On the Global Optimum Path Planning for Redundant Space Manipulators

Om P. Agrawal and Yangsheng Xu

Abstract—Robotic manipulators will play a significant role in the maintenance and repair of space stations and satellites, and other future space missions. Robot path planning and control for the above applications should be optimum, since any inefficiency in the planning may considerably risk the success of the space mission. This paper presents a global optimum path planning scheme for redundant space robotic manipulators to be used in such missions. In this formulation, a variational approach is used to minimize the objective functional. It is assumed that the gravity is zero in space, and the robotic manipulator is mounted on a completely free-flying base (spacecraft) and the attitude control (reaction wheels or thrust jets) is off. Linear and angular momentum conditions for this system lead to a set of mixed holonomic and nonholonomic constraints. These equations are adjoined to the objective functional using a Lagrange multiplier technique. The formulation leads to a system of Differential and Algebraic Equations (DAEs). A numerical scheme for forward integration of this system is presented. A planar redundant space manipulator consisting of three arms and a base is considered to demonstrate the feasibility of the formulation. The approach to optimum path planning of redundant space robots is significant since most robots that have been developed for space applications so far are redundant The kinematic redundancy of space robots offers efficient control and provides the necessary dexterity for extra-vehicular activity that exceeds human capacity.

I. Introduction

SPACE exploration is a new frontier in current science and engineering [1]. Benefits from the space exploration are enormous, however, the stake is also very high. Space missions are hazardous to astronauts [2] because of extremes of temperature and glare, and possibly high level of radiation. The extra-vehicular activity also consumes considerable time and may need the dexterity and high load handling capacity that astronauts can not provide. Therefore, using robots in space is beneficial for extending on-orbit time of space shuttle and increasing productivity of space mission.

The use of robotic manipulators in space applications introduces several new problems which do not arise in ground base robot applications. For example, the motion of a space manipulator can cause the base (satellite or spacecraft) of the manipulator to move and disturb the trajectory of the spacecraft [1]–[4]. This can severely affect the spacecraft performance specially when the mass and the moment of inertia of the manipulator arms and the payloads are not negligible in comparison to the manipulator base.

One solution to the above problem is to use reaction (thrust) jets to control the attitude of the spacecraft (or the robot base) [5]–[6]. To meet this requirement, the spacecraft must carry additional reaction jet fuel. Since a spacecraft can carry only a limited load, this approach may force removal of other facilities of considerable importance for the success of the mission. Thus it may alter the goal of the mission. Furthermore, exhaust from the reaction jets may interfere with proper operation of the instruments on the board. For example. the exhaust may reduce the vision distance of a camera if the camera must see along the reaction jets, and in some extreme cases the deposition of the exhaust on the camera lenses may completely block the vision of the camera.

Another solution to the above problem is to keep the reaction jets off and to move the space manipulator arms in such a fashion that it accomplishes the desired space tasks and yet maintains the stability and the overall trajectory of the spacecraft and/or the satellite. This approach will considerably reduce the reaction jet fuel needed for attitude control and increase the life span of the spacecraft and the manipulator.

It is clear that the second approach is far superior than the first. For this reason. most investigators in this area have focused their research interests on the second approach [1]–[10]. Lindberg. Longman. and Zedd [1] address various issues related to fm-flying space manipulators and provide a comprehensive review of several papers on the subject. Umetani and Yoshida [7] present a Jacobian matrix formulation for the study of kinematics and control of a free-flying space manipulator. Their formulation includes both the linear and the angular momentum conservation conditions. It should be noted that the pseudo Jacobian inverse solution of the velocity constraint equations provides only local minimum of the generalized velocity norm which may not be the global minimum.

Recently, Vafa and Dubowsky [2]have presented the concept of a Virtual Manipulator (VM) for space robots and have shown that a space robot and the corresponding virtual manipulator give identical kinematic response. They have also shown that if the net linear momentum of the space robotic system is zero. then the location of the virtual ground remains fixed. Thus, using the virtual manipulator concept, the studies and results for ground based manipulators may be extended to space manipulator systems. Papadopoulos and Dubowsky [8] have used this concept to study the singularity of space manipulators.

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Longman, Lindberg, and Zedd [3] present a reaction wheel compensation method for attitude control of the spacecraft (or the space manipulator base). Since the reaction wheels use photo-voltaic energy, this method uses considerably less control jet fuel than a reaction jet control method. Reaction wheels, however, do not control the translational disturbances of the spacecraft. Reference [9] provides the kinematics and workspace analyses of a satellite-mounted robotic system.

Spofford and Akin [10] presented a control algorithm for the coordinated control of a free-flying redundant space manipulator. The approach combines the Jacobian pseudo-inverse and reaction compensation trajectory control methods to minimize the fuel consumption and maintain the manipulator configurations. The formulation is based on a differential approach and it uses only local information about the path.

References [1]–[10] largely discuss techniques that accomplish certain tasks without disturbing the stability of the systems. Fernandes. Gurvits, and Li [11] present a method for near optimum attitude control of space manipulator using internal motion. This formulation considers a two point boundary value problem but it does not consider the problem of the end-effector following a path. As a result. the holonomic and nonholonomic momentum conservation constraints are homogeneous and the solution for the velocity equations lie in the null space of the Jacobian matrix. This may not be true if the end-effector must follow certain trajectory.

Nakamura and Hanafusa [12] present a Ponuyagin's maximum principle based scheme to solve a set of global redundancy control problems. The method leads to optimal trajectory if the trajectory exists. Martin *et al.* [13] present a Lagrangian approach to the redundant manipulator constrained optimization problem described by an integral objective function and a set of kinematic constraints. Using a set of examples they show that there are families of locally optimal solutions to the boundary value problems which fail to be globally optimal. Local and Global optimization in redundancy resolution of robotic manipulators has also been discussed by Kazerounian and Wang [14]. For further discussions, readers may wish to examine references [1]–[14] and the references therein.

In this paper. we pnsent a global optimum path planning scheme for a redundant space robotic manipulator flying-freely in a zero gravity space with reaction jets off. *Global* here implies that the fonnulation accounts for minimization of the functional for the entire path and not for the local point. The extra degrees of freedom of the redundant manipulator enlarge the usable workspace of the manipulator and provide dexterity for extra-vehicular activity that exceeds the capacity of astronauts. Furthermore. these extra degrees of freedom may be used to optimize certain functionals and avoid collision truss work and orbit-replacement unit or other facilities on space stations. The formulation accounts for the holonomic and nonholonomic constraints arising from the momentum conservation conditions. A variational approach is used to obtain a set of differential and algebraic equations that may be solved using one of the several numerical schemes. A numerical scheme for forward integration of this system is presented. A planar redundant space manipulator consisting of three joints and a base is considered to demonstrate the feasibility of the formulation.

II. Momentum Conservation and Kinematic Conditions

In order to develop the mathematical formulation for global optimum path planning for space robotic manipulators, consider a freely floating space manipulator shown in Fig. 1. Let $\mathbf{q} = \{q_1, \ldots, q_n\}^T$ be a vector of generalized coordinates defining the configuration of the system. Here n is the number of generalized coordinates. Selection of generalized coordinates is not the major issue in this paper. However, it is worth mentioning that these coordinates must be chosen very carefully; otherwise. it may lead to mathematical singularity, the resulting code may be computationally inefficient, and in some cases it may fail to give the desired solution. In literature, one of the favorite choices for these coordinates has been to use a set of translational and rotational coordinates of the manipulator base and relative rotation (or translation) of an arm with respect to its predecessor body. Base rotational coordinates may be represented by Euler angles. Euler like angles, a set of three appropriate parameters, or the Euler parameters. If the Euler parameters arc used. then one must impose an additional Euler parameter normalization condition.

