Impulsive Manipulation

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Abstract

In this paper, we examine a little-studied method of manipulation — manipulation by striking an object and letting it slide. There are two parts to this problem: *The Inverse Sliding Problem*, determining the velocities required to send an object to a desired configuration, and *The Impact Problem*, determining how to strike the object in order to achieve those velocities. We will present a solution to these two problems for the class of rotationally symmetric objects and conclude with some observations about this method of manipulation.

1 Introduction

Most robots today manipulate an object by grasping it, moving, and then releasing the object. A few robots make use of pushing strategies to manipulate objects. However, there has been very little exploration of another method of manipulation, striking an object and letting it slide until it comes to rest—a method of manipulation which is not uncommon in our daily lives.

Imagine a two dimensional world in which a robot manipulates objects using this method. The robot strikes the object, which gives it some initial translational and angular velocity. The object then slides, slowing down due to Coulomb friction, until it comes to rest.

The planning problem for a robot in this world consists of two parts:

1. *The Inverse Sliding Problem*: What initial translational and rotational velocities are required in order to send an object to a desired final configuration?

2. *The Impact Problem*: How can the robot strike the object in order to generate those velocities?

After reviewing some related work, we will present solutions to these two problems for the class of rotationally symmetric objects. We will close with some discussion on our observations about this method of manipulation.

2 Related Work

In regard to the Inverse Sliding Problem, Voyenli and Eriksen have examined the motion of a homogeneous circular ring or disk sliding on a plane under the action of Coulomb friction in [4]. They make some interesting observations regarding the evolution of this system in state space which we build upon in this paper.

In [1], Goyal et al. formulate a limit surface representation which is useful for visualizing the relationship between the force and torque due to friction and the translational and angular velocities of an object.

The solution to the Impact Problem makes straightforward use of Routh's graphical method for computing the impulse resulting from a collision. See [5] for a discussion of Routh's method and related issues.

In regard to potential applications of this method of manipulation, see [2]. In this paper, Higuchi presents work using an electromagnetic coil to deliver an impulse to an object in order to do linear micropositioning. His technique could be extended to do micropositioning in position and orientation using the analysis in this paper.

3 The Inverse Sliding Problem

The Inverse Sliding Problem can be formally stated as follows:

A rigid planar object of known geometry, support distribution, and frictional properties slides on a uniform surface, slowing down and coming to rest only due to Coulomb friction. Given an initial configuration (position and orientation) of the object, the problem is to determine the initial translational and angular velocities required in order for it to come to rest at a desired final configuration.

This paper addresses the class of rotationally symmetric objects — those objects whose pressure and mass distribution are invariant with respect to rotation. Rotationally symmetric objects, such as rings or discs, have the property that the net force due to friction is:

- independent of orientation
- always parallel to the translational velocity

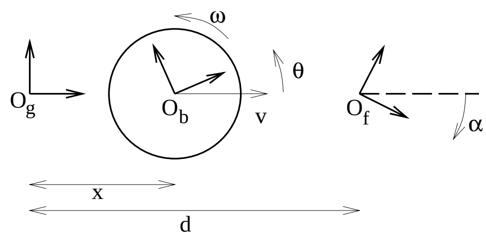

Figure 1: Notation: The frames $\mathcal{O}_g$ and $\mathcal{O}_f$ are the initial and final configurations for the object. The body coordinate frame $\mathcal{O}_b$ translates and rotates with the object as it travels from $\mathcal{O}_g$ to $\mathcal{O}_f$. The object's position and orientation are specified by x and θ (with respect to $\mathcal{O}_g$), its velocities by v and ω. The final configuration can be expressed as a distance d and an orientation α with respect to $\mathcal{O}_g$.

Though the second property may seem obvious, it is only true for rotationally symmetric objects. Non-rotationally symmetric objects will generally have some component of the net frictional force perpendicular to the velocity. See section 5.1 for further discussion.

3.1 Notation

Given an initial and final configuration, place the global coordinate frame $\mathcal{O}_g$ at the center of mass (COM) of the object in the initial configuration, so that its x axis points to the COM of the object in the final configuration.

Attach the body frame $\mathcal{O}_b$ to the object so that it coincides with the global frame $\mathcal{O}_g$ in the initial configuration. Place the final coordinate frame $\mathcal{O}_f$ so that it coincides with the body frame when the object is in the final configuration. See figure 1.

