An Exploration of Nonprehensile Two-Palm Manipulation

Michael Erdmann
School of Computer Science and the Robotics Institute
Carnegie Mellon University
5000 Forbes Avenue
Pittsburgh, PA 15213-3891

1 Introduction

This paper describes our current research into *nonprehensile palm manipulation*. The term "palm" refers to the use of the entire device surface during manipulation, as opposed to use of the fingertips alone. The term "nonprehensile" means that the palms hold the object without wrapping themselves around it, as distinguished from a force/form closure grasp often employed by a fingered hand. Indeed, nonprehensile operations such as purposeful sliding and constrained dropping constitute important palm primitives.

We have implemented a system for orienting parts using two palms. The system consists of a planner and an executive. As input, the system expects a geometric description of a part, its center of mass, the coefficients of friction between the part and each of the palms, and a start and goal configuration of the part in stable contact with one of the palms. As output, the system computes and executes a sequence of palm motions designed to reorient the part from the specified start to the specified goal configuration.

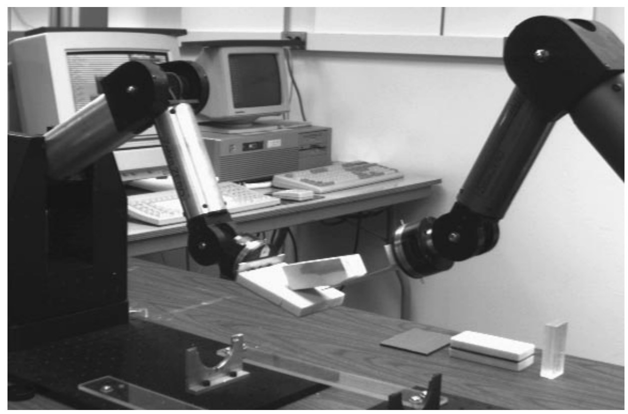

Figure 1: The experimental setup consists of two Zebra robots, with flat palms attached to each wrist. The Zebras are shown manipulating an irregular block.

1.1 System Specifics

For hardware we are using two Zebra Zero robots. We replaced the standard grippers on these arms with two flat surfaces; these are the "palms". Figure 1 shows the experimental setup. We used three different materials for the surfaces of these palms, namely aluminum, aluminum covered with duct tape, and aluminum covered with a hard impact-absorbing foam. These three surfaces provided us with a range of friction coefficients.

The system expects polyhedral parts that are extruded convex polygonal shapes. The actual parts oriented consisted of several rectangular blocks as well as one irregularly-shaped block. The irregular block's two-dimensional cross-section consisted of a five-sided polygon with three stable edges, one unstable edge, and one marginally stable edge.[1] Again, we used a variety of materials for the parts, namely hard foam, hard plastic, and wood.

The planner is planar, that is, it assumes that the parts have two translational and one rotational degrees of freedom. Hence the 3D parts oriented were extruded 2D shapes. The current version of the planner focuses on Type-B contacts, to use the classification of [Lozano-Pérez 1983, Section V]. In other words, the planner focuses on contacts involving the palm surfaces but not the boundaries of those surfaces. Viewed in 2D, this class of contacts includes both vertex and line contacts of the part with the palm lines. However, it excludes Type-A contacts, that is, again viewed in 2D, it excludes contacts between a line segment of the part and a vertex of the palms. Such contacts arise in the typical "point finger contact" problems that have been studied extensively

[1] An edge is "stable" if the part can rest horizontally on the edge without tipping over.

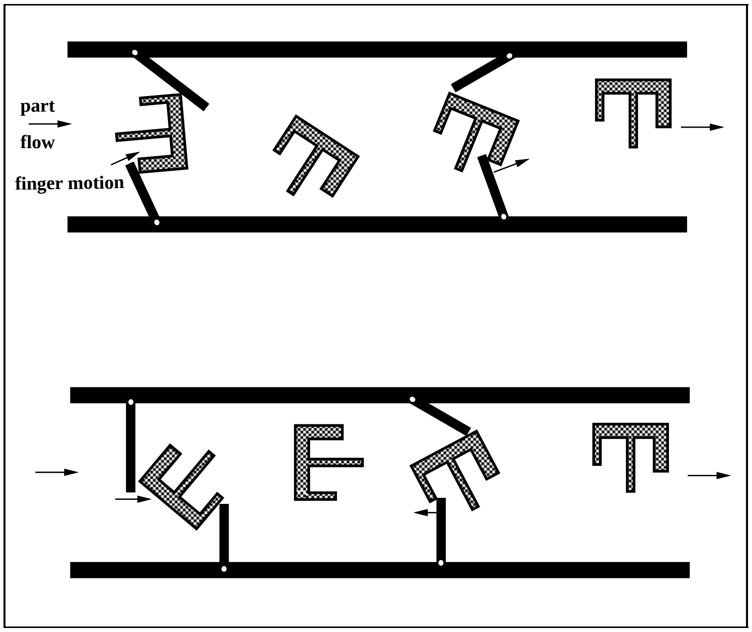

Figure 2: Cartoon of a feeder that uses local fingers to reorient objects. Each finger can rotate and slide back and forth a little. Here are two possible paths a part might take through such a feeder. A camera might sense the initial part configuration, then strategies similar to those presented in this paper might reorient the part.

in the past. In principle our planner can be extended to include such contacts, and indeed, more general curved objects. However, since our prime interest was to study the feasibility of palm manipulation proper we have not implemented this more general version of the planner.

1.2 Choices

A comment on our choice of hardware. We are using the Zebra robots because they provide a convenient testbed. The two arms together have twelve degrees of freedom. A planar rigid object has three degrees of freedom. Clearly there is a mismatch, and any practical system would not wish to waste so many degrees of freedom. In fact, we only use six of the twelve degrees of freedom, namely three degrees of freedom for each palm. Moreover, we could make do with as little as four. Four degrees of freedom would enable us to implement a two palm system, in which one palm rotates about a fixed point, and the other palm translates and rotates. We view the work here as a step towards building low-degree-of-freedom devices for parts orienting, such as the feeder system depicted in Figure 2. However, rather than commit ourselves to particular low-degree-of-freedom hardware during this exploratory stage, it seemed best to use two general purpose robots.

Finally, a word about sensing. The plans are sensorless. The planner takes careful account of object geometry, friction, and gravity, including knowledge of the object's center of mass. The resulting plans execute without any sensing of the part by the robots. Our goal in this study was to focus on the mechanics of contact, in order to develop the necessary analytic tools and explore the motions and manipulations possible. A practical system dealing with objects of unknown shape and mass distribution would need to focus much more on sensing and object recognition. Our planner and sensorless approach would not carry over to such a more general system. However, we believe that the underlying mechanical analysis tools presented in this paper would carry over. Moreover, the system presented here provides a baseline capability.

1.3 Contributions

This research makes two main contributions:

- **Automatic Two-Palm Manipulation.** We have demonstrated the feasibility of automatic non-prehensile palm manipulation, by implementing both a planner and an executive. The system reorients extruded polyhedral parts with two co-operating robot palms.

- **Contact Analysis Tools.** We have developed simple analysis tools for determining the different modes by which two palms can manipulate an object. These modes include holding the object, rotating the object, and slipping one palm or other against the surface of the object.

2 Examples

Figures 3 and 4 show two plans produced by our planner. We will refer to the left palm as "Palm 1" and the right palm as "Palm 2". The first plan reorients a rectangular block. The block was plastic, Palm 1 was aluminum, and Palm 2 was hard foam. The second plan reorients the irregular five-sided block mentioned earlier. The block was wood, Palm 1 was aluminum, and Palm 2 was aluminum covered with duct tape. The coefficients of friction may be found in the figure captions. The frames correspond to snapshots before and after actions planned by our planner.

Both plans have a similar overall character, but the precise actions differ. Both plans start with a TILT of Palm 1 to an angle that causes the block to slide to the right. The planner takes the tilt angle in each case to be 3 degrees greater than the angle of friction for that case. As part of the TILT, Palm 2 catches the block. With that, the reorienting process starts. Here are some highlights:

- **Slide.** A SLIDE operation consists of some action by the palms in which one palm slides relative to the part, or vice-versa. Each plan happens to include one such SLIDE operation, beyond the initial tilt. In Figure 3, Palm 2 drags the block to the right between Frames 5 and 6, moving tangentially to Palm 1. In Figure 4, Palm 2 drags the block down and to the right between Frames 2 and 3, again moving tangentially to Palm 1. The basic operation is the same in both cases, but the angles of the palms are different, in order to account for the different coefficients of friction.

  A different plan for the wood block, shown in Figure 5, contains three different SLIDE actions. Between Frames 3 and 4, Palm 2 drags the block to the right, much as in the other two plans discussed. Between Frames 9 and 10, Palm 2 partially slips out from under the block, and between Frames 14 and 15, Palm 2 slides back down, reestablishing its configuration relative to the block. Despite how precarious these SLIDE operations may appear, especially the action between Frames 14 and 15, the Zebra robots executed all three SLIDE actions successfully.

- **Regrip.** Both plans contain a number of REGRIPS. A REGRIP is an action in which a palm rotates about its contact point with the part. The plans call for such REGRIP operations in order to strategically configure the palms for some subsequent action. For instance, between Frames 4 and 5 of Figure 3, Palm 2 reorients itself to a shallow angle in order to then drag the block to the right. Having dragged the block to the right, Palm 2 then reorients itself to a steeper angle in order to perform the rotation from Frame 7 to Frame 8. Similarly, the actions between Frames 3 and 15 of Figure 4 basically constitute a dance of REGRIP and ROTATE operations required to turn the block upright without permitting it to slip at the contacts or otherwise lose equilibrium.

- **Rotate.** No matter what else the palms do, ultimately they must ROTATE the blocks from their start to their goal configurations. In the plans shown, this is generally accomplished by moving Palm 2 so that its contact point traverses an arc of a circle about Palm 1's contact point. It is up to the planner to choose the amount of rotation so the part does not slip. Both plans contain a number of ROTATE operations. For instance, in Figure 3 there is a small ROTATE operation between Frames 2 and 3, and a large one between Frames 7 and 8. In Figure 4, there is a tiny ROTATE between Frames 4 and 5, just enough to bring the small unstable edge of the block into flush contact with Palm 1. Two other rotations occur between Frames 6 and 7 and between Frames 10 and 11, each of which brings Palm 2 into flush edge contact with the block. (There are several more ROTATE actions subsequently.)

