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Variable structure model reference adaptive control of robot manipulators

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Abstract

By making use of all known parameters in the robot manipulator model and introducing variable structure control laws to adjust the remaining unknown parameters, a novel model reference adaptive control system is developed for robot motion control. With the new method applied, the tracking errors of the joint variables converge to zero irrespective of whether there is persistent excitation, the transient response can be prescribed in advance, and performance is basically not affected by input disturbances. The superior properties are demonstrated by a case study.

1 Introduction

The model-based nonlinear scheme popularly known as the computed torque method [1] has been used effectively to control robot manipulators. The requirements for successfully implementing the computed torque method are fast computation of the dynamic model and perfect knowledge of the actual system. This paper is devoted to solving the practical problem in the face of imprecise knowledge of system parameters.

Adaptive schemes as effective means to control complex systems have been applied to robot manipulators for years, based largely on either classical model reference adaptive control (MRAC) [2] or self-tuning regulator (STR) [3] theory. These methods are hence useful to slow motion only. The assumptions or approximations employed in the adaptive methods, such as local linearization, time-invariance, and decoupled dynamics, have been relaxed by some researchers. Slotine and Li [4] use a modified Lyapunov function with a modified output error when all dynamic parameters are estimated by identification technique. Craig et al. [5] take advantage of all known parameters and estimate only the remaining unknown parameters. The crucial issues which remain in all these adaptive schemes are relatively slow response, persistent excitation requirement and undesirable sensitivity to disturbance.

Variable structure control (VSC) provides an alternative to robot control with unknown parameters [6]. The main advantage of VSC is its robustness to input disturbances once the sliding surface is reached [7], but it seems that not enough attention has been paid to achieving fast response.

In this paper an adaptive control scheme combining the variable structure and model reference methods is presented. Using the variable structure technique, all known parameters of the robot system are fully used while the unknown parameters are adaptively adjusted. The overall control system maintains the basic structure of the computed torque controller, but incorporates adaptive components in the system. The method removes the requirement for persistent excitation, essential to traditional adaptive schemes for satisfactory operation. The proposed control algorithm ensures robustness of the controlled system with respect to disturbance, since the tracking error will always converge to zero theoretically, rather than to an ill-defined residual set as found in other adaptive schemes. Using the proposed method, the transient response can be prescribed in advance. Thus all the outstanding issues in adaptive control are directly treated. Simulation analysis for a two degree-of-freedom robot is conducted to compare the proposed method with the classical model reference method and computed torque method.

2 Computed torque control

We consider a robot manipulator with n degrees of

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freedom described by the following compact dynamic equation [1]:

$$\mathbf{T} = \mathbf{M}(\theta)\ddot{\theta} + \mathbf{Q}(\theta,\dot{\theta}) \qquad (1)$$

where T is the $n\times 1$ vector of joint forces or torques supplied by the actuators, and θ is the $n\times 1$ vector of joint variables. The matrix $\mathbf{M}(\theta)$ is an $n\times n$ manipulator inertia matrix. The vector $\mathbf{Q}(\theta,\dot{\theta})$ represents generalized forces arising from centrifugal, Coriolis, gravitational and frictional effects.

The *i*th element of equation (1) can be expressed as

$$\tau_i = \sum_{j=1}^{a_j} m_{ij} f_{ij}(\theta,\ddot{\theta}) + \sum_{j=1}^{b_j} q_{ij} g_{ij}(\theta,\dot{\theta}), \qquad (2)$$

where m_{ij} and q_{ij} are parameters involving link masses, link inertia constants, link lengths, friction coefficients and the gravitational acceleration constant. $f_{ij}(\theta,\ddot{\theta})$ and $g_{ij}(\theta,\dot{\theta})$ are functions of the joint variables and their derivatives. The structure of these functions is assumed known while the parameters may or may not be unknown. The uncertainty bounds on the parameter values are, however, assumed known.