Since the net frictional force will be parallel to the translational velocity of the object, the COM of the object will travel in a straight line along the x axis of the global frame. Therefore, the state of the object can be represented by its position along the x axis, its orientation, θ with respect to the global frame, and its velocities, $v = \dot{x}$ and $\omega = \dot{\theta}$.

The final configuration can be represented as translation of d along the x axis and an orientation of α with respect to the global frame.

The object is of mass M and has a moment of inertia I about the COM.

3.2 Preliminaries

The velocity of a point $\vec{r}$ on the object is given by:

$$\vec{u} = v\hat{x} + \omega\hat{z} \times \vec{r} \tag{1}$$

The direction of this velocity is:

$$\hat{u} = \frac{v\hat{x} + \omega\hat{z} \times \vec{r}}{|v\hat{x} + \omega\hat{z} \times \vec{r}|} = \frac{\hat{x} + \frac{\omega}{v}\hat{z} \times \vec{r}}{|\hat{x} + \frac{\omega}{v}\hat{z} \times \vec{r}|} \tag{2}$$

Note that the direction of this velocity is dependent only upon the the ratio of angular velocity to translational velocity, $\frac{\omega}{v}$, and the vector $\vec{r}$.

The differential force due to friction at this point (adopting the convention that $-\vec{df}$ is the force that acts on this point) is:

$$\vec{df} = \mu\, dm\, g\, \hat{u} \tag{3}$$

Since the net force will lie along the x axis, we can write:

$$F(\tfrac{\omega}{v}) = \left|\int \vec{df}\right| = \left|g \int \mu\, \hat{u}\, dm\right| \tag{4}$$

The net torque is given by:

$$T(\tfrac{\omega}{v}) = \left|\int \vec{r} \times \vec{df}\right| = \left|g \int \mu\, \vec{r} \times \hat{u}\, dm\right| \tag{5}$$

The equations of motion that govern this system are:

$$M\dot{v} = -F(\tfrac{\omega}{v}) \tag{6}$$
$$I\dot{\omega} = -T(\tfrac{\omega}{v}) \tag{7}$$
$$\dot{x} = v \tag{8}$$
$$\dot{\theta} = \omega \tag{9}$$

3.3 Monotonicity of Force and Torque

As noted in the previous subsection, the net force and torque are functions of only $\frac{\omega}{v}$. Mason [3] has shown that the net force is strictly monotonic decreasing and the net torque is strictly monotonic increasing (with respect to $\frac{\omega}{v}$).

Intuitively, we can see this from the following argument. The magnitude of the frictional force at any given point is fixed; its direction is determined by the ratio $\frac{\omega}{v}$. For a fixed velocity v, when $\omega = 0$ ($\frac{\omega}{v} = 0$), all the frictional forces oppose translation, so there will be maximum force and zero torque. As ω increases ($\frac{\omega}{v}$ increases), the frictional forces will increasingly oppose the rotational motion. This results in increasing torque and decreasing force.

3.4 More Notation

We now turn our attention to the total distance and angle the object traverses. Let the total distance and angle traveled by the object be denoted by x_f and θ_f, which will be functions of the initial translational and angular velocities, v_0 and ω_0. Mathematically, this is:

$$x_f(v_0, \omega_0) = \int_0^{t_f} v\, dt \tag{10}$$

$$\theta_f(v_0, \omega_0) = \int_0^{t_f} \omega\, dt \tag{11}$$

where $v(t)$ and $\omega(t)$ are solutions to the equations of motion, subject to the initial conditions:

$$v(0) = v_0 \qquad \omega(0) = \omega_0 \tag{12}$$

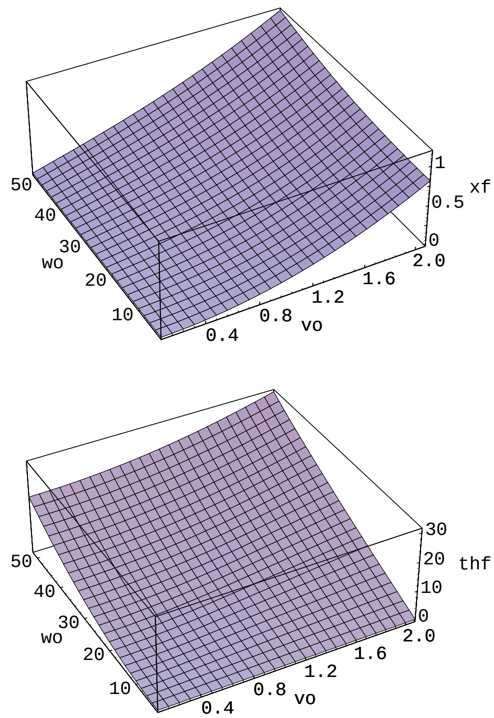

Figure 2: Graphs of x_f and θ_f for a disc of radius 0.05 meters and mass 0.1 kg; the coefficient of friction is 0.25. v_0 is in meters/sec, ω_0 in radians/sec, x_f in meters, and θ_f in radians.

and where t_f is defined as the time such that:

$$v(t_f) = 0 \qquad\qquad \omega(t_f) = 0 \tag{13}$$

Although x_f and θ_f cannot be computed analytically, we can deduce certain properties about them.

3.5 Monotonicity of x_f and θ_f

The functions $x_f(v_0, \omega_0)$ and $\theta_f(v_0, \omega_0)$ are monotonic in v_0 and ω_0. If we pick some point (v_0, ω_0) (where $v_0 > 0$ and $\omega_0 > 0$), the function x_f increases as we increase v_0, holding ω_0 constant, and vice versa. The same statement applies to the function θ_f. See figure 2 for an example of x_f and θ_f.

Intuitively, if you give the object a greater initial velocity it will slide further. This means that it will take more time for the object to come to rest, so it will also rotate more. Similarly, if you give the object a greater initial angular velocity, it will rotate more and also slide further.

More formally, we have:

$$v_{0_1} > v_{0_2},\ \omega_{0_1} = \omega_{0_2} \implies x_{f_1} > x_{f_2} \tag{14}$$
$$v_{0_1} > v_{0_2},\ \omega_{0_1} = \omega_{0_2} \implies \theta_{f_1} > \theta_{f_2} \tag{15}$$
$$v_{0_1} = v_{0_2},\ \omega_{0_1} > \omega_{0_2} \implies x_{f_1} > x_{f_2} \tag{16}$$
$$v_{0_1} = v_{0_2},\ \omega_{0_1} > \omega_{0_2} \implies \theta_{f_1} > \theta_{f_2} \tag{17}$$

We can prove this by reasoning about a pair of trajectories. If for some t_1 we have

$$v_1 > v_2 \qquad\qquad \omega_1 > \omega_2 \tag{18}$$

then for all $t > t_1$,

$$v_1 \geq v_2 \qquad\qquad \omega_1 \geq \omega_2 \tag{19}$$

The only way that equation 19 could be violated is if $v_1(t)$ crossed $v_2(t)$ or $\omega_1(t)$ crossed $\omega_2(t)$. Both of these crossings cannot happen simultaneously because the derivatives $\dot{v}$ and $\dot{\omega}$ are uniquely defined by the equations of motion. However it could be possible for $v_1(t)$ to cross $v_2(t)$ while $\omega_1(t) > \omega_2(t)$, or vice versa.

But in order for $v_1(t)$ to cross $v_2(t)$, at some point we must have $v_1 = v_2$. Since $\omega_1 > \omega_2$, we know that $\frac{\omega_1}{v_1} > \frac{\omega_2}{v_2}$. From monotonicity of $F(\frac{\omega}{v})$, we know that

$$F(\tfrac{\omega_1}{v_1}) < F(\tfrac{\omega_2}{v_2}) \tag{20}$$

This means there will be more force to decelerate object 2, so we will once again have the condition that $v_1(t) > v_2(t)$.

Similarly, if we consider $\omega_1(t)$ crossing $\omega_2(t)$ while $v_1 > v_2$, at the instant that $\omega_1 = \omega_2$, there will be more torque to decelerate the rotation of object 2, so we will once again have the condition that $\omega_1(t) > \omega_2(t)$.