- **Release.** The two plans exhibit two different release modes. In Figure 3, Palm 2 releases the plastic block by doing a very careful dance with Palm 1. Before Palm 2 lets go of the block and moves away, the block and Palm 1 are in their final configurations (see Frame 14). In contrast, in Figure 4, Palm 2 actually pushes the block past its stable point, and allows the block to rotate to a new stable equilibrium. This happens between Frames 16 and 17. The action between these frames is a ROTATE. Palm 2 rotates the block about its contact with Palm 1. As the center of mass passes over the contact point, the block

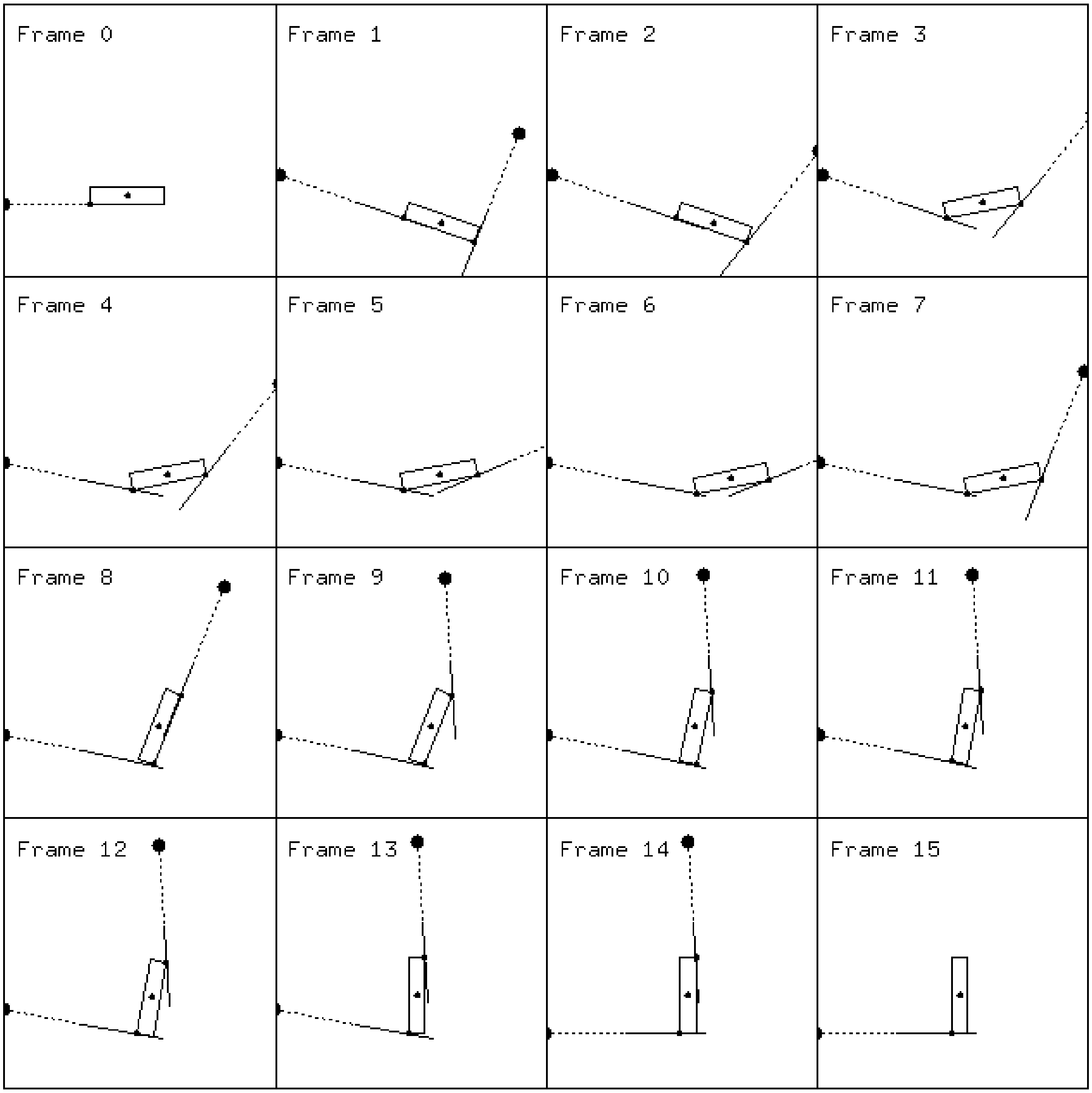

Figure 3: Plan for reorienting a rectangular block using two robot palms. The plan was generated automatically, given a geometric description of the part and the coefficients of contact friction. Two Zebra Zero robots executed this plan, each acting as one palm. — Friction is 0.287 at the left palm and 0.810 at the right palm. The block dimensions are 83mm by 18mm. The palm lengths are drawn to scale. The big black dots are the locations of the robot wrist joints; the dashed lines depict unreachable parts of the robot hands, such as the force sensor housings. The small dots indicate the primary vertex of contact for each palm. — The friction coefficients were computed by measuring the friction angles to about one degree of accuracy, then taking tangents of those angles. The three digits shown do not reflect three digit accuracy.

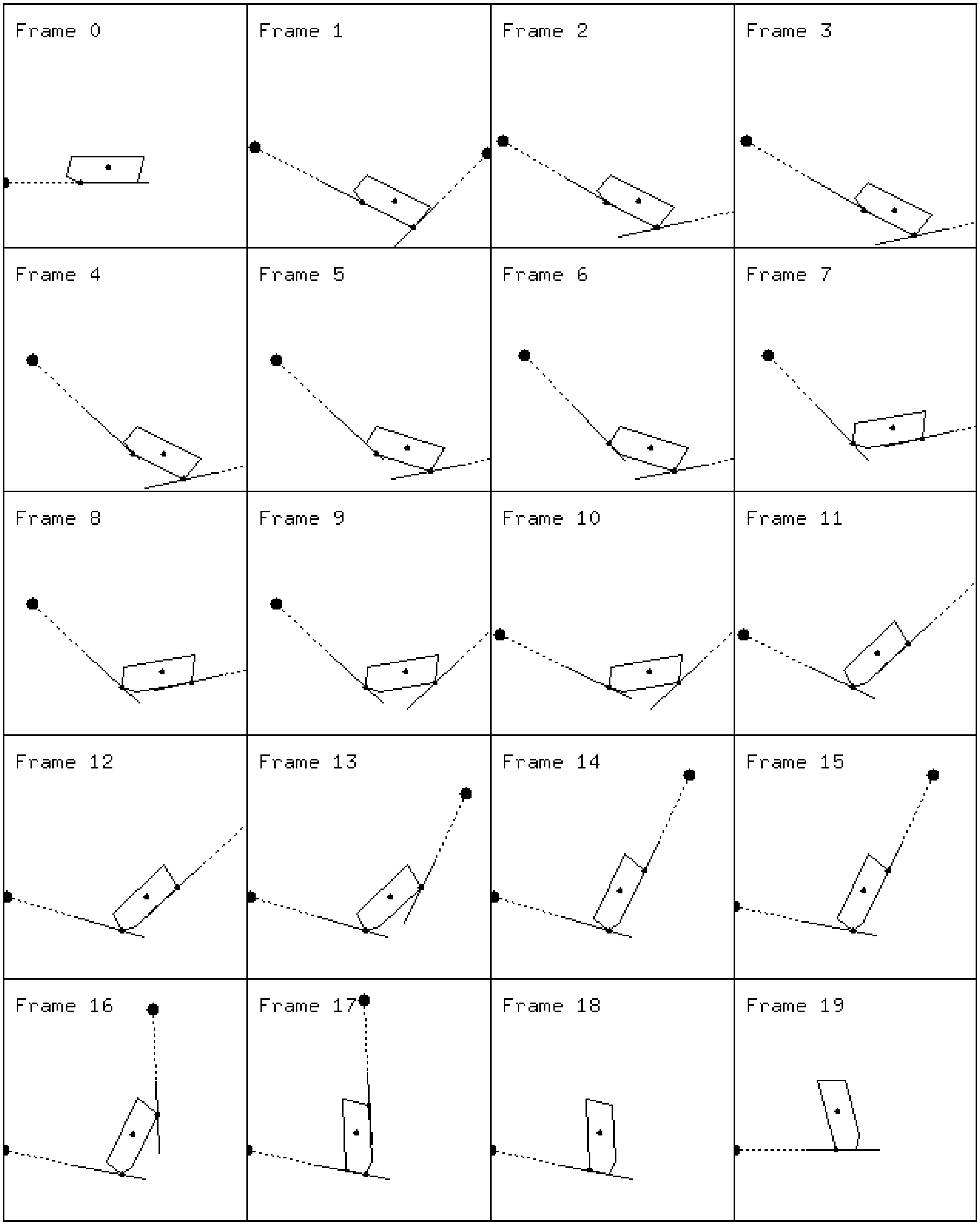

Figure 4: Plan for reorienting a five-sided block using two palms. Friction at the left palm is 0.445 and friction at the right palm is 0.554.