We assume that the gravity is zero in space and the thrust jets are off. Thus, there is no external force or torque acting on the system and the system is freely floating in the space. This implies that the linear and angular momentum of the system must be conserved. For simplicity in the discussion to follow. it is assumed that the linear momentum p and the angular momentum $\mathbf{L}$ of the system are zero. This is not the limitation of the formulation. Formulations for p and $\mathbf{L}$ not equal to zero may be developed on a similar line. However, in this case the formulation will be more involved. Following the above assumptions. the linear and angular momentum conservation conditions may be written as [15]

$$\mathbf{J}_1(\mathbf{q})\dot{\mathbf{q}} = 0 \tag{1}$$

$$\mathbf{J}_2(\mathbf{q})\dot{\mathbf{q}} = 0 \tag{2}$$

when $\mathbf{J}_1(\mathbf{q})$ and $\mathbf{J}_2(\mathbf{q})$ are the Jacobian matrices (of dimensions $k_1 \times n$ and $k_2 \times n$) associated with the linear momentum and the angular momentum respectively, and the period on a variable (.) denotes the total derivative of the variable (.) with respect to time t. Specific forms and the dimensions of these matrices depend on vector $\mathbf{q}$ selected and the system considered. For an example of a robot system, these matrices are demonstrated in the appendix.

Eq. (1) represents a set of holonomic conditions because its time integral may be written in a close form. We assume that the time integral is

$$\phi_1(\mathbf{q}, t) = 0 \tag{3}$$

Eq. (2) however. represents a set of nonholonomic conditions because its time integral cannot be written in a close form. Eq. (3) provides the linear momentum conditions in the configuration space. If the trajectory of the end-effector is

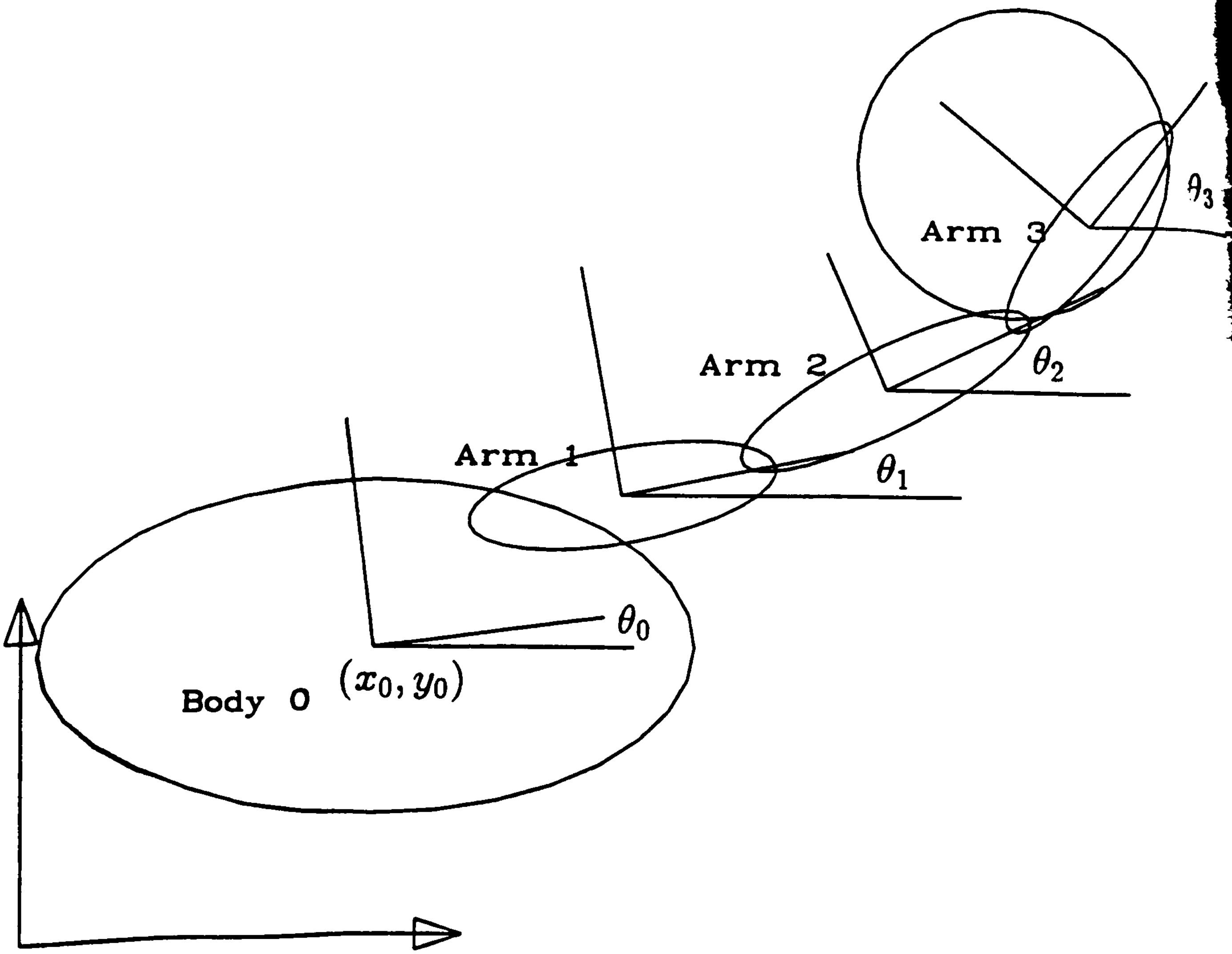

Fig. 1. Configuration of a two-dimensional space manipulator.

specified then the system must satisfy additional kinematic conditions. Let these conditions be given as

$$\phi_3(\mathbf{q}, t) = 0 \tag{4}$$

where ϕ_3 is a vector of dimension $k_3 \times 1$. The time derivative of (4) may be written as

$$\mathbf{J}_3(\mathbf{q}, t)\dot{\mathbf{q}} = \dot{\mathbf{X}}_3 \tag{5}$$

where $\mathbf{J}_3(\mathbf{q}, t) = (\partial\phi_3)/(\partial\mathbf{q})$ and $\dot{\mathbf{X}}_3 = -(\partial\phi_3)/(\partial t)$. Eqs. (1) to (5) play a significant role in the derivation to follow. For a system, these equations are demonstrated in the appendix.

Combining (1), (2) and (5), we obtain

$$\mathbf{J}(\mathbf{q}, t)\dot{\mathbf{q}} = \dot{\mathbf{X}} \tag{6}$$

where

$$\mathbf{J}(\mathbf{q}, t) = \begin{bmatrix} \mathbf{J}_1(\mathbf{q}) \\ \mathbf{J}_2(\mathbf{q}) \\ \mathbf{J}_3(\mathbf{q}, t) \end{bmatrix} \tag{7}$$

is the combined Jacobian matrix of the system of dimension $k \times n$, here $k = k_1 + k_2 + k_3$, and

$$\dot{\mathbf{X}} = \begin{bmatrix} 0 \\ 0 \\ \dot{\mathbf{X}}_3 \end{bmatrix} \tag{8}$$

For a redundant system, $\mathbf{J}$ is in general a rectangular matrix with $n > k$. The difference $(n - k)$, provides the degree of redundancy at the velocity level. Using the pseudoinverse of the Jacobian matrix, the general solution of (6) may be written as [16]

$$\dot{\mathbf{q}} = \mathbf{J}^{\#}\dot{\mathbf{X}} + (\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})\mathbf{u} \tag{9}$$

where $\mathbf{J}^{\#}$ is the pseudoinverse of the Jacobian matrix and $\mathbf{u}$ is an arbitrary unknown vector. The first part of this equation is the particular solution of (6). This is also the least square solution of the same equation. The second part of (9) is the complementary solution or the solution of the homogeneous equations [16].

From (6) and (9), the following two observations may be made: first, by setting $\mathbf{u} = 0$ in (9) one obtains only a lominimum for $\mathbf{q}$ which may or may not give a global minimum [14]. Furthermore, if $\|\dot{\mathbf{X}}\|$ is zero then (9) would lead to a trivial and undesirable solution. This condition appears if the trajectory of the end-effector is not specified. Second, if $\mathbf{X} \neq 0$ then the solution of (6) will not lie in the null space of the Jacobian matrix and the second part of (9) only will be insufficient to define the solution of (6). The present formulation directly works with (6) and therefore, it accounts for both the particular and the homogeneous solutions.

In the next section, we derive the necessary conditions for global optimum path planning for a space robotic manipulator using a variational approach [17].