To control the manipulators, we use the control law

$$\mathbf{T} = \hat{\mathbf{M}}(\theta)\ddot{\theta}^* + \hat{\mathbf{Q}}(\theta,\dot{\theta}) \qquad (3)$$

where $\hat{\mathbf{M}}(\theta)$ and $\hat{\mathbf{Q}}(\theta,\dot{\theta})$ are the estimates of $\mathbf{M}(\theta)$ and $\mathbf{Q}(\theta,\dot{\theta})$, and

$$\ddot{\theta}^* = \ddot{\theta}_d + \mathbf{K}_v\dot{\mathbf{E}} + \mathbf{K}_p\mathbf{E} \qquad (4)$$

where $\mathbf{E}$ is the servo error defined by

$$\mathbf{E} = \theta_d - \theta,\ \mathbf{E} = [e_1\ e_2\ \ldots e_n]^T \qquad (5)$$

$\mathbf{K}_v$ and $\mathbf{K}_p$ are $n\times n$ constant diagonal gain matrices with k_{vi} and k_{pi} on the diagonals. The controller (3) is that of the so-called computed torque method. The desired trajectory of the manipulator is described by known time functions of joint positions, velocities, and accelerations, $\theta_d(t)$, $\dot{\theta}_d(t)$ and $\ddot{\theta}_d(t)$.

When the estimates of parameters do not match the true parameter values, the error equation of the whole system becomes

$$\ddot{\mathbf{E}} + \mathbf{K}_v\dot{\mathbf{E}} + \mathbf{K}_p\mathbf{E} = \hat{\mathbf{M}}^{-1}(\theta)\,[\tilde{\mathbf{M}}(\theta)\ddot{\theta} + \tilde{\mathbf{Q}}(\theta,\dot{\theta})] \qquad (6)$$

where $\tilde{\mathbf{M}}(\theta) = \mathbf{M}(\theta) - \hat{\mathbf{M}}(\theta)$ and $\tilde{\mathbf{Q}}(\theta,\dot{\theta}) = \mathbf{Q}(\theta,\dot{\theta}) - \hat{\mathbf{Q}}(\theta,\dot{\theta})$ represent the mismatch errors between the estimated and true parameters. This may be rewritten in the form [5]

$$\ddot{\mathbf{E}} + \mathbf{K}_v\dot{\mathbf{E}} + \mathbf{K}_p\mathbf{E} = \hat{\mathbf{M}}^{-1}(\theta)\ \mathbf{W}(\theta,\dot{\theta},\ddot{\theta})\ \Phi \qquad (7)$$

where Φ is the $r\times 1$ vector of parameter errors defined by

$$\Phi = \mathbf{F} - \hat{\mathbf{F}} \qquad (8)$$

where $\mathbf{F} = [f_1 f_2 \ldots f_r]^T$ is the combination of all unknown r parameters in the system and $\hat{\mathbf{F}}$ is the estimate of $\mathbf{F}$,($\mathbf{W}(\theta,\dot{\theta},\ddot{\theta})$ being an $n\times r$ matrix of functions satisfying $\mathbf{W}(\theta,\dot{\theta},\ddot{\theta})\ \Phi = \tilde{\mathbf{M}}(\theta)\ddot{\theta} + \tilde{\mathbf{Q}}(\theta,\dot{\theta})$.)

Dropping all arguments of $\hat{\mathbf{M}}^{-1}$ and $\mathbf{W}$ thereafter, the error equation for the *i*th joint is

$$\ddot{e}_i + k_{vi}\dot{e}_i + k_{pi}e_i = (\hat{\mathbf{M}}^{-1}\ \mathbf{W}\ \Phi)_i \qquad (9)$$

where $(.)_i$ means the *i*th element of the $n\times 1$ vector $\hat{\mathbf{M}}^{-1}\ \mathbf{W}\ \Phi$.

In the following analysis $\hat{\mathbf{M}}^{-1}\ \mathbf{W}$ is assumed to be bounded all the time. Since $\mathbf{W}$ consists of bounded functions of the manipulator trajectory, it will remain bounded if the trajectory is bounded. The matrix $\hat{\mathbf{M}}^{-1}$ will remain positive definite and invertible if we ensure that all its parameters remain within a sufficiently small range near the actual parameters.