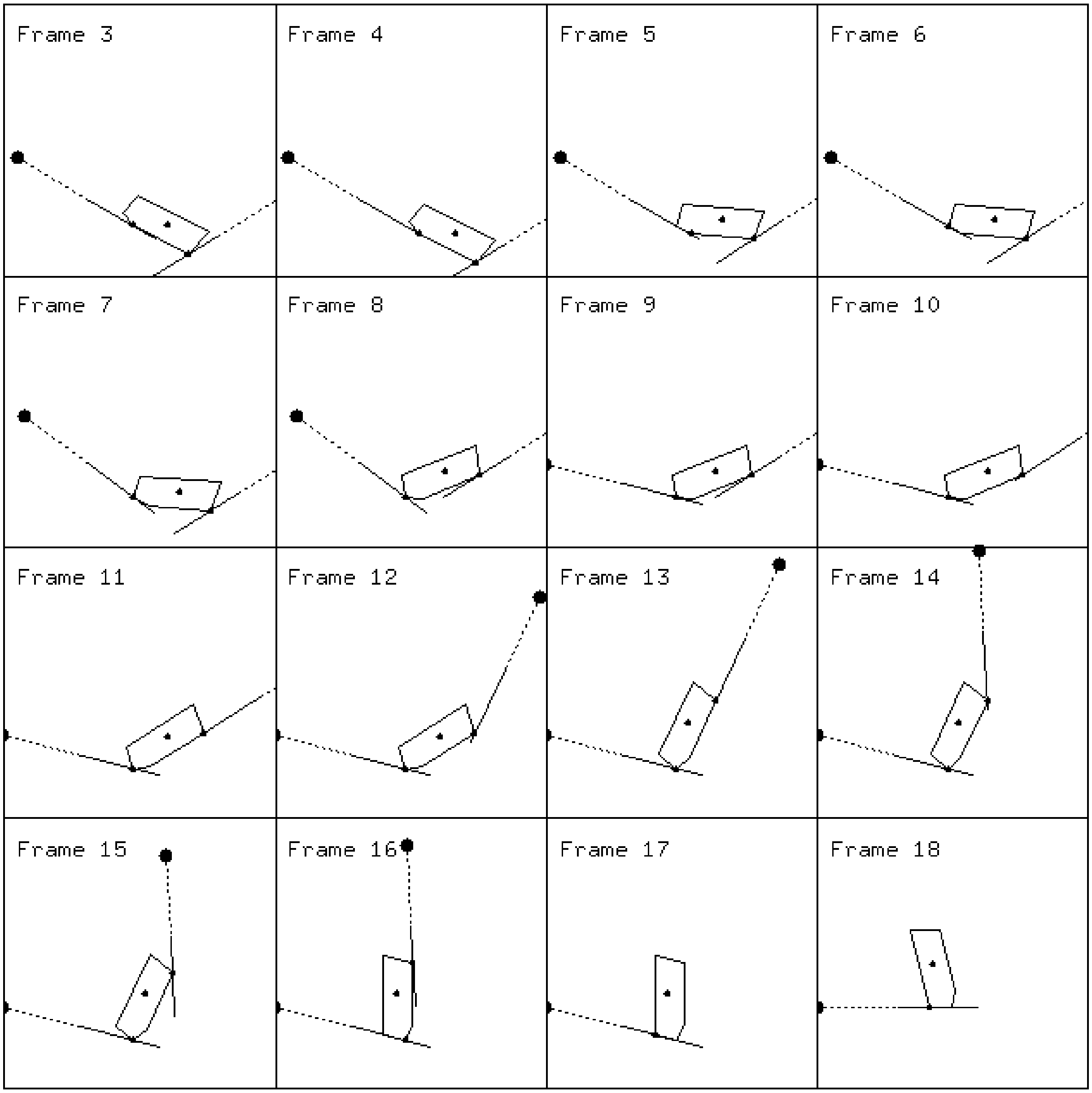

Figure 5: A different plan for the same block as in Figure 4 (frames 0–2 are not shown; they are similar to the frames in the previous two plans). This plan includes three different SLIDE actions. These occur between Frames 3 and 4, between Frames 9 and 10, and between Frames 14 and 15.

rotates on its own under the influence of gravity. By Frame 17, Palm 2 has again caught up with the block, but not before the block has rotated freely under the influence of gravity. This operation is very delicate. The planner had to choose the angle of Palm 1 sufficiently steep to prevent the block from rotating too far and tipping over. After all, the goal edge is barely stable. On the other hand, choosing the tilt of Palm 1 too steeply would have caused the block to slide and thus slip between the two palms, again falling over. Finally, once the block rotates to its goal edge, Palm 2 can let go (Frame 18), and Palm 1 can then tilt to horizontal (Frame 19).

The heart of this paper is a discussion of how the planner analyzes the possible motions of a two-palm contact.

3 Motivation

We are motivated to study palm manipulation for three basic reasons: simplicity, naturalness, and foundation.

Simplicity. Palm manipulation can reduce the mechanical complexity of devices needed to manipulate objects. We are interested in designing low degree of freedom devices for orienting and feeding parts to assembly workcells. In the past, feeder designs have suffered from the "frozen hardware" problem. Too much of the mechanics of the task has been compiled into hardware, in the form of fixed feeder gates and orienting shapes. At the other extreme, general purpose robots have suffered from too much generality and too little reliable software. An important intermediate architecture consists of a sequence of simple manipulators under software control. Since the configurations and motions of the devices are under software control and since the devices are easy to program by virtue of having low degrees of freedom, last-minute part changes may be accommodated by the feeder with relative ease. For the devices, we envision a series of palms, each with one or two degrees of freedom, situated along a feeder belt. For example, see Figure 2.

Naturalness. Nonprehensile palm manipulation is natural. Humans and animals often manipulate objects with a series of partial grasps, shuffling the object back and forth between various contacts. For instance, birds twirl seeds in their beaks, chipmunks roll acorns in their paws while shelling them with their teeth, and humans roll objects back and forth between their two palms in order to orient or inspect them. Turning over a large book is one everyday example. Lathering soap is another. These tasks include periods of prehensile grasps, but it is nonprehensile manipulation that provides the fluidity whereby one equilibrium grasp is dynamically transformed into another.

Even such a simple operation as *placing* an object must, by definition, end without prehension. Although one may transport the object using a force/form closure grasp, as one releases the object the interaction between the hand and the object becomes nonprehensile, often involving sliding contacts over an extended surface of the hand. The more quickly one wishes to release the object, the more significant this interaction. The placing operation effectively becomes a *slide-release* (see Section 5).

Moreover, the transport itself may involve a grasp that is merely an equilibrium grasp against gravity, without being prehensile. A waiter carrying a large tray on his palm is one example. A forklift raising a crate is another. Other natural examples of palm manipulation include scooping, cradling, smacking, and flipping.

We wish to understand and harness these natural operations, with the aim of simplifying robot programming. By building planners and executors we highlight weaknesses and calibrate competences, thereby increasing our understanding.

Foundation. Palm manipulation forms a key point in the spectrum of manipulation operations. At one end of this spectrum we have force/form closure grasps; at the other end we have free flight motions of thrown objects. Force/form closure grasps are characterized by complete control over the object and resultant insensitivity to environmental dynamics. Free flight motions are characterized by an impulse of initial control followed by strong sensitivity to environmental dynamics. In between these extremes lies a large class of nonprehensile manipulation operations. Research to date has explored only a small portion of this class. Some examples include quasi-static manipulation of an object on a planar support surface, such as pushing, tumbling, and pivoting. Other examples include controlled slip motion, reorienting by tipping, and object acquisition. The unstudied problems lie in two directions: (i) shaping and controlling constraint surfaces other than pure point-fingers or horizontal planes, and (ii) exploiting dynamics to take advantage of dynamic coupling, much as in the work on underactuated manipulators. Palm manipulation offers a simple domain in which to explore these two directions.

4 Related Work

Previous robotics research related to our work on palm manipulation falls into three broad categories: nonprehensile manipulation, grasping with sliding contacts, and studies of friction. This research also includes specific prior work on palm-like manipulation. More generally, the work on grasping strategies and dynamic manipulation is important. We now briefly review some of the work from these six interrelated areas.

4.1 Nonprehensile Manipulation

The importance of nonprehensile manipulation in robotics first makes its written appearance in Mason's Ph.D. thesis [Mason 1982] and the paper [Mason 1986]. Mason in turn points to work by Pingle, Paul, and Bolles from 1974, a conversation with Inoue (see also [Inoue 1974]), and general parts feeding and assembly approaches. The basic idea is for a robot to manipulate an object purposefully even though the robot does not have full control over the object. Mason looked at the problem of pushing objects in the plane. He analyzed the mechanics of pushing, including the classification of sliding and rolling contacts, and showed how pushing could be used to reduce an object's uncertainty. Mason's research spawned over a decade of further work on pushing. Peshkin analyzed the pushing problem [Peshkin 1986] and showed how to design a series of fences along a conveyor belt to orient parts on the conveyor [Peshkin and Sanderson 1988]. Peshkin's work was later extended by [Brokowski, Peshkin, and Goldberg 1993] to design curved fences that smoothly engaged and reoriented the parts on the conveyor. [Brost 1988] implemented a system that could orient parts through a series of pushing and squeezing operations. [Akella and Mason 1992] showed how to pose any polygon using only pushing strategies. [Lynch 1992] showed how to pose objects by pushing without relative slip, and [Lynch and Mason 1995] investigated various non-intuitive properties of pushing strategies. Finally, [Donald, Jennings, and Rus 1994] implemented a variety of pushing and reorienting strategies, using cooperating mobile robots to move large objects such as boxes and couches between rooms.

A number of systems have been studied that can orient parts without any sensing using nonprehensile operations. [Mani and Wilson 1985] implemented one such system, using pushing operations as primitives. [Erdmann and Mason 1988] implemented another such system, in which a robot would tilt a tray containing a planar part of known shape but unknown configuration. The robot would plan a sequence of tilts guaranteed to move the part to a known configuration. [Natarajan 1986] studied general algorithms for designing nonprehensile sensorless strategies. [Erdmann, Mason, and Vaněček 1993] developed a theoretical generalization of the planar tray-tilter to 3D objects resting on a tiltable table.

A number of researchers have investigated and implemented other ingenious forms of nonprehensile manipulation. [Sawasaki, Inaba, and Inoue 1993] considered the task of moving an object over a planar surface by tumbling it about various edges of contact. [Aiyama, Inaba, and Inoue 1993] considered the task of moving an object over a planar surface by pivoting it about various vertices, much as one might move a large refrigerator. These researchers observe that pushing, tumbling, and pivoting form a geometrically complete class of motions. Specifically, these operations preserve, respectively, plane, line, and vertex contact with a support surface. From this perspective, palm manipulation is a simple generalization of these operations, in which the support surface itself becomes moveable.

4.2 Palm-Like Manipulation

Turning now to palm manipulation and palm-like manipulation, we see that there has been considerable prior research in this area. The tray-tilting system of [Erdmann and Mason 1988] constitutes one simple example. The edges and corners of the rotating tray effectively act as moving palms, but with fixed relative orientations. In a recent example, [Akella, Huang, Lynch, and Mason 1995] use a single palm acting across a conveyor belt to configure parts moving along the conveyor. In another recent example, [Zumel and Erdmann 1996] use a pair of palms hinged in a "V" shape to reorient planar parts in a gravity field. This spectrum of palm manipulators is interesting in part because it shows how different degrees of freedom may be used to manipulate objects without prehension. The system described in this paper requires four degrees of freedom. It is implemented using six of the twelve degrees of freedom provided by the two Zebra robots. The "V" palms system uses two degrees of freedom, the tray-tilter used 1.5 degrees of freedom, and the palm-conveyor system uses a single degree of freedom. All systems rely on some external part actuation, either by gravity or a conveyor belt. Interestingly, [Akella, Huang, Lynch, and Mason 1995] prove that their system can, under mild conditions, configure any planar polygonal part in an arbitrary pose, a claim none of the other systems has made.