III. Global Optimization Formulation

A global optimization problem may be stated as follows:

$$\text{Minimize} \qquad I_f = \int_{t_0}^{t_f} f(\mathbf{q}, \dot{\mathbf{q}}, t)dt \tag{10}$$

subject to the conditions in (6). Here I_f is the objective functional. t_0 and t_f are the initial and the final times. and f is some given function. Terminal times t_0 and t_f may be fixed or free. Some of the choices for function f considered in the past are:

$$\text{First,} \qquad f(\mathbf{q}, \dot{\mathbf{q}}, t) = \frac{1}{2}\dot{\mathbf{q}}^T\dot{\mathbf{q}}, \tag{11}$$

$$\text{Second,} \qquad f(\mathbf{q}, \dot{\mathbf{q}}, t) = \frac{1}{2}\dot{\mathbf{q}}^T A(\mathbf{q}, t)\dot{\mathbf{q}} \tag{12}$$

and

$$\text{Third,} \qquad f(\mathbf{q}, \dot{\mathbf{q}}, t) = 1. \tag{13}$$

$f = (1/2)\dot{\mathbf{q}}^T\dot{\mathbf{q}}$ represents minimization of half of the Euclidean velocity norm or the unit-mass-kinetic energy, $f = (1/2)\dot{\mathbf{q}}^T A\dot{\mathbf{q}}$ represents minimization of the weighted Euclidean velocity norm or the kinetic energy if $\mathbf{A}$ is the mass matrix, and $f = 1$ represents minimization of executing time. In many space applications. f may also represent objectives of obstacle avoidance based on CAD database of space station, or minimization of the base reaction force of the robot.

Here we consider that t_0 and t_f are fixed. Using the calculus of variations, the variation of (10) at the optimum point may be written as

$$\delta I_f = \int_{t_0}^{t_f} \left[\left(\frac{\partial f}{\partial \mathbf{q}}\right) - \frac{d}{dt}\left(\frac{\partial f}{\partial \dot{\mathbf{q}}}\right)\right]\delta\mathbf{q}(t)dt + \left[\frac{\partial f}{\partial \dot{\mathbf{q}}(t_f)}\right]\delta\mathbf{q}(t_f) - \left[\frac{\partial f}{\partial \dot{\mathbf{q}}(t_0)}\right]\delta\mathbf{q}(t_0) = 0 \tag{14}$$

where $\delta\mathbf{q}(t)$ is the virtual displacement of $\mathbf{q}(t)$. Note that the coefficient of $\delta\mathbf{q}(t)$ in (14) in the present form can not be set qual to zero. This is because all components of $\mathbf{q}$ are not independent. For virtual displacement. $\delta\mathbf{q}(t)$ (6) leads to

$$\mathbf{J}(\mathbf{q}, t)\delta\mathbf{q} = 0 \tag{15}$$

Multiplying (15) with λ^T, when λ is the vector of Lagrange multipliers. and integrating the result with respect to time from t_0 to t_f, we obtain

$$\int_{t_0}^{t_f} \lambda^T\mathbf{J}(\mathbf{q}, t)\delta\mathbf{q}dt = 0. \tag{16}$$

Combining (14) and (16) we have

$$\int_{t_0}^{t_f} \left[\frac{\partial f}{\partial \mathbf{q}} - \frac{d}{dt}\left(\frac{\partial f}{\partial \dot{\mathbf{q}}}\right) - \lambda^T\mathbf{J}\right]\delta\mathbf{q}(t)dt + \left[\frac{\partial f}{\partial \dot{\mathbf{q}}(t_f)}\right]\delta\mathbf{q}(t_f) - \left[\frac{\partial f}{\partial \dot{\mathbf{q}}(t_0)}\right]\delta\mathbf{q}(t_0) = 0 \tag{17}$$

Using (17) and following the discussion presented in [17], we obtain the following Euler-Lagrange differential equation

$$\left(\frac{\partial f}{\partial \mathbf{q}}\right)^T - \frac{d}{dt}\left(\frac{\partial f}{\partial \dot{\mathbf{q}}}\right)^T - \mathbf{J}^T\lambda = 0 \tag{18}$$

and the following terminal conditions

$$\left[\frac{\partial f}{\partial \dot{\mathbf{q}}(t_b)}\right]\delta\mathbf{q}(t_b) = 0 \qquad t_b = t_0 \text{ or } t_f. \tag{19}$$

Solution of (18) that satisfies (3), (4), and (6) along with a set of boundary conditions which are consistent with (19) yield the desired global optimum joint trajectory. Note that some of the concepts presented here are similar to the approach presented by Kazerounian and Wang [14]. Their formulation, however, does not account for the nonholonomic constraints.

The above formulation is general and applicable to a large class of function f. In particular. for f given by (12), (18) and (19) reduce to

$$A(\mathbf{q}, t)q + \dot{A}(\mathbf{q}, t)q - \frac{1}{2}\left[\frac{[illegible]}{\partial \mathbf{q}}\right]^T \mathbf{q} + \mathbf{J}^T\lambda = 0, \tag{20}$$

and

$$\dot{\mathbf{q}}^T(t_b)A(\mathbf{q}(t_0), t)\delta\mathbf{q}(t_b) = 0 \qquad t_b = t_0 \text{ or } t_f \tag{21}$$

where it is assumed that A is a symmetric mamx. If not, then (12) can be modified such that the resulting matrix A is symmetric. A special case is when A is the identity matrix. This is equivalent to considering (11) for f. In this case, (20) and (21) reduce to

$$\ddot{\mathbf{q}} + \mathbf{J}^T\lambda = 0, \tag{22}$$

and

$$\dot{\mathbf{q}}^T(t_b)\delta\mathbf{q}(t_b) = 0 \qquad t_b = t_0 \text{ or } t_f \tag{23}$$

Eq. (20) (or (22)) is a set of n second order differential equations. This set is equivalent to $2n$ first order differential equations. Alternatively, one can use Pontryagin's maximum principle, which maximizes a certain Hamiltonian function, to obtain a similar set of $2n$ first order equations. Pontryagin's maximum principle is. essentially, an extension of the variational method and it is applicable to open as well as to closed sets of input variables [18]. The calculus of variations used hen results in simpler equations which are suitable for physical interpretations as well as numerical and symbolic manipulations [14].

IV. BOUNDARY CONDITIONS

Eq. (20) provides the necessary conditions for functional I_f in (10) to be optimum, Kazerounian and Wang [14] present four different sets of boundary conditions for a similar optimization problem. Although, the results of reference [14] are applicable in this formulation also, they must be used with proper understanding because they do not include the nonholonomic conditions. For the sake of completeness, we derive the boundary conditions and outline the essential differences between reference [14] and this formulation.

In order to obtain a proper set of boundary conditions, we return to (21). (Take (23) if $A = I$). Note that the generalized coordinates are not all independent. This implies that the virtual displacements are also not all independent, i.e., all components of $\delta\mathbf{q}(t_b)$, $t_b = t_0$ or t_f, in (21) (or (23)) cannot be varied freely and they must satisfy (15) at the end points. Discussion of reference [16] cannot be applied here in a straight forward manner because of the presence of nonholonomic constraints.

The necessary natural conditions may be evaluated as follows: First, evaluate (15) for $t = t_0$, second, multiply the result with μ^T, where μ is the vector of Lagrange multipliers of dimension $k \times 1$, third, subtract the result from (21) for $t_b = t_0$, and finally repeat the procedure for $t = t_f$. This leads to

$$[\dot{\mathbf{q}}^T(t_b)A(\mathbf{q}(t_b),t) - \mu^T(t_b)\mathbf{J}(\mathbf{q}(t_b),t_b)]\delta\mathbf{q}(t_b) = 0 \qquad t_b = t_0 \text{ or } t_f \tag{24}$$

Some remarks on μ will be made in the next section. In the discussion to follow, consider that no generalized coordinate is specified at the end points. In this case the coefficient of $\delta\mathbf{q}(t_b)$, $(t_b = t_0, t_f)$ in (24). can be set equal to zero for a proper vector μ and a proper set of independent virtual displacement (see reference [17]). Therefore. for this vector μ and the independent virtual displacements. (24) after some algebra, leads to

$$\dot{\mathbf{q}}(t_b) = A^{-1}(\mathbf{q}(t_b),t_b)\mathbf{J}^T(\mathbf{q}(t_b),t_b)\mu(t_b) \qquad t_b = t_0 \text{ or } t_f \tag{25}$$

Eq. (6) should be satisfied at time t_b also. This implies

$$\mathbf{J}(\mathbf{q}(t_b),t_b)\dot{\mathbf{q}}(t_b) = \dot{\mathbf{X}}(t_b) \tag{26}$$

From (25) and (26), we obtain

$$\mu(t_b) = [\mathbf{J}A^{-1}\mathbf{J}^T]^{-1}\dot{\mathbf{X}}(t_b) \tag{27}$$

Substituting the value of μ back in (31), we obtain the generalized velocity vector at time t_b as

$$\dot{\mathbf{q}}(t_b) = A^{-1}\mathbf{J}^T[\mathbf{J}A^{-1}\mathbf{J}^T]^{-1}\dot{\mathbf{X}}(t_b) \qquad t_b = t_0 \text{ or } t_f \tag{28}$$

Eq. (28) is referred to as the *natural* boundary conditions. Taking $A = (1/2)I$ in (28) we get the same set of natural boundary conditions as that of reference [14].