3 An MRAC algorithm for robot control

The form of adaptation is based on an MRAC algorithm proposed by Craig et al. [5]. The adaptive laws determine how to change the parameter estimates as a function of a filtered servo error signal. The filtered servo error $\mathbf{E}_1$ is defined as

$$\mathbf{E}_1 = \dot{\mathbf{E}} + \Psi\mathbf{E} \qquad (10)$$

where $\Psi = \mathrm{diag}(\psi_1\psi_2\ldots\psi_n)$ with $\psi_i > 0$. ψ_i is so chosen that the transfer function

$$G(f) = \frac{s+\psi_i}{s^2 + k_{vi}s + k_{pi}} \qquad (11)$$

is strictly positive real (SPR).

The filtered state-space error equation of the entire system is

$$\dot{X} = AX + B(\hat{M}^{-1} W \Phi)$$
$$E_1 = C^T X \tag{12}$$

where **A**,**B**, and **C** are all block diagonal (with A_i, B_i and C_i on the diagonals, respectively) and $X = [X_1 X_2 \ldots X_n]^T$, $X_i = [e_i, \dot{e}_i]^T$.

From the positive real lemma [2] there exist positive matrices P_i and Q_i, such that

$$A_i^T P_i + P_i A_i = -Q_i$$
$$P_i B_i = C_i \tag{13}$$

In order to design the adaptive laws, the Lyapunov function candidate is chosen as

$$v(X) = X^T P X + \Phi^T \Gamma^{-1} \Phi \tag{14}$$

with $\Gamma = \mathrm{diag}(\gamma_1 \gamma_2 \ldots \gamma_r)$, γ_i being nonnegative.

By specially choosing

$$\dot{\Phi} = -\Gamma W^T \hat{M}^{-1} E_1 \tag{15}$$

we can ensure that

$$\dot{v}(X) = -X^T Q X \leq 0 \tag{16}$$

The adaptive scheme is stable in the sense that all signals remain bounded and trajectory tracking errors **E** and $\dot{E}$ converge to zero [5]. Without persistent excitation, estimated parameters may not converge to their true values.The results are the same as those in conventional MRAC schemes [2]. Even if the desired trajectory is so chosen that the persistent excitation condition is satisfied, a relatively long period of adaptation is in general required in order to identify all the parameters.

4 New VS-MRAC algorithm

In order to remove the requirement for persistent excitation and to speed up the convergence rate to minimise tracking error, a new variable structure model reference adaptive control law is developed as follows.

The Lyapunov function candidate

$$v(X) = X^T P X \tag{17}$$

is non-negative in servo error. Its derivative can be computed by using (12) to give

$$\dot{v}(X) = -X^T Q X - 2\hat{F}^T(W^T \hat{M}^{-1} E_1) + 2F^T(W^T \hat{M}^{-1} E_1) \tag{18}$$

since $\Phi = F - \hat{F}$.

If we specifically choose the switching law

$$\hat{f}_i = f_i^* \mathrm{sign}(W^T \hat{M}^{-1} E_1)_i \ , i=1,\ldots r \tag{19}$$

where $f_i^* > |f_i|$, then it can be shown that $\dot{v}(X) < 0$. It is well known that with **X** governed by (17) and $\dot{v}(X) < 0$, $||X||$ will decrease at least exponentially to zero [8]. This is an important difference compared to the conventional adaptive methods [2,3]. In fact by using the above VS-MRAC algorithm, the exponential stability of the system is achieved independently of the excitation. In other words, persistent excitation is no longer required. The rate of convergence of $||X||$ to zero will be affected only by the minimum eigenvalue of $P^{-1}Q$ which is independent of the excitation [14]. Since (**P**,**Q**) is directly related to **A**, it is expected that the system transient is dictated by **A** in some way. The present approach is a practical extension of the method discussed in [8] intended for SISO systems only.

5 Transient analysis

We further study the transient analysis of robot motion control by using the above nonlinear-model-based VS-MRAC algorithm.