Probably the first to examine palm-like manipulation explicitly were Salisbury in one setting and Trinkle in another. [Salisbury 1987] proposed the idea of whole arm manipulation, in which an arm manipulates an object by making contact with the object anywhere on the arm's surface, as needed, rather than restricting contact to the fingertips of the hand. Around the same time, in a series of papers Trinkle and colleagues analyzed the mechanics of strategies for scooping up frictionless objects resting on a table, by enveloping them with two fingers [Trinkle 1992], [Trinkle, Abel, and Paul 1988], [Trinkle and Paul 1990]. Later Trinkle considered the dual problem, in which the hand points up, the object rests on the "palm" and the fingers manipulate the object from the sides [Trinkle and Hunter 1991], [Trinkle, Ram, Farahat, and Stiller 1993].

[Yoshikawa and Zheng 1993] analyzed the dynamics of a hybrid control scheme for manipulating objects with multiple manipulators, possibly in the presence of stationary environmental constraints. These researchers implemented a two-finger system for rolling a part against a constraint surface. In related work, [Nakamura, Nagai, and Yoshikawa 1989] analyzed the stability of multi-robot manipulation, focusing in particular on the stability of frictional contact. [Yokokohji, Yu, Nakasu, and Yoshikawa 1993] again looked at the problem of manipulating a constrained object with two fingers, in order to explore the differences between quasi-static and quasi-dynamic analyses.

Yun and colleagues looked at the problem of manipulating large objects by supporting them with two palms [Yun 1993], [Paljug and Yun 1993], [Paljug, Yun, and Kumar 1994]. These researchers developed and implemented control laws for stably moving and reorienting objects with non-sliding rolling contacts. Bicchi and colleagues analyzed the kinematics and manipulability of pairs of cooperating robots that manipulate objects with flat palm-like surfaces [Bicchi, Melchiorri, and Balluchi 1995], [Bicchi and Sorrentino 1995]. In one task, two palms roll an object between them, much as two hands might lather soap.

4.3 Slip

An important aspect of our system is the ability to purposefully slip one palm relative to the object being held, or, equivalently, to use one palm to drag or push the object relative to the other palm. There has been considerable prior work on slip. [Fearing 1984] first used slip to attain stable grasps on objects of unknown shape, as well as to twirl objects using three fingers. [Nguyen 1988] also felt that slip was a beneficial aspect of grasping and analyzed strategies for achieving grasps using slipping operations. The works by Mason and Trinkle and their colleagues cited earlier made extensive use of slip to orient objects and reduce uncertainty. [Brock 1988] used slip to reorient grasped objects in the presence of gravity. [Carlisle, Goldberg, Rao, and Wiegley 1994] used slip to reorient grasped objects by pushing them against other objects. [Cole, Hsu, and Sastry 1992] developed a dynamic control law for regrasping objects using sliding. Perhaps the most extensive investigation of slip originated with work by Kao and Cutkosky. See [Kao and Cutkosky 1989, 1992, 1993]. These researchers describe the trajectories of grasped objects using differential equations whose coefficients depend on the finger stiffnesses, contact kinematics, and actual finger motions. The fingers are allowed to slip along the surface of the grasped objects. One example task uses two fingers to rotate a card resting on a table, by slipping the fingers over the top surface of the card. Related work includes the work on slip using soft fingertips by [Xue and Kao 1994] and the work on tactile sensing by [Cutkosky and Hyde 1993] and by [Howe and Cutkosky 1993]. See also the work [Montana 1988] for a complete description of contact kinematics, including rolling and slipping, in terms of relative part curvatures. Finally, [Yoshikawa, Yokokohji, and Nagayama 1993] analyze three-fingered grasping of three-dimensional objects using slip motions.

4.4 Grasping

The work on slip leads naturally into the broader realm of grasping. This is much too large a field to cover in our current brief review, so we will mention only a few papers. Much work on grasping seeks to establish a prehensile grasp and in particular to establish force or form closure. That work generally points back to [Salisbury 1982] and [Cutkosky 1985] for seminal foundation work, to [Nguyen 1988] for first characterizing the geometry of force closure grasps, and to [Mishra, Schwartz, and Sharir 1987] for the first simple unified mathematical picture. For an overview with further pointers see [Murray, Li, and Sastry 1994]. We would also like to mention the algebraic formulation of force closure grasps found in [Ponce, Stam, and Faverjon 1993]. Although our planner uses purely numerical techniques, the basic idea of using cylindrical decompositions is similar to that presented in the work cited. Its roots in robotics go back to the motion planning work of [Schwartz and Sharir 1983]. Finally, work on reorienting grasped objects relates in a broad sense to our current research. Here are some examples. [Li and Canny 1990] used nonholonomic rolling at the fingertips to reorient objects.

[Hong, Lafferriere, Mishra and Tan 1990], [Omata and Nagata 1994], and [Teichmann and Mishra 1994] developed various finger gait algorithms for grasping and reorienting objects. And [Rus 1992] developed a number of algorithms for stably reorienting objects using multiple fingers, including the use of slip.

In the context of nonprehensile grasping, [Abell and Erdmann 1995] developed a planner for holding and reorienting planar parts stably, using only two fingers and gravity. [Zumel 1995] is developing a planner for manipulating objects using two palms. Again, the palms do not establish prehensile grasps. Moreover, the planner may call for the palms to push a part purposefully into an unstable configuration, in order to reorient the part.

4.5 Friction

A key component of most grasping strategies, including our palm manipulation system, is the ability to account for friction. There has been considerable work on friction in robotics. The heart of our planner is based on earlier work on friction found in [Erdmann 1994]. That paper describes a system for predicting object motions in the presence of static constraints by modelling friction in configuration space. It is easy to generalize that approach to moving and accelerating constraints, by including time dependencies in the *First and Second Variation Constraints* described in the paper. We implemented a configuration space simulator that takes account of moving and accelerating constraints. Our palm planner's inner loop calls this simulator to determine how the palms may move. See [Lozano-Pérez 1981, 1983] for seminal research into configuration space, and [Latombe 1991] for a comprehensive overview of configuration space techniques.

Work on friction in robotics appears intensely with the research into assembly strategies conducted at the Charles Stark Draper Laboratories in the 1970s. See, for instance, [Nevins et al. 1975], [Drake 1977], [Simunovic 1979]. This work lead to the highly successful *Remote Center of Compliance* devices for peg-in-hole insertions, along with the force-moment insertion diagrams described in [Whitney 1982]. Seminal work on modelling friction in terms of screw theory was conducted by [Ohwovoriole, Hill, and Roth 1980] and [Ohwovoriole and Roth 1981]. Generalizations to three dimensions of the Draper peg-in-hole diagrams and strategies may be found in [Caine 1985] and [Sturges 1988]. Most of the researchers discussed earlier in the context of grasping also delved into friction. General approaches for modelling friction and simulating motions of rigid bodies in the presence of friction and impact may be found in [Wang and Mason 1987], [Lötstedt 1982], [Haug, Wu, and Yang 1986], and [Trinkle, Pang, Sudarsky, and Lo 1995]. Of particular importance is the limit surface model proposed by [Goyal, Ruina, and Papadopoulos 1991]. This representation has been used extensively by Cutkosky and colleagues in their study of slip motions. Finally, our work on palm manipulation has benefited greatly from the planar friction cone model, and in particular the ideas underlying the work in [Brost and Mason 1989].

4.6 Dynamic Manipulation

Lastly, we would like to cite ongoing research into dynamic nonprehensile manipulation. The philosophy underlying general nonprehensile manipulation is: often a robot can exercise effective control over an object even though the robot does not actually have full control at all times. In this context, [Mason and Lynch 1993] observed that a robot can exploit dynamics to control an object while reducing the overall complexity of the robot. As an example, they showed how to effectively grasp an object by accelerating a one-degree-of freedom robot arm properly. They were able to grasp the object in configurations for which no constant velocity motion of the arm could possibly maintain contact with the object. They described further research for reorienting objects dynamically. Work on dynamic manipulation finds its dynamical roots in three basic areas, namely hopping [Raibert 1986], [Hodgins and Raibert 1990]; juggling [Bühler, Koditschek, and Kindlmann 1989], [Rizzi and Koditschek 1993], [Schaal and Atkeson 1993]; and the control of underactuated manipulators [Arai and Tachi 1991], [Arai, Tanie, and Tachi 1991]. Recent work on dynamic manipulation may be found in [Arai and Khatib 1994] and [Burridge, Rizzi, and Koditschek 1995]. The first paper shows how to reorient a block by accelerating a PUMA palm properly, based on the ideas underlying the control of underactuated joints. The second paper examines control behaviors, theoretically and experimentally, for achieving dynamic pick-and-place, based on ongoing juggling research.

Finally, we close this section with a dynamic manipulation example. Figure 6 shows how a single palm can reorient a block, if friction is high and the palm has high acceleration capability. This example was planned by hand using our configuration space simulator. Of course, the requirements of this task far exceed the capabilities of our Zebra robots, which are essentially quasi-static devices.