We are now in a position to obtain the proper boundary conditions. Traditionally, (24) is used to obtain the following four sets of boundary conditions:

Set 1. Generalized coordinates are free at both ends: In this case the optimum trajectory must satisfy the natural boundary conditions given by (28) at both end points.

Set 2. Generalized coordinates are given at $t = t_0$ but not at $t = t_f$: In this case $\delta\mathbf{q}(t_0) = 0$ and therefore (24) for $t_b = t_0$ is automatically satisfied and the optimum trajectory must meet the natural conditions at $t = t_f$ only.

Set 3. Generalized coordinates are given at $t = t_f$ but not at $t = t_0$: This set is identical to set 2, except that the roles of t_0 and t_f have interchanged. In this case $\delta\mathbf{q}(t_f) = 0$ and (24) for $t_b = t_f$ is automatically fulfilled and the optimum trajectory needs to agree with the natural conditions at $t = t_0$ only.

Set 4. Generalized coordinates are specified at both ends: In this $\delta\mathbf{q}(t_0) = \delta\mathbf{q}(t_f) = 0$ and (24) is automatically satisfied for both $t_b = t_0$ and $t_b = t_f$.

Sets 1, 2, and 4 correspond. respectively, to cases 1, 2. and 4, of reference [14]. In the above four sets, it is assumed that at each end either all (independent) generalized coordinates are prescribed or all of them (independent generalized coordinates) are free. This might not be the case. In reality, part of the (independent) generalized coordinates may be prescribed and part of them may be free. Therefore. there are many more possible combination sets which will also satisfy the optimality criteria. More specifically, if a holonomic system has n_d degrees of freedom, then there are 2^{2n_d} possible number of *sets* which will satisfy the optimality criteria. For a nonholonomic system. as is the case here, this number depends on the number of holonomic and nonholonomic constraints in a complex way.

At this stage, it is worth emphasizing the following point: The possible number of sets of independent coordinates is ${}^{n}C_{n_d}$, where C is a combination symbol. These sets correspond to only one set in the optimality criteria discussed above because, for a given set of independent coordinates. the other coordinates are all fixed. Therefore. a given set of independent coordinates can be uniquely mapped to another set of independent coordinates.

Among all possible sets, set 1 stated above gives the minimum value for the object function, because this set imposes no restriction on the system. In practice, it is also possible to have a set of boundary conditions which do not satisfy the optimality criteria One such set is when the independent generalized coordinates and velocities are specified at time $t = t_0$. This corresponds to case 3 of reference [14]. This set results in an initial value problem. Since. in this case, the natural boundary conditions at $t = t_f$ are ignored. the resulting path is a "weak" minimum. A strategy for obtaining strong minimum for this case is given in [14].

Note that the equations satisfying the optimality criteria lead to split boundary conditions. i.e. two point boundary value problems. A close form solution of these equations is generally not possible. Furthermore, these equations can not be solved in a straight forward manner. A common approach to this problem is as follows: 1) estimate the initial conditions, 2) numerically integrate the differential and algebraic equations, 3) use the numerical results at $t = t_f$ to update the initial conditions, and 4) repeat the steps 2 and 3 until the terminal conditions are satisfied or the number of iteration exceeds a prespecified value. A numerical scheme to integrate the differential and algebraic equations appearing in this formulation is given next.

V. Numerical Integration of Differential and Algebraic Equations

Global optimum path planning formulation presented above leads to a system of Differential and Algebraic Equations (DAEs). Although, the formulation leads to a two point boundary value problem, in this section we begin with some assumed initial conditions, i.e. we take some suitable values for $\mathbf{q}(t_0)$, and $\dot{\mathbf{q}}(t_0)$. An iterative shooting method [15] may be used to improve the initial conditions so that the resulting solution satisfies the boundary conditions at both ends. The objective of this section is to present a numerical scheme to advance the solution of the DAEs from a time grid point t_i to the next time grid point t_{i+1}. For a systematic development and ease of reference, the DAEs are rewritten below:
Differential (20):

$$A(\mathbf{q},t)\ddot{\mathbf{q}} + \mathbf{J}(\mathbf{q},t)^T\lambda = F(\mathbf{q},\dot{\mathbf{q}},t) \tag{29}$$

Holonomic Constraints:

$$\phi(\mathbf{q},t) = \begin{bmatrix} \phi_1(\mathbf{q}) \\ \phi_3(\mathbf{q},t) \end{bmatrix} = \mathbf{0} \tag{30}$$

Nonholonomic Constraints along with the time derivative of (30):

$$\mathbf{J}(\mathbf{q},\mathbf{t})\mathbf{q} = \mathbf{X} \tag{31}$$

Vector $F(\mathbf{q},\dot{\mathbf{q}},t)$ in (29) is given as

$$F(\mathbf{q},\mathbf{q},t) = \frac{1}{2}\left[\frac{\partial(A\dot{\mathbf{q}})}{\partial\mathbf{q}}\right]^T \dot{\mathbf{q}} - \dot{A}(\mathbf{q},t)\dot{\mathbf{q}} \tag{32}$$

Note that the Lagrange multipliers λ's are unknown. These multipliers may be eliminated from (29) as follows: First, differentiate (31) with respect to time. This leads to

$$\mathbf{J}(\mathbf{q},\mathbf{t})\mathbf{q} = \mathbf{X} - \dot{\mathbf{J}}(\mathbf{q},t)\dot{\mathbf{q}} \tag{33}$$

Second. substitute the value of $\mathbf{q}$ from (29) into (33), and solve for A. This gives

$$\lambda = \left[JA^{-1}J^T\right]^{-1}(JA^{-1}F - \mathbf{X} + \mathbf{Jq}) \tag{34}$$

Finally, substitute the expression for λ back into (29). After some algebra. this leads to

$$\mathbf{q} = A^{-1}F - A^{-1}J\left[JA^{-1}J^T\right]^{-1}(JA^{-1}F - \mathbf{X} + \mathbf{Jq}) \tag{35}$$

Before we proceed further, note that A ((34)) at $t = to$ (or t_f) is different from $\mu(t_0)$ (or $\mu(t_f)$). In reality, λ and μ are entirely two different vectors and they should be treated so.

The right hand side of (35) contains no unknown terms. Therefore. given $\mathbf{q}(t_i)$, $\dot{\mathbf{q}}(t_i)$, and t_i, (35) may be used to compute $\ddot{\mathbf{q}}(t_i)$. Another approach to compute $\ddot{\mathbf{q}}(t_i)$ is as follows: Eqs. (29) and (33) may be written in combined form as

$$\begin{bmatrix} A & \mathbf{J}^T \\ \mathbf{J} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \ddot{\mathbf{q}} \\ \lambda \end{bmatrix} = \begin{bmatrix} F \\ \mathbf{X} - \dot{\mathbf{J}}\dot{\mathbf{q}} \end{bmatrix} \tag{36}$$

In this equation, the square mamx and the right hand vector can be computed numerically. Vectors $\ddot{\mathbf{q}}$ and λ then can be solved using a numerical scheme such as the Gaussian elimination technique. Thus, vector q may be obtained using either (35) or (36). Eq. (36) has two major advantages over (35). First, (36) preserves the sparsity of the mamces and therefore a sparse mamx code may be used to store the mamx elements and solve the resulting equations, and second, if the eigenvalues of $\mathbf{J}$ vary widely then (35) results in ill condition mamces which cause numerical instability. For the same Jacobian matrix, (36) leads to a relatively well behaved problem.