We will show that $E_1(t)$ can go to zero in a finite time t_2 if

$$||\hat{M}^{-1} W|| > c > 0, \forall t > t_1 \tag{20}$$

for an arbitrary positive constant c, and $t_2 > t_1$.

From (12) we obtain

$$E_1^T \dot{E}_1 = E_1^T C^T A X + E_1^T C^T B(\hat{M}^{-1} W \Phi)$$
$$= E_1^T C^T A X + E_1^T C^T B \hat{M}^{-1} W F - E_1^T C^T B \hat{M}^{-1} W \hat{F}$$
$$\leq ||E_1|| [k_1 ||X|| - k_2 ||\hat{M}^{-1} W||] \tag{21}$$

where $k_1, k_2 > 0$ are appropriate constants.

Provided that (20) is satisfied, since $||X||$ tends to zero exponentially, there exists $t_2 > t_1$, such that

$$\mathbf{E}_1^T\dot{\mathbf{E}}_1 < 0, \qquad \forall t > t_2 \tag{22}$$

With reference to [7], therefore, for $t > t_2$, $\mathbf{z} = \mathbf{E}_1(t) = \mathbf{C}^T\mathbf{X} = \mathbf{0}$ is guaranteed to be a sliding surface. The sliding surface will be reached in a finite time.

$$\begin{aligned}\dot{e}_{1i} &= \mathbf{C}_i^T\dot{\mathbf{X}}_i \\ &= \mathbf{C}_i^T\mathbf{A}_i\mathbf{X}_i + \mathbf{C}_i^T\mathbf{B}_i(\hat{\mathbf{M}}^{-1}\,\mathbf{W}\,\Phi)_i\end{aligned} \tag{23}$$

Since $||\mathbf{X}||$ tends to zero, and in view of (20), there exists $k_3 > 0$ such that

$$|\dot{e}_{1i}| > k_3, \quad \forall t > t_3 \tag{24}$$

(22) and (24) describe the transient property and suggest that $\mathbf{E}_1(t)$ goes to zero in a finite time and then a sliding mode starts on the surface $\mathbf{E}_1 = \mathbf{0}$.

6 Robustness to bounded disturbances

Recently robustness to disturbances has been extensively studied. It can been shown that for linear plants with some unstructured uncertainty, a loss of stability or unbounded parameter estimates could result.

Craig et al. [5] show that the presence of bounded disturbances will not result in loss of stability or unbounded estimates whenever a priori bounds on the parameter values are assumed and "resetting" rules are implemented as part of the adaptive algorithm. However, there are some adverse effects on the adaptive scheme. The servo errors do not converge asymptotically to zero, but rather, converge to a bounded residual set. Parameter estimates can only converge to another residual set if a certain persistent excitation condition, which it is difficult to state in terms of the desired trajectory, can be satisfied.

To analyse the effect of disturbances, we now assume that the modelling error has two components, one due to parameter mismatch Φ and the other due to disturbances $\mathbf{d}'$. Therefore,

$$\tilde{\mathbf{M}}(\theta)\ddot{\theta} + \tilde{\mathbf{Q}}(\theta,\dot{\theta}) = \mathbf{W}(\theta,\dot{\theta},\ddot{\theta})\,\Phi + \mathbf{d}' \tag{25}$$

where the $n \times 1$ vector function $\mathbf{d}'$ is in general unknown but bounded.

We modify the nonlinear-model-based controller to

$$\mathbf{T} = \hat{\mathbf{M}}(\theta)(\ddot{\theta}^* + \mathbf{T}_0) + \hat{\mathbf{Q}}(\theta,\dot{\theta}) \tag{26}$$

where $\ddot{\theta}^*$ is defined by (4) and $\mathbf{T}_0$ is a $n \times 1$ control input vector, to be defined later.