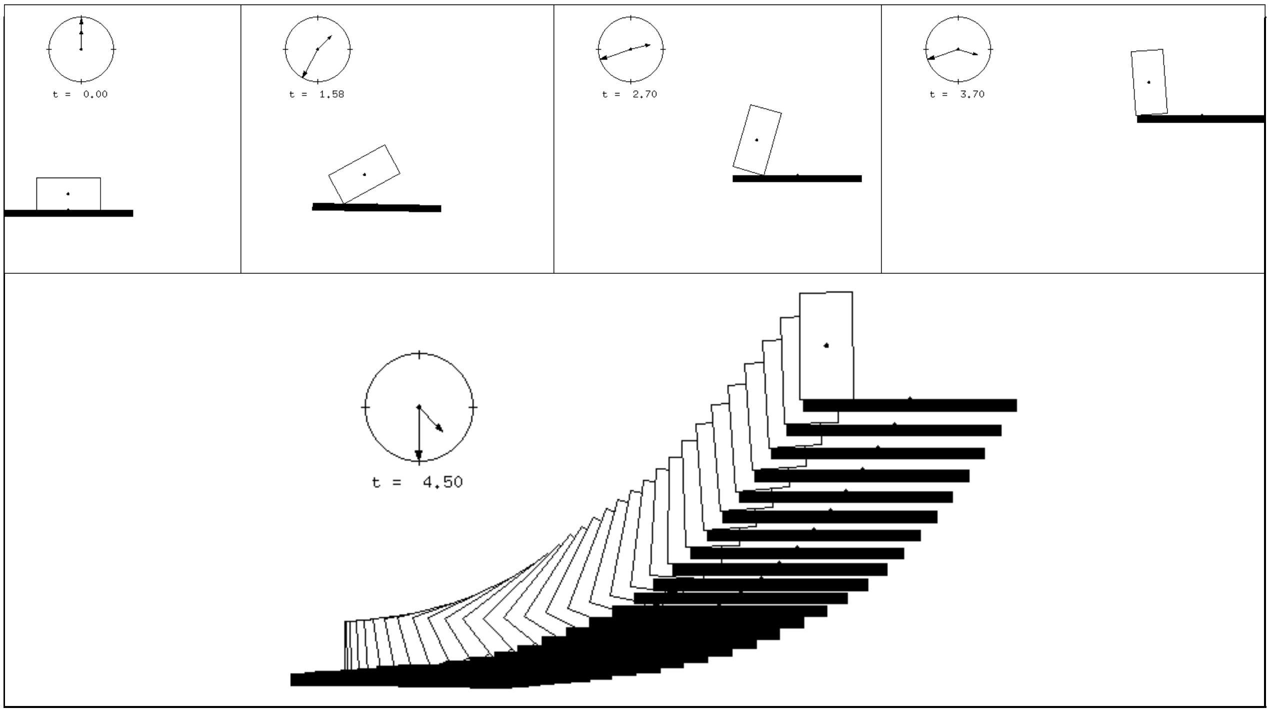

Figure 6: An accelerating palm reorients a block. The palm first tilts slightly downward. Then it accelerates sharply to the right, in order to turn the block upright. Finally, the palm accelerates upward to stabilize the block, including a short period of leftward acceleration. (Although not shown, once the block has been stabilized, the hand can slow down.) — Scale: The palm is 20mm long. Time is in hundredths of a second. Gravity acts down at 10m/sec^2. Friction is large, with coefficient $\mu = 10$. The acceleration magnitudes peak at 10 G's. Significantly smaller accelerations tend not to reorient the block, while significantly larger ones tend to slip the palm out from under the block.

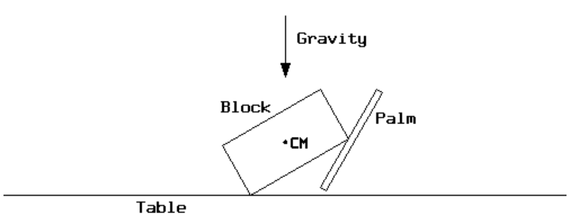

Figure 7: Block in contact with a table and a palm.

5 Slide-Release

When two hands manipulate an object, there are three basic operations the hands perform: holding or rotating the object in an equilibrium grasp, allowing the object to fall in disequilibrium, and sliding part of one hand against the object. In this section we focus on the sliding operation.

5.1 Single-Handed Manipulation

Even single-hand manipulation gains scope and competence by judicious use of sliding. Imagine for instance placing a large dictionary on a table, using one hand. Shortly before release, one edge of the book will be in contact with the table, while one's hand is holding an opposite edge. In all likelihood, one's fingers are supporting the bottom face of the book, while one's thumb is wrapped around the top side. In fact, the thumb is superfluous. Sooner or later one will release one's grip on the book, slide the fingers out from underneath, and allow the book to fall into place. This is a classic *slide-release*. It demonstrates one form of relative sliding, in which the hand slides relative to the object, while the object remains stationary in the world frame. In the example, the fingers slide out from under the book, while the book does not slide relative to the table. Of course, once the hand has slid

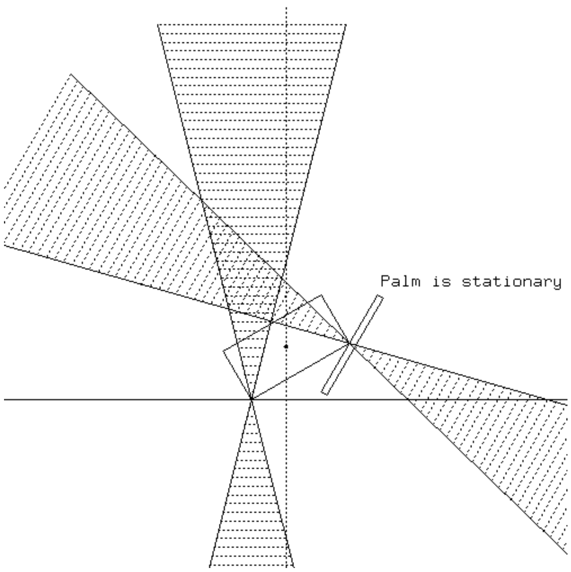

Figure 8: Initial configuration of block, table, and palm, along with the two two-sided contact friction cones and the vertical center-of-mass line. Friction is 0.25 at each contact. Observe that the friction cones *do not* intersect anywhere along the center-of-mass vertical.

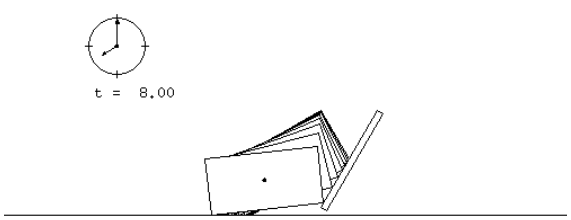

Figure 9: Simulation of motion starting from the configuration in Figure 8. The block falls.

all the way out from under the book, the book drops into place on the table, which is the whole point of a slide-release.

There is another form of sliding, dual to the one described, in which the hand moves and the object remains fixed to the hand, sliding in the world frame while tracking the hand. Such sliding can be used prior to a release to position the object in the world frame. In the example, we might move our hand so as to position the dictionary precisely on the table, and only thereafter release the book. Since we have a thumb, we would probably perform this fine positioning by grasping the raised end of the dictionary. However, again, the thumb is really unnecessary. One can both push and pull the dictionary simply by choosing carefully the angle with which one's palm or fingers support the raised end of the dictionary. Friction can substitute for the force closure provided by a thumb.

5.2 The Book-Placing Task

In the remainder of this section we will examine the book placing example in detail. We will develop a simple test for determining when sliding is possible and what type of sliding is possible. This test constitutes the central tool used by our two-palm planner for carving up its search space.

The basic problem is shown Figure 7. The figure depicts a block in contact with a horizontal table and a tilted palm. Gravity acts straight down. We assume that friction arises from dry Coulomb friction. We assume further that there is no difference between the static and dynamic coefficients of friction.

5.3 Equilibrium Conditions

Let us look at the conditions under which equilibrium is possible. Figure 8 again shows the block in contact with the table and the palm, but now with the contact friction cones depicted as well. We have depicted the cones as two-sided friction cones in order to emphasize the lines of action of the reaction forces. We have also drawn a dashed vertical line through the center of mass of the block. This line depicts the line of action of the force of gravity.

First, we ask whether the block will fall or remain stationary. It turns out the block will fall; Figure 9 shows the result after 8 centiseconds, as determined by our configuration space friction cone simulator.

In contrast, consider Figure 10. In that figure, the coefficient of friction between the block and the table is now $\mu_1 = 0.5$, larger than it was in Figure 8. This time the block remains stationary; it is in equilibrium. See Figure 11.

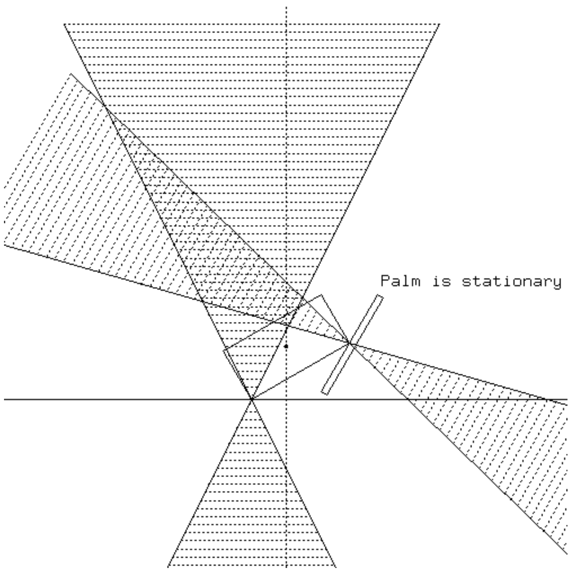

Figure 10: Another initial block-table-palm configuration, along with the two-sided friction cones and the vertical center-of-mass line. In this case friction is still 0.25 at the palm contact, but it is now 0.5 at the table contact. Observe that in this example the friction cones *do* intersect along the center-of-mass vertical.

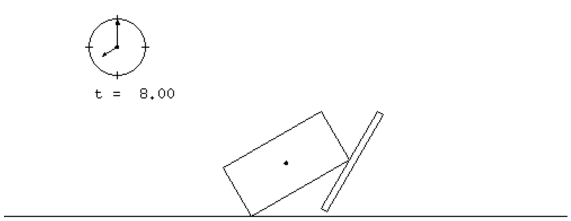

Figure 11: Simulation of motion starting from the configuration in Figure 10. The block remains stationary.

Mechanically, the difference between the examples of Figure 8 and Figure 10 is that the contact reaction forces can balance gravity in the second case but not the first. Geometrically, two conditions must be satisfied. First, the negative of the gravity vector must lie in the positive-span of the two contact reaction forces. Second, the lines of action of the two contact reaction forces and the gravity vector must all intersect at a common point. In other words, to use Brock's terminology [Brock 1988], the *force focus* of the three forces must exist. Together, the two conditions ensure that force-torque balance is physically possible.

We will focus on the second condition in the remainder of this paper, that is, the *force focus test*. Nonetheless, we caution the reader not to forget the first condition, that is, the *positive-span test*.

Next, we ask ourselves whether we can tell instantly from Figures 8 and 10 if the force focus test is satisfied. The answer is yes. We need merely intersect the two two-sided friction cones and compare the resulting geometric object to the vertical line drawn through the object's center of mass. If the intersection of the two two-sided friction cones overlaps the vertical line, then it is possible to satisfy the force focus test. Conversely, if the intersection of the friction cones does not overlap the vertical line then it is not possible to satisfy the force focus test.

5.4 Sliding Contact Conditions

A robot expands its effective workspace when it can slide its hands relative to the objects it is manipulating. By strategically sliding the hands, the robot can reposition the objects in collision-free and mechanically advantageous locations on the hands.

Consider Figure 12. The figure again shows the vertical line through the object's center of mass and the two two-sided friction cones. Suppose the palm moves to the right, tangential to the table. Will the block track the palm or fall? Figure 13 shows the simulation results after 8 centiseconds. We see that the block falls. In contrast, consider Figures 14 and 15. In that example the block tracks the hand.