Let the solution for vector $\ddot{\mathbf{q}}$ be written symbolically as

$$\ddot{\mathbf{q}} = g(\mathbf{q},\dot{\mathbf{q}},t) \tag{37}$$

The vector function $\mathbf{g}$ is never computed explicitly but only numerically. Eq. (37) may be reduced to a set of first order equations as

$$\dot{\mathbf{y}} = \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} g(\mathbf{q},\mathbf{u},t) \\ \mathbf{u} \end{bmatrix} = g_1(\mathbf{y},t) \tag{38}$$

Eq. (38) may now be integrated using a direct integration scheme. This scheme may lead to numerical problems because the components of $\mathbf{q}$ arc not all independent due to the presence of nonlinear holonomic and nonholonomic constraints. These constraints nonlinearly transform the small numerical integration errors which act as feedback to the system causing large constraint violation and numerical instability. Furthermore, since the dimension of $\mathbf{q}$ may be much larger than the number of independent coordinates. these schemes require integration of a much larger set of differential equations.

The DAEs in this formulation are similar to those in multi-body dynamics. In recent years. several methods have appeared in the area of multibody dynamics to solve such DAEs. Some of these methods attempt to satisfy the constraints explicitly and some implicitly, and some take a hybrid approach. A brief review of these methods appear in reference [19]. From theory of differential geometry, it is clear that the solution of these DAEs lies on the manifolds defined by these constraints. It is possible to define coordinate systems on these manifolds and reduce the number of differential equations to its minimum (see reference [16] and the references therein). In this case, the resulting generalized coordinates are no longer the original set but the combination of original coordinates with time varying coefficients. In this section we shall present a numerical algorithm to solve the DAEs in terms of a set of coordinates which is a subset of the original set of generalized coordinates.

In order to discuss the numerical scheme to follow, let $\mathbf{q}_{I1}(t_i)$ and $\dot{\mathbf{q}}_{I2}(t_i)$ be the vectors of independent generalized coordinates and velocities at any time t_i. These two vectors completely define the state of the system. because they may be used to solve the dependent generalized coordinates and velocities. Note that the dimensions of these two vectors arc not the same because of the presence of nonholonomic constraints. Once these vectors have been identified, the numerical scheme to solve the DAEs may be given as follows:

Numerical Algorithm:

Step 1. Use $\mathbf{q}_{I1}(t_i)$ and (30) to solve the unknown dependent coordinate. This requires solution of a set of nonlinear equations. Newton-Raphson method and its variants may be used for this purpose.

Step 2. Use $\dot{\mathbf{q}}_{I2}(t_i)$, (31) and the vector $\mathbf{q}(t_i)$ obtained in step 1 to solve for the dependent velocities. This requires solution of a set of linear equations. Any of the many schemes such as Gaussian elimination, etc. may be used for this purpose.

Step 3. Solve for $\ddot{q}(t_i)$ using either (35) or (36) and identify vector $\ddot{\mathbf{q}}_{I2}(t_i)$ from this vector.

Step 4. Use vectors $\ddot{\mathbf{q}}_{I2}(t_i)$ and $\dot{\mathbf{q}}_{I2}(t_i)$ in an integration subroutine to obtain vectors $\dot{\mathbf{q}}_{I2}(t_{i+1})$ and $\mathbf{q}_{I1}(t_{i+1})$.

Step 5. Repeat steps 1 to 4 until final time has reached.

Steps 1 and 2 insure that the kinematic conditions are satisfied. Also note that the number of equations that needs to be integrated is much less than the number of differential equations in (38). In this approach, however, one must solve additional linear and nonlinear equations.

VI. Numerical Results

In order to demonstrate the feasibility of the formulation, we consider a two-dimensional space manipulator consisting of a base (body 0) and three arms (bodies 1, 2, and 3) as shown in Fig. 1. The link lengths are $L_1 = L_2 = 7.0$ m, and $L_3 = 4.0$ m. Initially, the inertia properties considered are $m_0 = 100$ kg., $m_1 = m_2 = 7.0$ kg., $m_3 = 4.0$ kg., $I_0 = 20.0$ (kg$-m^2$), $I_1 = I_2 = 28.583$ (kg$-m^2$), and $I_3 = 5.333$ (kg$-m^2$). The revolute joint between the base and arm 1 is considered at the center of mass of the base. The tip of the vector is required to move along a circular trajectory defined as

$$\mathbf{X} = \begin{bmatrix} x_p(t) \\ y_p(t) \end{bmatrix} = \begin{bmatrix} x_c + r\cos(\alpha) \\ y_c + r\sin(\alpha) \end{bmatrix} \quad (39)$$

where (x_c, y_c) is the center of the circle, r its radius, and α a specified time dependent parameter. The center is considered at $x_c = 11.0$ m and $y_c = 6.0$ m. and the radius $r = 3.0$ m. $\alpha(t)$ considered is

$$\alpha(t) = \frac{2\pi}{9} - \sin\left(\frac{2\pi t}{9}\right) \quad (40)$$

The objective is to find an optimal trajectory that minimizes the unit-mass-kinetic energy for a set of specified initial generalized coordinates. Also, the initial and the final velocities should be zero. The initial conditions, which are consistent with the constraints, are $x_0 = -1.0213$, $y_0 = -0.7124$, $\theta_0 = 0.0$, $\theta_1 = 0.93478$, $\theta_2 = 0.0064397$, and $\theta_3 = 0.262$, and all initial velocities are zero.

The numerical results of the system are given below. Figs. 2 to 5 show x_0 and y_0, and θ_1, and θ_3, and their time derivative as a function of time. Fig. 6 shows four configurations of the space manipulator at various times. From these figures, it is clear that the motion of the arms causes the

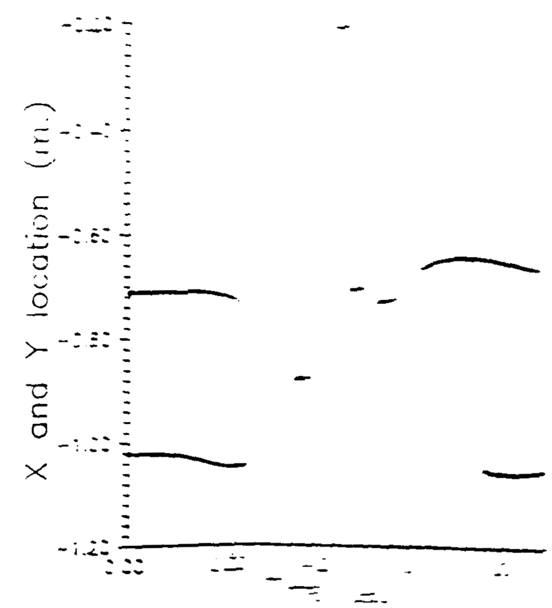

Fig. 2. T[illegible]

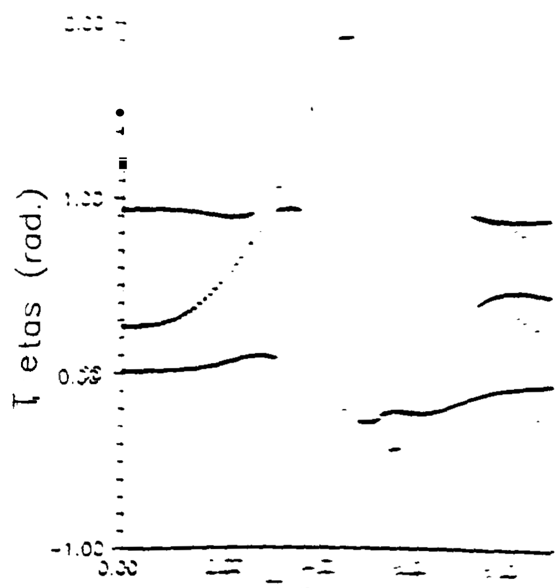

Fig. 3. Orientations [illegible] of time.

base to translate and [illegible] for this response were also [illegible]. It is observed that the [illegible] and the response of the system [illegible].