The system error equations become

$$\begin{aligned}\dot{\mathbf{X}} &= \mathbf{A}\mathbf{X} + \mathbf{B}(\hat{\mathbf{M}}^{-1}\,\mathbf{W}\,\Phi) + \mathbf{B}\mathbf{T}_0 + \mathbf{d} \\ \mathbf{E}_1 &= \mathbf{C}^T\mathbf{X}\end{aligned} \tag{27}$$

where $\mathbf{d} = \mathbf{B}\,\hat{\mathbf{M}}^{-1}\mathbf{d}'$ is a $2n \times 1$ bounded vector, i.e.

$$||\mathbf{d}(t)|| < d_{max} \tag{28}$$

Choosing the same Lyapunov function as before,

$$v(\mathbf{X}) = \mathbf{X}^T\mathbf{P}\mathbf{X} \tag{29}$$

and taking by design

$$\hat{f}_i = f_i^*\mathrm{sign}(\mathbf{W}^T\,\hat{\mathbf{M}}^{-1}\,\mathbf{E}_1)_i\ ,i=1,\ldots r \tag{30}$$

$$\mathbf{T}_{0j} = -d^*\mathrm{sign}(e_{1j}),\ j=1,\ldots n \tag{31}$$

where d^* is a constant, we obtain

$$\dot{v}(\mathbf{X}) < -\mathbf{X}^T\mathbf{Q}\mathbf{X} + \mathbf{E}_1^T\mathbf{T}_0 + \mathbf{X}^T\mathbf{P}\mathbf{d} \tag{32}$$

Similar to (19), we choose $f_i^* > |f_i|$.

Since $\mathbf{X}^T\mathbf{P}\mathbf{d} < c_1 d_{max}||\mathbf{X}||$, where c_1 is an appropriate constant, we further choose

$$d^* > c_2 > c_1 d_{max}||\mathbf{X}||(||\mathbf{E}_1||)^{-1} \tag{33}$$

so that from (32), $\dot{v}(\mathbf{X}) < 0$.

From (27) we can obtain

$$\dot{\mathbf{E}}_1 = \mathbf{C}^T\mathbf{A}\mathbf{X} + \mathbf{C}^T\mathbf{B}(\hat{\mathbf{M}}^{-1}\,\mathbf{W}\,\Phi) + \mathbf{C}^T\mathbf{B}\,\mathbf{T}_0 + \mathbf{C}^T\mathbf{d} \tag{34}$$

and

$$\begin{aligned}\mathbf{E}_1^T\dot{\mathbf{E}}_1 &= \mathbf{E}_1^T\mathbf{C}^T\mathbf{A}\mathbf{X} + \mathbf{E}_1^T\mathbf{C}^T\mathbf{B}\,\hat{\mathbf{M}}^{-1}\,\mathbf{W}\,\mathbf{F} \\ &\quad - \mathbf{E}_1^T\mathbf{C}^T\mathbf{B}\,\hat{\mathbf{M}}^{-1}\,\mathbf{W}\,\hat{\mathbf{F}} + \mathbf{E}_1^T\mathbf{C}^T\mathbf{B}\,\mathbf{T}_0 + \mathbf{E}_1^T\mathbf{C}^T\mathbf{d} \\ &\le ||\mathbf{E}_1||[k_1||\mathbf{X}|| - k_2||\hat{\mathbf{M}}^{-1}\,\mathbf{W}|| - k_3 + k_4 d_{max}]\end{aligned} \tag{35}$$

It has been shown that $||\mathbf{X}||$ tends to zero exponentially. Hence, if in addition we choose a constant $c = (k_4 d_{max} - k_3)/k_2$ such that

$$||\hat{\mathbf{M}}^{-1}\,\mathbf{W}|| > c,\ \forall t > t_1 \quad \text{for some finite } t_1, \tag{36}$$

then there exists $t_2 \ge t_1$ such that

$$\mathbf{E}_1^T\dot{\mathbf{E}}_1 < 0 \qquad \text{for all } t > t_2 \tag{37}$$

Hence, for $t > t_2$, $\mathbf{z} = \mathbf{E}_1(t) = \mathbf{C}^T\mathbf{X} = \mathbf{0}$ is guaranteed to

be a sliding surface.

We can also conclude from (35) and (36) that $|\dot{e}_{1i}| > k_3 > 0$, $\forall t > t_3$. Therefore again, $\mathbf{E}_1(t)$ goes to zero in a finite time and then a sliding mode starts on the surface $\mathbf{E}_1 = \mathbf{0}$. Robustness to disturbances is achieved after the sliding surface is reached. Satisfactory transient response is thus ensured.