The difference between these two examples may be explained and predicted using similar reasoning as we used with the equilibrium examples of Section 5.3, only we must be more careful in describing the source of the reaction forces. The reaction force at the sliding contact cannot arise arbitrarily from within its contact friction cone. Instead, the reaction force must be on an edge of the friction cone, namely the edge that opposes the sliding motion. The reaction force at the other contact, the non-sliding contact, can of course potentially arise from anywhere within its

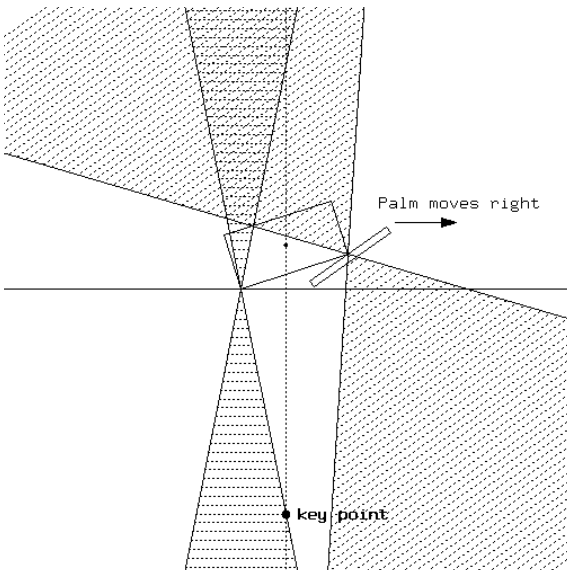

Figure 12: Initial configuration of block, table, and palm. The palm is about to move straight to the right. Friction is 0.2 at the table contact and 0.8 at the palm contact. The "key point" is the intersection of the table's left friction cone edge with the center-of-mass vertical. Observe that the key point lies *outside* the palm's friction cone.

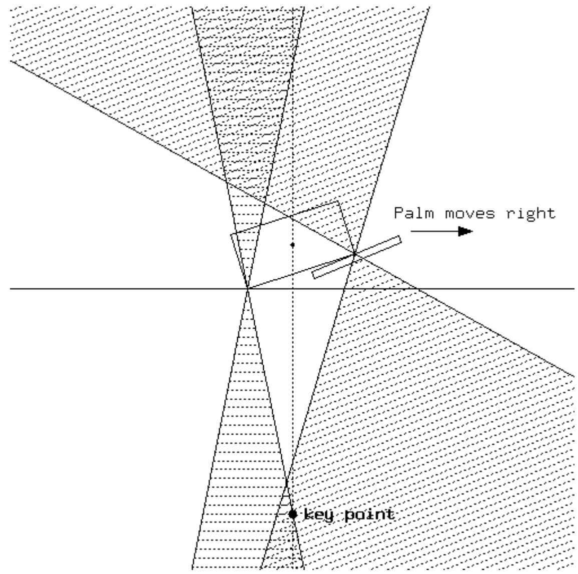

Figure 14: Initial configuration of block, table, and palm. The palm is about to move straight to the right. Friction is 0.2 at the table contact and 0.8 at the palm contact. Observe that the palm is at a shallower angle than it was in Figure 12. Consequently, the "key point" now lies *inside* the palm's friction cone.

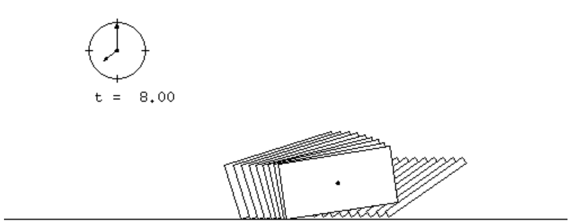

Figure 13: Simulation of motion starting from the configuration in Figure 12. As the palm moves, the block slips and rotates down.

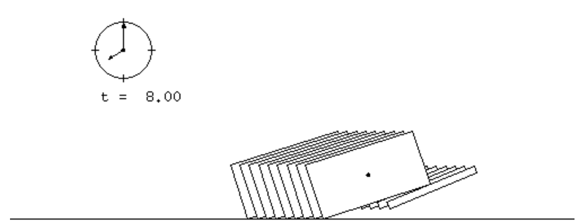

Figure 15: Simulation of motion starting from the configuration in Figure 14. As the palm moves, the block tracks the palm without rotating.

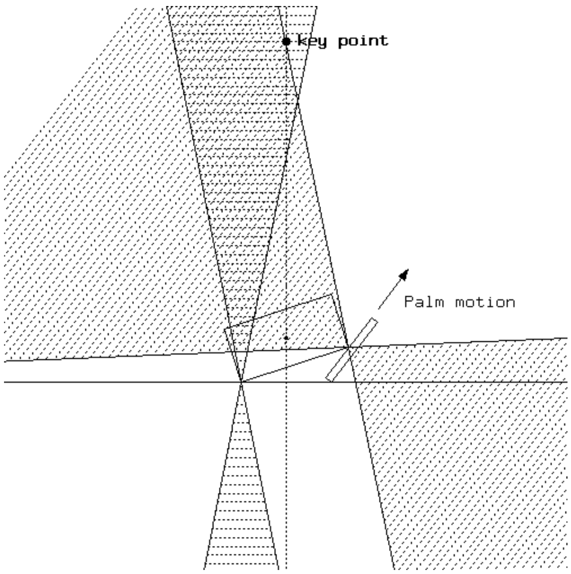

Figure 16: Initial configuration of block, table, and palm. The palm is about to move towards the upper right, tangentially to itself. Friction is 0.20 at the table contact and 0.85 at the palm contact. The "key point" is the intersection of the palm's right friction cone edge with the center-of-mass vertical. Observe that the key point lies *inside* the table's friction cone.

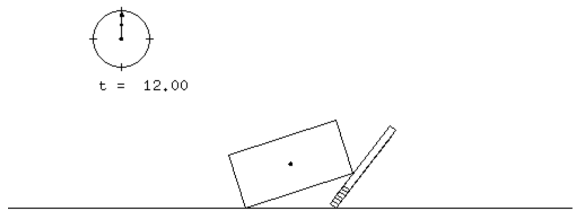

Figure 17: Simulation of motion starting from the configuration in Figure 16. As the palm moves, the block remains undisturbed.

contact friction cone.

For the example of Figures 12 and 14, the reaction force at the block-table contact must lie on the left edge of its friction cone. In deciding whether quasi-static dragging is possible, we must therefore intersect the line through the left edge of the friction cone with the vertical line through the object's center of mass.[2] If this intersection point lies within the two-sided friction cone anchored at the block-palm contact, then quasi-static sliding is indeed possible. Otherwise it is not.

The conditions for pushing the block to the left, or for sliding the palm relative to the block without disturbing the block, are similar. For example, Figures 16 and 17 show a configuration in which Palm 2 can slip out from under the block without disturbing it.

5.5 Summary

To decide whether static equilibrium is possible one checks whether the two friction cones have a common overlap with the vertical line drawn through the center of mass. To decide whether quasi-static sliding is possible one checks whether an edge of one friction cone has a common overlap with the other friction cone on the vertical. The specific edge used depends on the particular sliding mode. The two tests are really the same, merely constrained differently to account for the different object motions.

5.6 Planning Tools

It is easy to generalize these equilibrium and sliding tests into planning tools. For a fixed block configuration and a fixed set of contacts, the truth value of an equilibrium or sliding test can only change at certain discrete orientations of the palm. These critical orientations are those angles at which a friction cone edge of one friction cone intersects a friction cone edge of the other friction cone at a point lying on the center-of-mass vertical (or lying at infinity), as in Figure 18. In short, the mechanics of the task splits the orientation space of the palm into a finite set of sectors yielding qualitatively similar object behaviors.

6 The Planner

6.1 Carving up the Configuration Space

The fundamental planning stage for the planner is a two-dimensional space that encodes the possible orientations ϕ_1 and ϕ_2 of the two palms for a fixed orientation θ of the object. (See Figure 19 for a description of these terms.) The basic role of the two-palm planner is to carve up (ϕ_1, ϕ_2) space into regions within which the permissible palm motions are qualitatively identical. The planner does this by first computing certain critical curves. The critical curves describe the palm orientations (ϕ_1, ϕ_2) at which two

[2] By an "edge" of a friction cone we mean the entire right or left line of the two-sided friction cone, not just the ray of physically feasible forces.

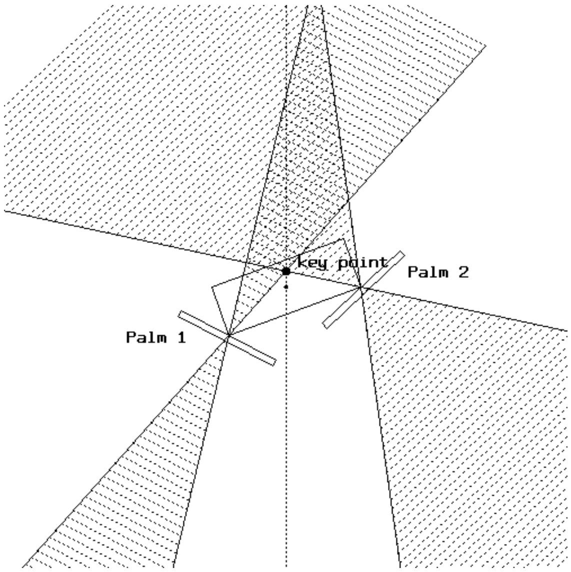

Figure 18: Two palms holding a block. Friction at the left contact (Palm 1) is 0.25; friction at the right contact (Palm 2) is 0.70. The figure shows a singular configuration, in which Palm 1's right friction cone edge intersects Palm 2's left friction cone edge right on the center-of-mass vertical.

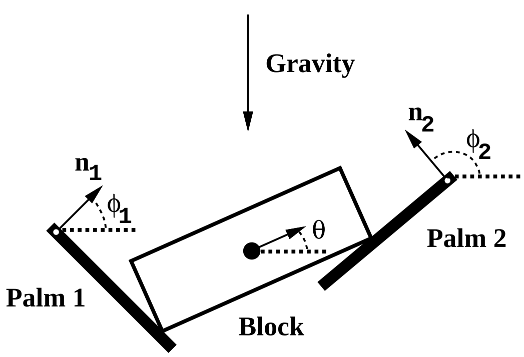

Figure 19: Two palms hold a block in the presence of gravity and contact friction. Some notation: θ is the orientation of the block, ϕ_1 is the orientation of the contact normal of Palm 1, and ϕ_2 is the orientation of the contact normal of Palm 2.

friction cone edges intersect at the same spot on the vertical center-of-mass line, or at which the positive-span test might change truth value. Finally, the planner creates a cylindrical decomposition based on these critical curves. The permissible behavior of the palms and the object are invariant across palm orientations within any region thus constructed.