In a further investigation [illegible] each arm were increased [illegible] three times. Corresponding [illegible] $= x_0(t) - x_0(0)$, $\Delta\theta_3 =$ [illegible] in Figs. 7 to 11. It can be [illegible] response of Δx_0 and $\Delta\theta_3$ increase [illegible] response for other generalized [illegible]. This is in agreement with the [illegible]

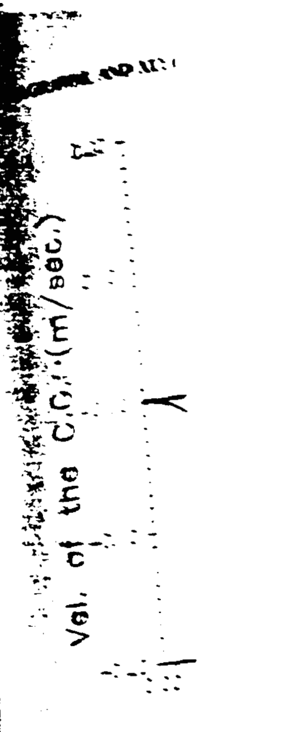

Fig. [illegible]

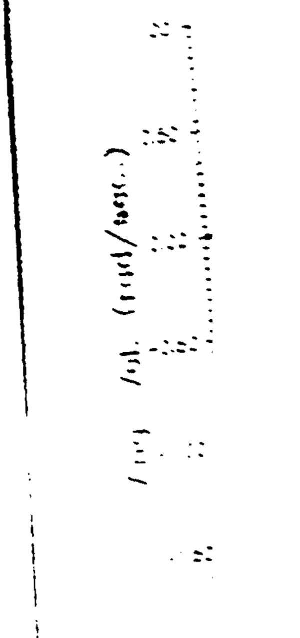

Fig. 5. [illegible]

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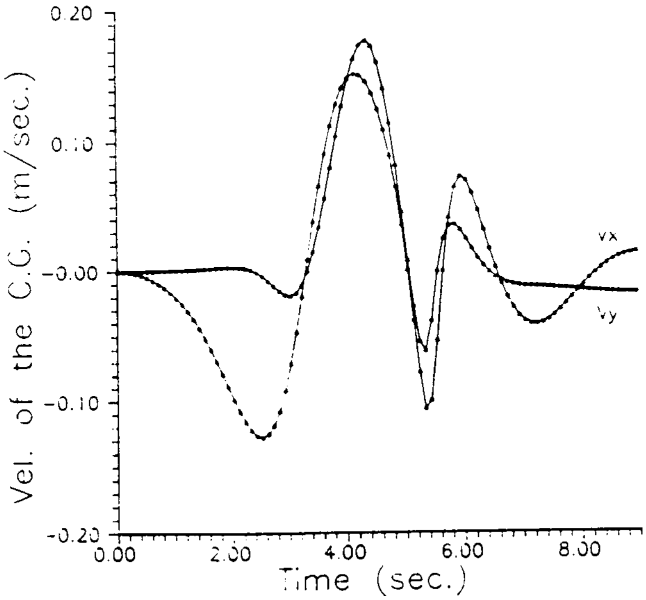

Fig. 4. Velocities ($\dot{x}$, $\dot{y}$) of the center of mass as a function of time.

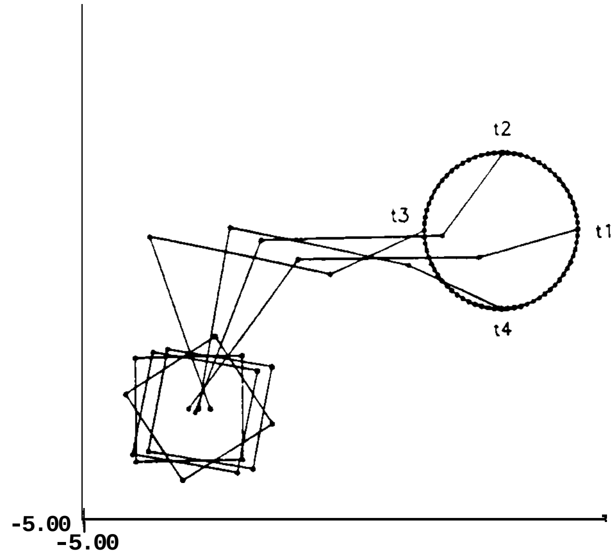

Fig. 6. Configuration of the manipulator at four different times.

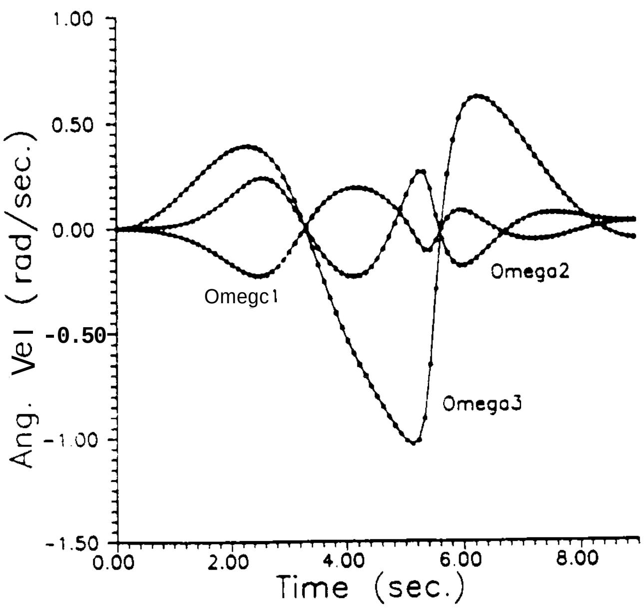

Fig. 5. Angular velocities ($\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{\theta}_3$) of the manipulator arms as a function of time.

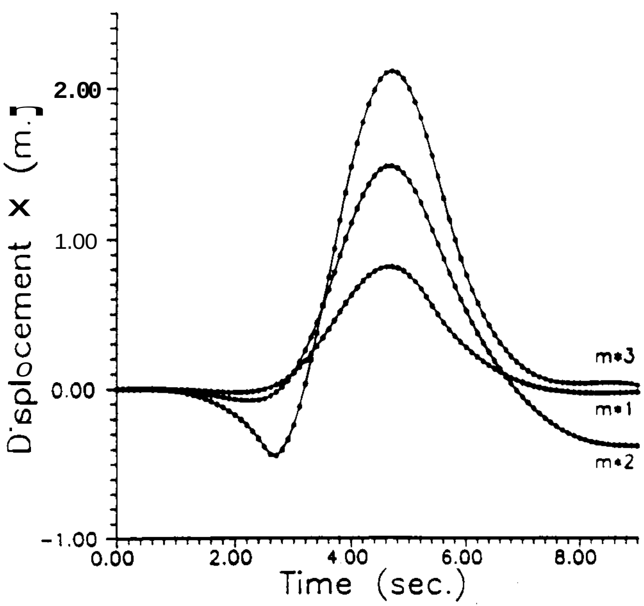

Fig. 7. Effect of the change of masses on the relative displacement Δx.

VII. CONCLUSION

An optimum path planning formulation for a free floating space robotic manipulator based on a variational approach has been presented. The formulation shows that the moment conservation conditions result in a system of holonomic and nonholonomic constraints. A method to incorporate these constraints has also been given. This leads to a system of differential and algebraic equations, and two-point boundary conditions if the initial conditions are not specified. A numerical algorithm to solve the resulting DAEs has been proposed. The formulation has been used to obtain the response of a two-dimensional redundant space robot. The numerical results show that the motion of the arms can considerably affect the response of the base. The approach is of significance in many space applications since most current space robots are redundant. This redundancy provides the necessary dexterity and enhance performance that is required for extra-vehicular activity in space stations.