When $v_{0_1} > v_{0_2}$ and $\omega_{0_1} = \omega_{0_2}$, we know that at $t = 0^+$, we will have $v_1 > v_2$ and $\omega_1 > \omega_2$. For all time after this, we will have $v_1 \geq v_2$ and $\omega_1 \geq \omega_2$. These inequalities, taken altogether, imply:

$$\int_0^\infty v_1(t)\,dt > \int_0^\infty v_2(t)\,dt \tag{21}$$
$$\int_0^\infty \omega_1(t)\,dt > \int_0^\infty \omega_2(t)\,dt \tag{22}$$

or

$$x_f(v_{0_1}, \omega_{0_1}) > x_f(v_{0_2}, \omega_{0_2}) \tag{23}$$
$$\theta_f(v_{0_1}, \omega_{0_1}) > \theta_f(v_{0_2}, \omega_{0_2}) \tag{24}$$

which proves equations 14 and 15. Similar reasoning can be employed to prove equations 16 and 17.

3.6 Level Curves & Solutions

In addition to knowing that the functions x_f and θ_f are monotonic increasing, we can easily determine their values when $v_0 = 0$ or $\omega_0 = 0$. For these cases, the coupling between translational and rotational motion disappears, the problem is reduced to elementary physics, and we get:

$$x_f(0, \omega_0) = 0 \tag{25}$$
$$\theta_f(v_0, 0) = 0 \tag{26}$$
$$x_f(v_0, 0) = \frac{\frac{1}{2} M v_0^2}{\mu M g} \tag{27}$$
$$\theta_f(0, \omega_0) = \frac{\frac{1}{2} I \omega_0^2}{\mu g \int |\vec{r}| dm} \tag{28}$$

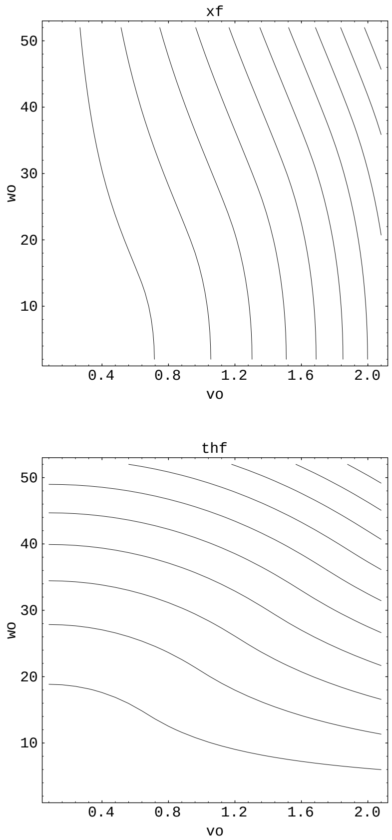

Figure 3: Level curves of x_f and θ_f from figure 2.

Although we cannot solve for level curves of x_f and θ_f algebraically, we can deduce some facts about them from monotonicity and from equations 25 – 28. To find a level curve of x_f at a height d, we start on the v_0 axis. There, we can solve $d = x_f(v_{0_d}, 0)$ for v_{0_d} using equation 27. We know that x_f increases as we move upwards, parallel to the ω_0 axis, and since x_f is 0 along the ω_0 axis and is increasing along the positive v_0 axis, the level curve must lie to the left of the line $v_0 = v_{0_d}$. Furthermore, we also can deduce that this curve, given by $d = x_f(v_{0_d}, 0)$, is monotonic in the sense that as its v_0 coordinate decreases, its ω_0 coordinate increases. See figure 3.

By similar reasoning, we can deduce that the level curve of θ_f at some height α must lie below the line $\omega_0 = \omega_{0_\alpha}$ where we can solve the equation $\alpha = \theta_f(0, \omega_{0_\alpha})$ for ω_{0_α} using equation 28.

These two level curves must cross somewhere in the region below the line $\omega_0 = \omega_{0_\alpha}$ and to the left of the line $v_0 = v_{0_d}$. The coordinates of this intersection point are the desired initial conditions to achieve a displacement of d and a rotation of α.

We can find the coordinates of this intersection point by employing a variation on bisection. For any point, we can do forward integration of the equations of motion. Comparing x_f with d and θ_f with α tells us whether this point is above or below each of the level curves. Using this information, we can eliminate certain regions of $\omega_0 v_0$ space.