Figure 18 shows two palms supporting a block in the presence of gravity and contact friction. The configuration is a critical configuration; the right edge of friction cone 1 intersects the vertical line at the same point as does the left edge of friction cone 2. If we rotate either palm slightly the resulting contact would become non-critical. Separating such generic contact states is a critical curve in (ϕ_1, ϕ_2) space. At each point on this curve, the right edge of Palm 1's friction cone meets the left edge of Palm 2's friction cone right on the vertical through the block's center of mass. The planner constructs all such critical curves, for all possible contacts of the two palms with vertices of the block.

For completeness, let us derive the form of the critical curve corresponding to the configuration depicted in Figure 18.

Let $\alpha_1 = \tan^{-1}\mu_1$ and $\alpha_2 = \tan^{-1}\mu_2$, where μ_1 and μ_2 are the coefficients of friction between the block and the two palms. Then the right edge of Palm 1's friction cone makes angle $\phi_1 - \alpha_1$ with the x-axis, and the left edge of Palm 2's friction cone makes angle $\phi_2 + \alpha_2$ with the x-axis. Let us also take the object's center of mass to be at the origin, and let us write the coordinates of the contact point with Palm 1 as (a, b) and the coordinates of the contact point with Palm 2 as (c, d). For simplicity, let us assume that $a < 0$, that $c > 0$, and that neither friction cone edge is vertical.

Let $y_1(\phi_1)$ be the y-coordinate of the point at which Palm 1's right friction cone intersects the vertical. The equation for $y_1(\phi_1)$ is:

$$y_1(\phi_1) = b - a\,\tan(\phi_1 - \alpha_1).$$

Similarly, if $y_2(\phi_2)$ is the y-coordinate of the point at which Palm 2's left friction cone intersects the vertical, then

$$y_2(\phi_2) = d - c\,\tan(\phi_2 + \alpha_2).$$

On the critical curve, $y_1(\phi_1) = y_2(\phi_2)$. Solving for ϕ_2 in terms of ϕ_1 we get:

$$\phi_2 = \tan^{-1}\left(\frac{a\,\tan(\phi_1 - \alpha_1) - b + d}{c}\right) - \alpha_2.$$

Here $\tan^{-1}$ is bi-valued. The critical curve thus has two components, representing Palm 2 in one configuration, and the same configuration rotated by π radians. Of course, for physical contact with a particular vertex of a convex polygon, at most one of these components will be physically feasible. The planner only looks at those portions of the curves that are physically feasible.

6.2 Planning Motions

The planner uses labelled subdivisions of the form shown in Figure 20 to plan motions of the palms to move the part from the start to the goal. The planner uses yet another critical event analysis, this time in θ, to construct (ϕ_1, ϕ_2) slices at a small number of generic part orientations θ.

The planner searches the (θ, ϕ_1, ϕ_2) configuration space from start to goal by searching its collection of (ϕ_1, ϕ_2) slices. Moving between different slices is equivalent to rotating the part from one θ orientation to another. For a fixed part orientation, the palms can potentially perform two types of motions: (i) they can rotate about their contact points, or (ii) they can slide left or right. Rotating about a contact point changes a palm's orientation and thus corresponds to a motion within a given (ϕ_1, ϕ_2) slice. Sliding a palm left or right does not change the system's representative point in (θ, ϕ_1, ϕ_2) space. However, the contact point of the part on the palm moves. The planner maintains a separate record of this location for each palm, in order to avoid exceeding the finite palm limits. The labelled subdivisions in each (ϕ_1, ϕ_2) slice tell the planner which palm motions are legal. Specifically, the boundaries of a region in a (ϕ_1, ϕ_2) slice tell the planner how far the palms may reorient themselves before either changing region classification or making palm-edge contact with the part. And the region classification itself tells the planner whether a given palm can slide left or right without disturbing the part.

7 Discussion

7.1 Experimental Results

Initially we tested our system on rectangular blocks, made of plastic and/or foam. Once we had ironed out all the basic bugs, we then tested the system on a new object, namely the irregularly shaped wood block mentioned in Section 1.1. At that point we realized that we needed to take better account of potential energy during the planning phase. Once we had incorporated that addition to the planner we went back and verified that the planner still produced executable plans with the old parts. Since that time we have added extra capabilities to the system. One such capability is for a palm to rotate a part while itself rotating, rather than merely translating along an arc. Another capability is for one palm to push an object resting on a table, or to tip the object over onto the other palm. This allows the palms to pick up an object off a table, then reorient it. We have also improved the robustness of the execution, by accounting for the effects of various errors, such as positioning errors of the palms, during planning. Finally, we have added a high-level planner on top of the existing planner. This planner assumes zero friction. With zero friction one can very quickly compute a plan to reorient a part. The plan is not practically executable, but the contacts that appear in this plan focus the frictional planner. This has helped speed up planning time. Planning time, running Lisp on a Sparc 20, now takes about 11 minutes for a plan such as that shown in Figure 3 and about 20 minutes for a plan such as that shown in Figure 4. This time is down from one to three hours originally. Execution time is about 1 minute.

Throughout this exploration, plans tended either to work almost all the time or not to work at all. The plans that did not work led us to incorporate stability tests, potential energy tests, and so forth. The plans that generally worked would occasionally fail, we think for one of three reasons: (1) initial calibration errors; (2) changes in the coefficient of friction due to repeated wear; and (3) underlying Zebra errors. Our latest plan for the wood block has run fifty times with four failures. I would not extrapolate this error rate to other parts, but it provides some hope that robust manipulation strategies can be built using the tools outlined in this paper.

7.2 Generalizations

In Section 1.1, we suggested that our method could be generalized to more complicated shapes. The primary change is best described by referring to the decomposition of (ϕ_1, ϕ_2) space, as in the diagram of Figure 20. The idea of using critical curves determined by friction cone edges to subdivide this space remains unchanged. The change is in the axes of the diagram. The key is to find a parameterization that encodes all the possible primitive contacts the palms can have with the object being manipulated. For general smooth objects, instead of using the palm orientations ϕ_1 and ϕ_2, one would use arclength encodings of the two contacts. These parameters specify the locations of the contacts on the object boundary. As another example, consider extending the existing representation to handle Type-A contacts, that is, contacts of object edges with palm vertices. In this

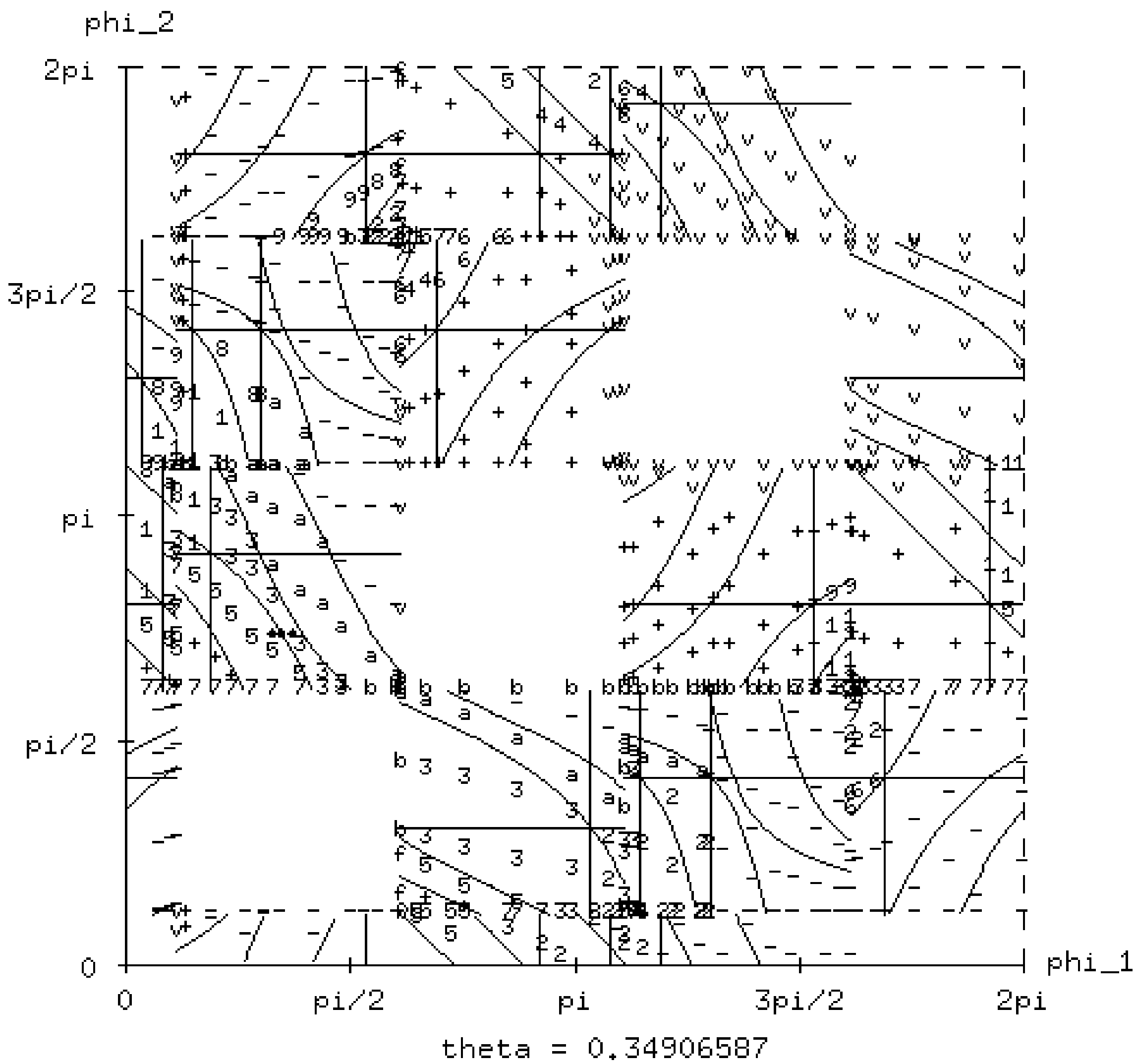

Figure 20: The planner subdivides the space (ϕ_1, ϕ_2) of palm orientations for a fixed object orientation θ into regions of invariant contact mode, using a cylindrical decomposition. This figure shows the location of the representative state of each invariant region. The block orientation is fixed at 20 degrees (0.349 radians), as in Figure 18. The palms can make contact with any vertex of the block. Of course, not all contacts yield equilibrium states. — Each state is depicted by a letter. The letters are internal codes used by the planner to describe the types of motions possible at that state. For instance, the planner uses a one letter hexadecimal representation to encode the sliding modes possible when the block is in equilibrium. And a "v" means that the block is falling without rotating. — Note the three dots in the left central area. The middle dot corresponds to the critical configuration depicted in Figure 18. The underlying curve describes the orientations (ϕ_1, ϕ_2) at which the right edge of Palm 1's friction cone meets the left edge of Palm 2's friction cone right on the center-of-mass vertical. — Note also the vertical and horizontal lines. These represent palm orientations at which the *positive span test* might change truth value. — Finally, the solid white areas represent invalid orientations at which both palms would be in simultaneous contact with the same block vertex.

case, the representation along each axis of Figure 20 would become mixed, specifying orientations of the palm normals for Type-B contacts at the object vertices and specifying contact locations on the boundary of the object for Type-A contacts. We note in passing that the curves in Figure 20 are not closed curves. This is because we are only dealing with Type-B contacts, that is, contacts of the palm edges with vertices of the object. For palm contacts with smooth objects, or for Type-A-and-B contacts with polygonal objects, these curves would appear as closed curves in the corresponding diagram.