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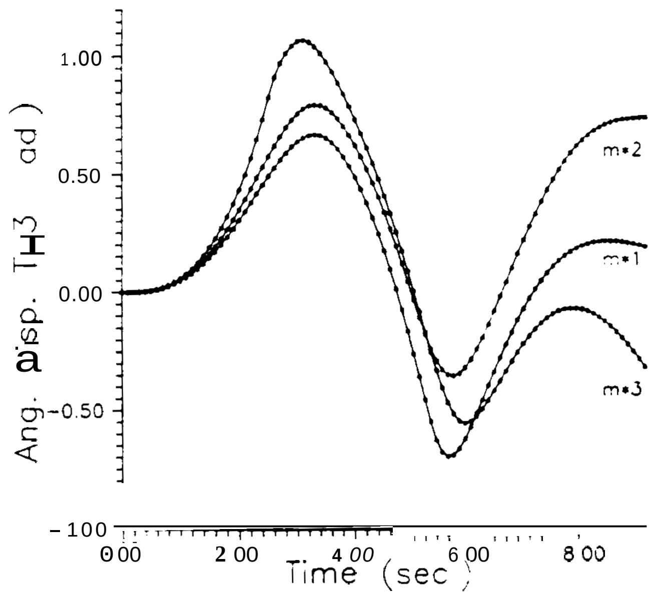

Fig. 8 Effect of the change of masses on the relative angular displacement $\Delta\theta_3$

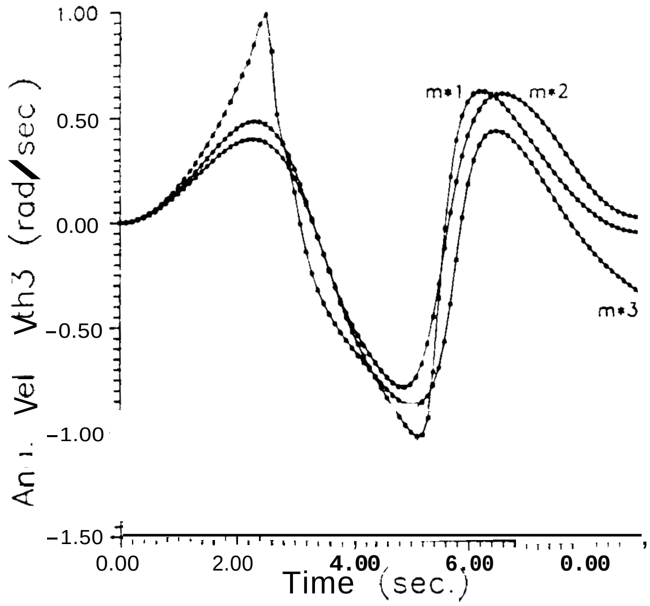

Fig. 10. Effect of the change of masses on the relative angular velocity $\Delta\dot{\theta}_3$

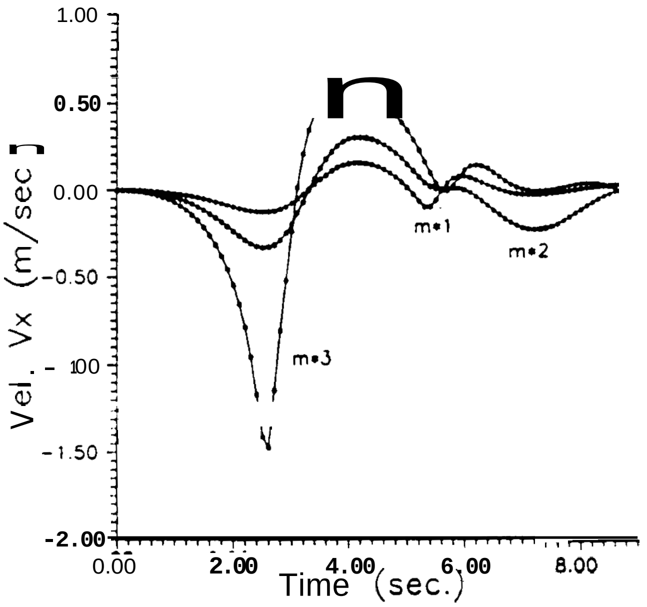

Fig. 9. Effect of the change of masses on the relative velocity $\Delta\dot{x}$.

IX. APPENDIX

Eqs. (1) to (5) are the main equations in the derivation for global optimum path planning for a redundant space robotic manipulator. Once these equations are known, the other equations may be obtained using the formulation presented in this paper. In this section we demonstrate these equations for a two dimensional space manipulator system.

In order to accomplish the above stated objective, consider the two dimensional space manipulator consisting of one base (body 0) and three arms (bodies 1, 2, and 3) as shown in Fig. 1. The configuration of this system is defined using two sets of frames, namely an inertial frame and four body frames. The origin of each body frame is rigidly attached to the center of mass of the respective body. For simplicity, it is assumed that the total linear and angular momentums of the system are zero and the origin of the inertial frame coincides with the center of mass of the overall system. For this system, the vector of generalized coordinates $\mathbf{q}$ taken here is

$$\mathbf{q} = [\theta_1 \quad \theta_2 \quad \theta_3 \quad x_0 \quad y_0 \quad \theta_0]^T \tag{A1}$$

where θ_i is the orientation of the z-axis of body frame i with respect to the z-axis of the inertial frame, and (x_0, y_0) is the location of center of mass with respect to the inertial frame. Let m_i and I_i be the mass and moment of inertia of body i and m be the total mass of the system; i.e.

$$m = m_0 + m_1 + m_2 + m_3 \tag{A2}$$

The linear and angular momentum conservation conditions for this system lead to

$$\sum_{i=0}^{3} m_i \dot{\mathbf{r}}_i = 0 \tag{A3}$$

$$\sum_{i=0}^{3} (I_i \dot{\theta}_i + m_i \mathbf{r}_i \times \dot{\mathbf{r}}_i) = 0 \tag{A4}$$

where $\mathbf{r}_i$ is the position vector of center of mass of body i, and the period on (*) denoted total time derivate of (*). Vectors $\mathbf{r}_i$ $(i = 0, 1, 2, 3)$ may be written in tenns of $\mathbf{q}$ directly from Figure 1. Substituting the expression for $\mathbf{r}_i$ in (A3) and (A4), we obtain (1) and (2). For this case. the components of matrices $\mathbf{J}_1$ and $\mathbf{J}_2$ are as follows:

$$[\mathbf{J}_1]_{11} = -\left(\frac{m_1}{2} + m_2 + m_3\right) * L_1 * \sin(\tag{A5}$$

$$[\mathbf{J}_1]_{12} = -\left(\frac{m_2}{2} + m_3\right) * L_2 * \sin(\theta_2) \tag{A6}$$

$$[\mathbf{J}_1]_{13} = -\frac{m_3}{2} * L_3 * \sin(\theta_3) \tag{A7}$$

$$[\mathbf{J}_1]_{14} = m \tag{A8}$$

$$[\mathbf{J}_1]_{15} = 0 \tag{A9}$$

$$[\mathbf{J}_1]_{16} = (m - m_0) * (-\xi * \sin(\theta_0) - \eta * \cos(\theta_0)) \tag{A10}$$

$$[\mathbf{J}_1]_{21} = \left(\frac{m_1}{2} + m_2 + m_3\right) * L_1 * \cos(\theta_1) \tag{A11}$$

$$[\mathbf{J}_1]_{22} = \left(\frac{m_2}{2} + m_3\right) * L_2 * \cos(\theta_2) \tag{A12}$$

$$[\mathbf{J}_1]_{23} = \frac{m_3}{2} * L_3 * \cos(\theta_3) \tag{A13}$$

$$[\mathbf{J}_1]_{24} = 0 \tag{A14}$$

$$[\mathbf{J}_1]_{25} = m \tag{A15}$$

$$[\mathbf{J}_1]_{26} = (m - m_0) * (\xi * \cos(\theta_0) - \eta * \sin(\theta_0)) \tag{A16}$$

$$\begin{aligned}[\mathbf{J}_2]_{11} = I_1 &+ \left(\frac{m_1}{2} + m_2 + m_3\right) L_1 (x_0 \cos(\theta_1) \\ &+ y_0 \sin(\theta_1) + \xi \cos(\theta_1 - \theta_0) \\ &+ \eta \sin(\theta_1 - \theta_0)) + \left(\frac{m_1}{4} + m_2 + m_3\right) L_1^2 \\ &+ \left(\frac{m_2}{2} + m_3\right) L_1 L_2 \cos(\theta_1 - \theta_2) \\ &+ \frac{m_3}{2} L_1 L_3 \cos(\theta_1 - \theta_3)\end{aligned} \tag{A17}$$

$$\begin{aligned}[\mathbf{J}_2]_{12} = I_2 &+ \left(\frac{m_2}{2} + m_3\right) L_2 (x_0 \cos(\theta_2) \\ &+ y_0 \sin(\theta_2) + \xi \cos(\theta_2 - \theta_0) + \eta \sin(\theta_2 - \theta_0)) \\ &+ \left(\frac{m_2}{2} + m_3\right) L_1 L_2 \cos(\theta_2 - \theta_1) \\ &+ \left(\frac{m_2}{4} + m_3\right) L_2^2 + \frac{m_3}{2} L_2 L_3 \cos(\theta_2 - \theta_3)\end{aligned} \tag{A18}$$