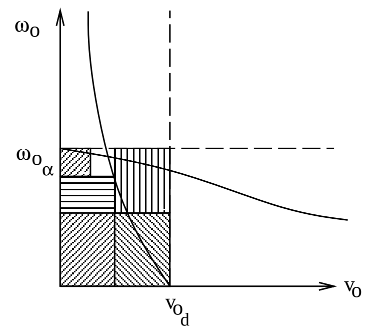

Figure 4: Determining the Initial Conditions: We can find the coordinates of the intersection point of the two level curves by employing a variation on bisection. The coordinates of this point correspond to the required initial conditions to traverse a distance d and an angle α.

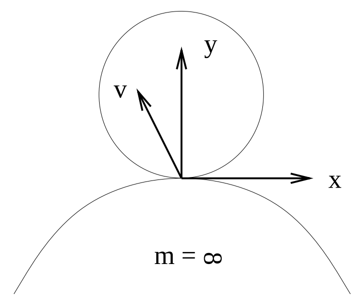

Figure 5: Notation for the Impact Problem. A striker of infinite mass strikes the object (initially at rest) with some velocity $\vec{v}$. We attach a coordinate frame to the point of impact such that the x axis is tangential to the surfaces and the y axis normal.

By repeating this process, we can zero in on the coordinates of the intersection point. See figure 4. Like bisection, convergence is linear.

4 The Impact Problem

Once we know the initial velocity and moment required, we want to determine whether we can attain those initial conditions by striking the object.

Assume we use a striker of infinite mass to strike the object. See figure 5. The object starts at rest, and the striker has some velocity $\vec{v}$. We attach a coordinate frame to the point of impact so that its x axis is tangential to the surfaces at the impact point and so its y axis is normal.

Using Routh's method for analyzing collisions, as described in [5], we have calculated that the condition on the impulse we can achieve is:

$$|P_x| \leq \mu P_y \tag{29}$$

where P_x and P_y are the impulses generated along the x and y axes, and μ is the coefficient of friction between the striker and the object. The velocities of the striker required to generate that impulse are:

$$v_{s_x} = \left(\frac{1}{m} + \frac{r^2}{I}\right) P_x \tag{30}$$

$$v_{s_y} = \frac{1}{m(1+e)} P_y \tag{31}$$

where m, r, and I are the mass, radius, and moment of inertia of the object, and e is the coefficient of restitution between the object and the striker.

This impulse will give the object its initial velocities:

$$v_{0_x} = \frac{P_x}{m} \tag{32}$$

$$v_{0_y} = \frac{P_y}{m} \tag{33}$$

$$\omega_0 = \frac{P_x r}{I} \tag{34}$$

We can transform these equations into a constraint in the $\omega_0 v_0$ plane. The desired initial angular velocity will determine P_x by equation 34. P_x in turn determines v_{0_x} (equation 32), so we can write:

$$v_{0_x} = \frac{\omega_0 I}{rm} \tag{35}$$

Combining equations 29, 32, and 33, we get:

$$v_{0_y} \geq \frac{|v_{0_x}|}{\mu} \tag{36}$$

So, the total velocity is:

$$v_0 = \sqrt{v_{0_x}^2 + v_{0_y}^2} \geq |v_{0_x}| \sqrt{1 + \left(\frac{1}{\mu}\right)^2} \tag{37}$$

Substituting from equation 35, we get:

$$v_0 \geq |\omega_0| \frac{I}{rm} \sqrt{1 + \left(\frac{1}{\mu}\right)^2} \tag{38}$$

This equation describes a region in $\omega_0 v_0$ space bounded by two lines whose slopes are only dependent upon the parameters of the object and the friction between the striker and the object. See figure 6 for an illustration.

Because x_f and θ_f cannot be expressed analytically, we cannot translate this constraint directly into a relation between x_f and θ_f. However, it does show us what regions of $\omega_0 v_0$ space are accessible, so we could modify our search for initial conditions (to achieve a desired x_f and θ_f) accordingly.