7 Case study

A case study is presented to show the computation of the proposed algorithm and its feasibility in robot motion control subject to unknown system parameters and strong disturbance. By comparing the simulation results using the proposed method (VS-MRAC) with the computed torque method (CTM) and the classical model reference adaptive control (MRAC), we will show that the VS-MRAC method provides in general a fast response, and a high degree of robustness. The computation of the method is not expensive since all known parameters are fully used.

For simplicity, we use a two-DOF revolute manipulator with the link lengths denoted by l_1 and l_2. A lumped-parameter model is assumed with point masses m_1 and m_2 at the ends of each link. The gravity term is considered, and the viscous damping coefficients v_i and the Coulomb friction coefficients k_i are modelled.

The link lengths and gravitational constant g are known. The unknown parameters to be considered are masses of links and load, viscous damping coefficients, and Coulomb friction constants. Thus

$$[f_1\ f_2\ f_3\ f_4\ f_5\ f_6] = [m_1\ m_2\ k_1\ v_1\ k_2\ v_2]$$

and the $\mathbf{W}$ matrix governed by $\mathbf{W}\,\Phi = \tilde{\mathbf{M}}\,\ddot{\theta} + \tilde{\mathbf{Q}}$ is

$$\mathbf{W} = \begin{bmatrix} W_{11} & W_{12} & W_{13} & W_{14} & 0 & 0 \\ 0 & W_{22} & 0 & 0 & W_{25} & W_{26} \end{bmatrix}$$

where

$$W_{11} = l_1^2\ddot{\theta}_1 + l_1 g s_1$$

$$W_{12} = (l_1^2 + l_2^2 + 2l_1 l_2 c_2)\ddot{\theta}_1 + (l_2^2 + l_1 l_2 c_2)\ddot{\theta}_2 + l_1 g s_1 + l_2 g s_{12} - l_1 l_2 s_2 \dot{\theta}_2^2 - 2l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2$$

$$W_{13} = \mathrm{sign}(\dot{\theta}_1)$$

$$W_{14} = \dot{\theta}_1$$

$$W_{22} = (l_2^2 + l_1 l_2 c_2)\ddot{\theta}_1 + l_2^2\ddot{\theta}_2 + l_1 l_2 s_2 \dot{\theta}_1^2 + l_2 g s_{12}$$

$$W_{25} = \mathrm{sign}(\dot{\theta}_2)$$

$$W_{26} = \dot{\theta}_2$$

To compare the proposed VS-MRAC method with the computed torque method (CTM) and basic MRAC method, we use the following common set of conditions:

$$\theta_{1d} = 0.1(\sin(t) + \sin(2t) + 2)$$

$$\theta_{2d} = 0.1(\cos(4t) + \cos(6t) + 2)$$

$$k_v = 20$$

$$k_p = 100$$

$$\mathbf{E}_1 = \dot{\mathbf{E}} + 5\,\mathbf{E}$$

In order to make the comparison more favorable to the basic MRAC method, a persistent excitation is provided deliberately by θ_{1d} and θ_{2d}.

We first consider the ideal case without any input disturbance. When complete knowledge of the system is available and hence perfect compensation can be provided by the CTM, the CTM response shown in Fig.1(a) and (b) is extremely satisfactory. The VS-MRAC transient response is not much inferior and is distinctly faster than the basic MRAC response.

When a disturbance in the form of 2*RND(0,1) is introduced to the input of the second joint torque, the error responses appear as shown in Fig.2. The steady-state offset becomes evident in the CTM but the VS-MRAC method appears unsusceptible at all to the disturbance. The basic MRAC scheme takes a long time to converge, whether there is disturbance or otherwise, although it exhibits stability generally.