$$\begin{aligned}[\mathbf{J}_2]_{13} = I_3 &+ \frac{m_3}{2} L_3 \Big((x_0 \cos(\theta_3) + y_0 \sin(\theta_3) \\ &+ \xi \cos(\theta_3 - \theta_0) \\ &+ \eta \sin(\theta_3 - \theta_0)\Big) + L_1 \cos(\theta_3 - \theta_1) \\ &+ L_2 \cos(\theta_3 - \theta_2) + \frac{1}{2} L_3\Big)\end{aligned} \tag{A19}$$

$$\begin{aligned}[\mathbf{J}_2]_{14} = &-m y_0 - (m - m_0)(\xi \sin(\theta_0) + \eta \cos(\theta_0)) \\ &- \left(\frac{m_1}{2} + m_2 + m_3\right) L_1 \sin(\theta_1) \\ &- \left(\frac{m_2}{2} + m_3\right) L_2 \sin(\theta_2) \\ &- \frac{m_3}{2} L_3 \sin(\theta_3)\end{aligned} \tag{A20}$$

$$\begin{aligned}[\mathbf{J}_2]_{15} = &\, m x_0 + (m - m_0)(\xi \cos(\theta_0) - \eta \sin(\theta_0)) \\ &+ \left(\frac{m_1}{2} + m_2 + m_3\right) L_1 \cos(\theta_1) \\ &+ \left(\frac{m_2}{2} + m_3\right) L_2 \cos(\theta_2) \\ &+ \frac{m_3}{2} L_3 \cos(\theta_3)\end{aligned} \tag{A21}$$

$$\begin{aligned}[\mathbf{J}_2]_{16} = I_0 &+ (m - m_0)(\xi^2 + \eta^2) + ((m - m_0) x_0 \\ &+ \left(\frac{m_1}{2} + m_2 + m_3\right) L_1 \\ &+ \left(\frac{m_2}{2} + m_3\right) L_2 \cos(\theta_2) \\ &+ \frac{m_3}{2} L_3 \cos(\theta_3))(\xi \cos(\theta_0) - \eta \sin(\theta_0)) \\ &+ ((m - m_0) y_0 + \left(\frac{m_1}{2} + m_2 + m_3\right) L_1 \sin(\theta_1) \\ &+ \left(\frac{m_2}{2} + m_3\right) L_2 \sin(\theta_2) \\ &+ \frac{m_3}{2} L_3 \sin(\theta_3))(\xi \sin(\theta_0) + \eta \cos(\theta_0))\end{aligned} \tag{A22}$$

Eqs. (A20) and (A21) follow from linear momentum conservation conditions. Integrating (A3) and using the fact that the center of mass of the system lies at (0,0), we obtain (3) as

$$\phi_1(\mathbf{q}) = \begin{bmatrix}\phi_{11}(\mathbf{q}) \\ \phi_{12}(\mathbf{q})\end{bmatrix} = \begin{bmatrix}0 \\ 0\end{bmatrix} \tag{A23}$$

where functions $\phi_{11}(\mathbf{q})$ and $\phi_{12}(\mathbf{q})$ are given as

$$\begin{aligned}\phi_{11}(\mathbf{q}) &= m x_0 + (m - m_0)(\xi \cos(\theta_0) - \eta \sin(\theta_0)) \\ &\quad + \left(\frac{m_1}{2} + m_2 + m_3\right) L_1 \cos(\theta_1) \\ &\quad + \left(\frac{m_2}{2} + m_3\right) L_2 \cos(\theta_2) + \frac{m_3}{2} L_3 \cos(\theta_3) \\ &= 0\end{aligned} \tag{A24}$$

$$\begin{aligned}\phi_{12}(\mathbf{q}) &= m y_0 + (m - m_0)(\xi \sin(\theta_0) + \eta \cos(\theta_0)) \\ &\quad + \left(\frac{m_1}{2} + m_2 + m_3\right) L_1 \sin(\theta_1) \\ &\quad + \left(\frac{m_2}{2} + m_3\right) L_2 \sin(\theta_2) + \frac{m_3}{2} L_3 \sin(\theta_3) \\ &= 0\end{aligned} \tag{A25}$$

Eqs. (A24) and (A25) lead to $[\mathbf{J}_2]_{14} = [\mathbf{J}_2]_{14} = 0$. Let $\mathbf{X}_3 = [x_p(t) \quad y_p(t)]^T$ be the specified trajectory of the end effector. Then the trajectory constraint (4) is given as

$$\phi_3(\mathbf{q}, t) = \begin{bmatrix}\phi_{31}(\mathbf{q}, t) \\ \phi_{32}(\mathbf{q}, t)\end{bmatrix} = \begin{bmatrix}0 \\ 0\end{bmatrix} \tag{A26}$$

where functions $\phi_{31}(\mathbf{q}, t)$ and $\phi_{32}(\mathbf{q}, t)$ are given by the following equations:

$$\begin{aligned}\phi_{31}(\mathbf{q}, t) &= x_0 + \xi * \cos(\theta_0) - \eta * \sin(\theta_0) + L_1 * \cos(\theta_1) \\ &\quad + L_2 * \cos(\theta_2) + L_3 * \cos(\theta_3) - x_p(t) \\ &= 0\end{aligned} \tag{A27}$$

$$\begin{aligned}\phi_{32}(\mathbf{q}, t) &= y_0 + \xi * \sin(\theta_0) + \eta * \cos(\theta_0) + L_1 * \sin(\theta_1) \\ &\quad + L_2 * \sin(\theta_2) + L_3 * \sin(\theta_3) - y_p(t)\end{aligned} \tag{A28}$$

In this study, the trajectory is a circle specified as

$$\mathbf{X}_3 = \begin{bmatrix}x_p(t) \\ y_p(t)\end{bmatrix} = \begin{bmatrix}x_c + r \cos(\alpha(t)) \\ y_c + r \sin(\alpha(t))\end{bmatrix} \tag{A29}$$

where (x_c, y_c) and r are, respectively, the coordinate of the center and the radius of the circle, and $\alpha(t)$ is a specified time dependent parameter. In this study, $\alpha(t)$ is considered as

$$\alpha(t) = \frac{2\pi}{9} - \sin\left(\frac{2\pi t}{9}\right) \tag{A30}$$

Time derivative of (A27) and (A28) gives (5). The components of the Jacobian matrix $\mathbf{J}_3$ are as follows:

$$[\mathbf{J}_3]_{11} = -L_1 * \sin(\theta_1) \tag{A31}$$

$$[\mathbf{J}_3]_{12} = -L_2 * \sin(\theta_2) \tag{A32}$$

$$[\mathbf{J}_3]_{13} = -L_3 * \sin(\theta_3) \tag{A33}$$

$$[\mathbf{J}_3]_{14} = 1 \tag{A34}$$

$$[\mathbf{J}_3]_{15} = 0 \tag{A35}$$

$$[\mathbf{J}_3]_{16} = -\xi * \sin(\theta_0) - \eta * \cos(\theta_0) \tag{A36}$$

$$[\mathbf{J}_3]_{21} = L_1 * \cos(\theta_1) \tag{A37}$$

$$[\mathbf{J}_3]_{22} = L_2 * \cos(\theta_2) \tag{A38}$$

$$[\mathbf{J}_3]_{23} = L_3 * \cos(\theta_3) \tag{A39}$$

$$[\mathbf{J}_3]_{24} = 0 \tag{A40}$$

$$[\mathbf{J}_3]_{25} = 1 \tag{A41}$$

$$[\mathbf{J}_3]_{26} = \xi * \cos(\theta_0) - \eta * \sin(\theta_0) \tag{A42}$$

and the vector $\mathbf{X}_3$ is given as

$$\mathbf{X}_3 = \frac{2\pi}{9}\left(1 - \cos\left(\frac{2\pi t}{9}\right)\right)\begin{bmatrix} -r\sin(\alpha) \\ r\cos(\alpha) \end{bmatrix} \tag{A43}$$

Thus, (1) to (5) are known for the system.

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