the case of homogeneous media, when a ray intersects an object the reflecting ray and the antenna center ray are calculated (Figure 16). If the reflecting ray intersects the send antenna's aperture, then the energy of the send antenna is placed in the data array as previously described. The changes for heterogeneous medium area conceptually straight forward (Figure 17). Ray intersections are done recursively. At each interface two rays are calculated: the reflected and the refracted rays (using the model described in section 2). There are two end conditions for the recursion: a ray intersect the send antenna's aperture and the angle β is within the user specified limits or a ray intersects the z-plane of the send antenna without intersecting the aperture. A model of the electrical parameters for each subsurface layer will also be needed since the index of refraction (used to calculate the angles of reflection and refraction) is a function of the medium's permittivity and the conductivity.

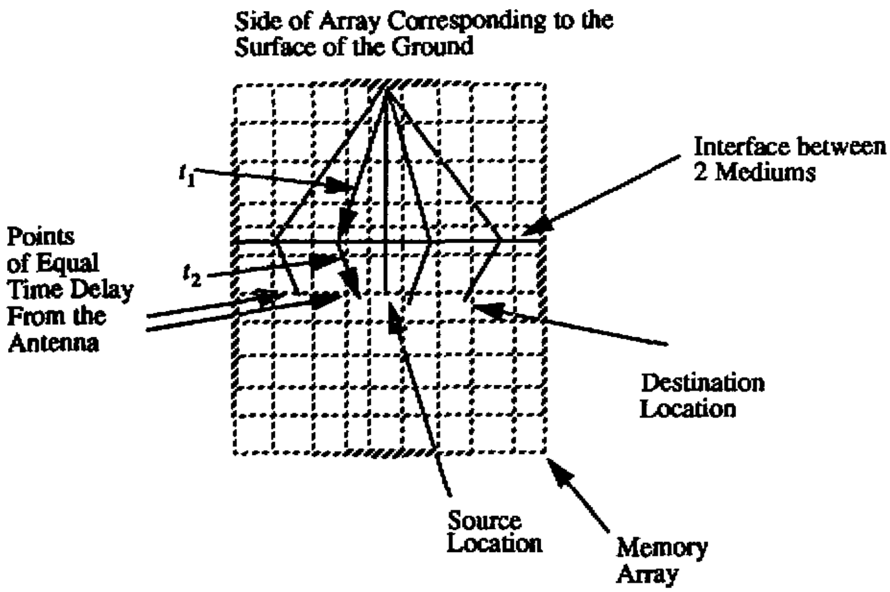
FIGURE 16 **Forward Modelling In Heterogeneous Medium**



6.2 Migration

The template for migration in homogeneous media consists of a series of arcs; every point on each arc is equidistant from the ground surface's data element (Figure 17). This is true when the velocity of the radar waves is constant throughout the medium, but it is not a valid assumption for the case of heterogeneous medium. The template takes on some arbitrary, perhaps irregular, geometry such that every point on each arc is the same time delay from the corresponding surface data element. Ray tracing will be used to determine the coordinates for this set of points. In the homogeneous case, a template was calculated in a preprocessing step, therefore saving computational time. Because an heterogeneous algorithm needs to work for any distribution of electrical parameters in the subsurface, a preprocessing step is not possible. This means the template must be calculated for each data element corresponding to the surface of the ground.

FIGURE 17 **Migration Template for Heterogeneous Medium**



7.0 Discussion and Recommendations

7.1 Discussion

Since the current focus of this research 1 is subsurface mapping, the ability to accurately locate the boundaries of buried waste is crucial. The test site used for this evaluation is a sandbox with the dimensions - 2 by 2 by 1.2 meters. A cylinder of length 700 millimeters and radius 175 millimeters was buried with one of its ends coinciding with one end of the sandbox (Figure 1). The left and right edges of the cylinder were predicted (using a GPR image) with less than 5% error. A vertical slice of unprocessed data taken from the midpoint of the cylinder is shown in Figure 2. Data from the same location, after deconvolution and three dimensional migration, is shown in Figure 3. Unprocessed data for a forward modelled sphere with the same radius is shown in Figure 4. After three dimensional migration, the same data slice is shown in Figure 5. The coordinates of the center of the sphere are (1000, 600).

A significant contribution of this research is a methodology for implementing three dimensional migration and forward modelling of GPR data. This contribution is divided into three areas: accurate modelling of the antenna beam's attenuation pattern, accurate modelling of the subsurface, and computationally fast ray tracing.

Digitized radar reflections in actual GPR data consist of positive and negative values. The migration operation causes positive reinforcement in the area where reflections due to subsurface objects or interfaces occurred. The operation causes negative reinforcement in the areas where there are no reflections due to subsurface objects or interfaces. Of course, in areas where there are only weak reflections the reinforcement caused by the migration operation is not complete. All the data generated by the forward modelling algorithm will have positive values. When the migration operation is used on data produced by the forward modelling algorithm, the positive reinforcement works but the negative reinforcement does not.

7.2 Recommendations

Implementation of three dimensional heterogeneous migration and forward modelling is the next important step. Its algorithm is outlined in the previous section. Because the velocity of the radar waves vary significantly in different medium, the accuracy of the located subsurface objects should significantly improve. The path of the traced rays will change significantly at many interfaces. Since the accurate location of subsurface objects is of utmost importance, a quantitative method is needed to measure this accuracy. A computerized method is needed to compare the actual location and geometry of the object (apriori knowledge assumed) with the location and geometry obtained from the GPR images. This could be done in three steps. Store the actual location and geometry of each object in a file, display it in an overlay on the GPR image (computer screen), and use visual inspection to determine the image's accuracy. A better method is to eliminate visual inspection by using an edge detection algorithm on the GPR image before comparing it with the computer model.

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