8 Conclusion

In this paper, a new adaptive control scheme is proposed. It makes full use of all known parameters and adaptively adjusts the remaining unknown parameters using variable structure model reference adaptive control laws. The method does not rely on persistent excitation and generally improves the convergence condition compared with other existing methods. The transient performance of the control scheme is studied for the first time, as applied to the adaptive control of robot manipulators. The tracking errors of robot motion will decay exponentially in theory and converge to zero instead of to an ill-defined residual set, even under bounded disturbance input conditions.

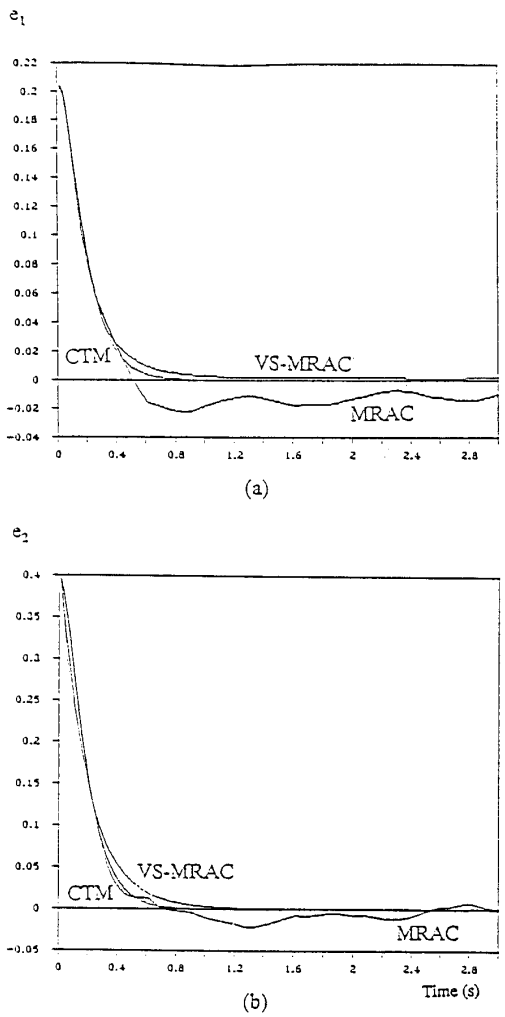

Fig.1 Transient response comparison

(a) $\theta_{1d} - \theta_1$ (b) $\theta_{2d} - \theta_2$

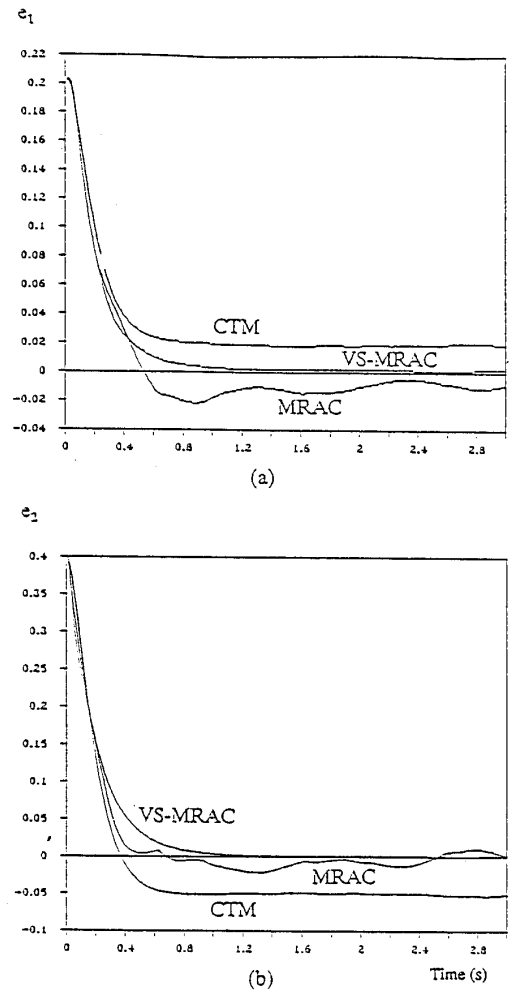

Fig.2 Transient response with disturbance

(a) $\theta_{1d} - \theta_1$ (b) $\theta_{2d} - \theta_2$

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