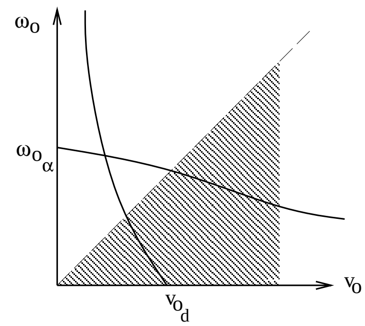

Figure 6: Impact constraint in $\omega_0 v_0$ space ($\omega_0 \geq 0$).

5 Discussion

5.1 Non-rotationally Symmetric Objects

The Inverse Sliding Problem becomes more difficult for the case of non-rotationally symmetric objects. This version of the problem is of a higher dimension than the rotationally symmetric case. More importantly, the monotonicity properties that were used to solve the rotationally symmetric case do not carry over to the non-rotationally symmetric case.

However, in simulations of a 2D barbell (a massless rigid rod with a point masses at each end), we have observed that the net frictional force perpendicular to the velocity tends to be very small and has a correspondingly small effect on the total distance and angle traversed. In order to develop more insight into the non-rotationally symmetric case, we have begun some physical experiments.

5.2 Separation of Task and Actuator Mechanics

By grasping an object, a robot can exert total control over that object. The price of that control is that the kinematics and dynamics of the robot are altered by holding that object.

A robot that strikes an object in order to manipulate it does not exert total control over the object; the object is left to evolve according to the task mechanics of sliding. Consequently, we need to consider the mechanics of the actuator only when solving the Impact Problem, not the Inverse Sliding Problem.

5.3 The Effect of Error

With the mechanics of this method of manipulation in hand, we can examine the effects of errors in our actions and in the object parameters.

By tracing through the Inverse Sliding Problem, we can translate bounds on μ (still assuming μ is constant) into bounds on x_f and θ_f.

Errors in the direction and magnitude of the velocity of the striker are more complicated to analyze but can also be translated into bounds on x_f and θ_f. However, in this case, the direction of the initial translational velocity of the object will also vary, increasing the range of possible final configurations.

For these two types of errors, the resulting bounds on x_f and θ_f are proportional to the magnitude of x_f and θ_f, respectively. Since x_f and θ_f increase at least quadratically with ω_0 and v_0, the smaller the amount of energy transferred to the object, the smaller the error.

5.4 On Performable Tasks

Given the mechanics of this world and how errors propagate through these mechanics, we can determine what tasks are performable by a robot using this method of manipulation.

For a robot using a single strike to accomplish a positioning task, there are three answers to the question of performability:

1. *No*: The desired final configuration requires an initial translational and angular velocity that we cannot achieve by striking the object.
2. *Maybe*: There are two cases.
   (a) The bound of the error is greater than the tolerance of the task.
   (b) The bounds on the initial translational and angular velocity straddle the division between achievable and unachievable velocities.
3. *Yes*: The bound of the error is less than or equal to the tolerance of the task.

One manipulation strategy in this world is to use one object to strike another object. However, it is not clear whether this adds any capabilities to a robot in this world. We suspect it does not because the game of pool would be much easier to play if you could shoot balls into the pockets directly, without using the cue ball.

Multiple step strategies are another dimension to consider. If there were no errors, we believe that multiple step strategies would not add to the capabilities of a robot in this world, except that all configurations could then be reached in two steps.

However, if there are errors, the error can be minimized by using a sequence of smaller impacts, as noted in the previous subsection. Now, there is a different set of three answers to the question of performability:

1. *Maybe*: Multiple steps do not guarantee that the task will ever be completed.
2. *Eventually*: We will eventually complete the task, but the number of steps is not bounded, perhaps because of an error governed by a probability distribution.
3. *Yes*: The task can be done in a bounded number of steps.

6 Summary

In this paper, we have examined a little-explored method of manipulation—manipulation by striking an object and letting it slide. We have presented a solution to the Inverse Sliding Problem and the Impact Problem for the class of rotationally symmetric objects. Although the Inverse Sliding Problem cannot be solved analytically, we have described a method for computing what initial velocities are required for an object to slide to a given final configuration. The Impact Problem revealed that there are certain initial velocities that cannot be achieved by striking the object.

We discussed how some types of errors and how manipulation strategies affect what tasks are performable by a robot using this method of manipulation. Some open questions in this area are the case of non-rotationally symmetric objects and the effect of allowing inter-object collisions. We have begun some physical experiments in order to gain further insight into this domain.

Acknowledgements

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