Generalizations to three dimensions are more difficult. Ongoing research (for instance, [Ponce, Sullivan, Boissonnat, and Merlet 1993] and [Yoshikawa, Yokokohji, and Nagayama 1993]) in three-dimensional grasping may help. As a practical matter, treating three-dimensional objects as planar may be a good choice. For instance, objects arriving on a conveyor as in Figure 2 may be treated as a collection of two-dimensional cross-sections. There is one cross-section for each stable resting configuration of the part on the conveyor. Each such cross-section represents the contacts possible with the surrounding fingers when the part is resting on the conveyor in the given configuration. The problem is not completely planar of course, since one must still be careful to avoid out-of-plane motions under this scheme.

In short, the analytical tools presented in this paper generalize to more complicated shapes. However, these global analytical tools only make sense if the object shape, its center of mass, and the coefficients of friction are known. Determining these in general settings will be very difficult. It is quite likely that a truly autonomous system will use intensive feedback to manipulate an object of unknown shape and mass by purely local computations. We refer to [Teichmann and Mishra 1994] and [Erdmann 1995] for a discussion of some of the issues involved and related work. We suspect that some of the analytic tools described in this paper will be useful locally whenever information regarding apparent mass and slip for a particular configuration have been obtained by the available sensors. For instance, an observed slip can give an estimate of the coefficient of friction. This in turn specifies an estimate of how the contact friction cones overlap the center-of-mass vertical, thereby determining what *other* relative slip motions are possible. Moreover, given the local contact geometry, the shape of the friction cone overlap also tells the robot how it might change its grasp locally in order to change the type of motions that are possible. Recall that changing a grasp is equivalent to moving the representative point in the diagram of Figure 20, and recall that the grasp type changes as the representative point crosses a critical curve.

7.3 Future Directions

Although reasonably automatic, the system is far from truly autonomous. It is easy to design tasks for which the planner produces unexecutable plans. Low friction tasks are prime candidates. We have probably reached the limits of the current planner's competence. One way to increase its competence would be to expand its mastery of mechanics. There are four directions future work should pursue:

1. Incorporate principled tests for contact stability.
2. Incorporate frictional analyses that distinguish between static and dynamic coefficients of friction.
3. Build planners that can manipulate parts without requiring continuous stability or equilibrium.
4. Build closed-loop systems with tactile sensing.

Items (3) and (4) are probably the most interesting directions to explore. Our current planner has a modicum of nonequilibrium motion, namely the release operation discussed in Section 2. However, much more is possible. [Zumel 1995] is currently pursuing this direction, also in the domain of palm manipulation, focusing on tasks with very low friction. Additionally, [Mason and Lynch 1993] are pursuing this direction with their work on dynamic manipulation.

Finally, a planner that assumes good open-loop control of the part will find itself limited by the hardware's inherent inaccuracies. The plans it produces will necessarily be brittle. Another approach is to instrument the palms. This approach is likely in the long run to be the method of choice for fast autonomous manipulation. It is also currently the most elusive. As we have indicated, such a system would need good sensors to ascertain slip, impending slip, contact normals, and mass properties. Although existing sensors are far from delivering such information, it is a rich topic of research. For an excellent survey of touch sensing see [Howe and Cutkosky 1992].

8 Acknowledgments

Many thanks to Craig Johnson for his excellent work with the Zebra Robots. This research benefited greatly from discussions with Tamara Abell, Bruce Donald, Kevin Lynch, Matt Mason, and Nina Zumel.

Financial support for this research was provided in part by Carnegie Mellon University through a Faculty Development Award, in part by an equipment grant from AT&T, and in part by the National Science Foundation through the following grants: NSF Presidential Young Investigator award IRI-9157643 and NSF Grants IRI-9213993 and IRI-9503648.

9 Bibliography

Abell, T., and Erdmann, M. 1995. Stably Supported Rotations of a Planar Polygon with Two Frictionless Contacts. *Proceedings of the 1995 IEEE/RSJ International Workshop on Intelligent Robots and Systems*, Pittsburgh, Pennsylvania, Vol 3, pp. 411–418.

Aiyama, Y., Inaba, M., and Inoue, H. 1993. Pivoting: A New Method of Graspless Manipulation of Object by Robot Fingers. *Proceedings of the 1993 IEEE/RSJ International Workshop on Intelligent Robots and Systems*, Yokohama, Japan, pp. 136–143.

Akella, S., and Mason, M. T. 1992. Posing Polygonal Objects in the Plane by Pushing. *Proceedings of the 1992 IEEE International Conference on Robotics and Automation*, Nice, France, pp. 2255–2262.

Akella, S., Huang, W. H., Lynch, K. M., and Mason, M. T. Planar Manipulation on a Conveyor with a One Joint Robot. *Seventh International Symposium on Robotics Research*, October 21–24, 1995, Herrsching, Germany, pp. 265–276.

Arai, H., and Tachi, S. 1991. Position Control of a Manipulator with Passive Joints Using Dynamic Coupling. *IEEE Transactions on Robotics and Automation.* **7**(4):528–534.

Arai, H., Tanie, K., and Tachi, S. 1991. Path Tracking Control of a Manipulator with Passive Joints. *Proceedings of the 1991 IEEE/RSJ International Workshop on Intelligent Robots and Systems*, Osaka, Japan, pp. 923–928.

Arai, H., and Khatib, O. 1994. Experiments with Dynamic Skills. *Proceedings 1994 Japan-U.S.A. Symposium on Flexible Automation*, Kobe, Japan, pp. 81–84.

Bicchi, A., Melchiorri, C., and Balluchi, D. 1995. On the Mobility and Manipulability of General Multiple Limb Robots. *IEEE Transactions on Robotics and Automation.* **11**(2):215–228.

Bicchi, A., and Sorrentino, R. 1995. Dexterous Manipulation Through Rolling. *Proceedings of the 1995 IEEE International Conference on Robotics and Automation*, Nagoya, Japan, Vol 1, pp. 452–457.

Brock, D. L. 1988. Enhancing the Dexterity of a Robot Hand Using Controlled Slip. *Proceedings of the 1988 IEEE International Conference on Robotics and Automation*, Philadelphia, Pennsylvania, pp. 249–251.

Brokowski, M., Peshkin, M., and Goldberg, K. 1993. Curved Fences for Part Alignment. *Proceedings of the 1993 IEEE International Conference on Robotics and Automation*, Atlanta, Georgia, Vol 3, pp. 467–473.

Brost, R. C. 1988. Automatic Grasp Planning in the Presence of Uncertainty. *International Journal of Robotics Research.* **7**(1):3–17.

Brost, R. C., and Mason, M. T. 1989. Graphical Analysis of Planar Rigid-Body Dynamics. *Fifth International Symposium on Robotics Research*, Tokyo.

Bühler, M., Koditschek, D. E., and Kindlmann, P. J. 1989. Planning and Control of Robotic Juggling Tasks. *Fifth International Symposium on Robotics Research*, August 28–31, 1989, Tokyo, Japan.

Burridge, R. R., Rizzi, A. A., and Koditschek, D. E. 1995. Toward a Dynamical Pick and Place. *Proceedings of the 1995 IEEE/RSJ International Workshop on Intelligent Robots and Systems*, Pittsburgh, Pennsylvania, Vol 2, pp. 292–297.

Caine, M. E. 1985. Chamferless Assembly of Rectangular Parts in Two and Three Dimensions. M.S. thesis, Massachusetts Institute of Technology, Department of Mechanical Engineering.

Carlisle, B., Goldberg, K., Rao, A., and Wiegley, J. 1994. A Pivoting Gripper for Feeding Industrial Parts. *Proceedings of the 1994 IEEE International Conference on Robotics and Automation*, San Diego, California, pp. 1650–1655.

Cole, A. A., Hsu, P., Sastry, S. S. 1992. Dynamic Control of Sliding by Robot Hands for Regrasping. *IEEE Transactions on Robotics and Automation.* **8**(1):42–52.

Cutkosky, M. R. 1985. *Robotic Grasping and Fine Manipulation.* Boston: Kluwer Academic Publishers.

Cutkosky, M. R., and Hyde, J. M. 1993. Manipulation Control with Dynamic Tactile Sensing. *Sixth International Symposium on Robotics Research*, October 1–5, 1993, Hidden Valley, Pennsylvania, pp. 143–153.

Donald, B. R., Jennings, J., and Rus, D. 1994. Analyzing Teams of Cooperating Mobile Robots. *Proceedings of the 1994 IEEE International Conference on Robotics and Automation*, San Diego, California, pp. 1896–1903.

Drake, S. H. 1977. Using Compliance in Lieu of Sensory Feedback for Automatic Assembly. Sc.D. thesis. Massachusetts Institute of Technology, Department of

Mechanical Engineering.

Erdmann, M. A. 1994. On a Representation of Friction in Configuration Space. *International Journal of Robotics Research.* **13**(3):240–271.

Erdmann, M. A. 1995. Understanding Action and Sensing by Designing Action-Based Sensors. *International Journal of Robotics Research.* **14**(5):483–509.

Erdmann, M., and Mason, M. T. 1988. An Exploration of Sensorless Manipulation. *IEEE Journal of Robotics and Automation.* **4**(4):369–379.

Erdmann, M., Mason, M. T., and Vaněček, G., Jr. 1993. Mechanical Parts Orienting: The Case of a Polyhedron on a Table. *Algorithmica*. **10**:226–247.

Fearing, R. S. 1984. Simplified Grasping and Manipulation with Dextrous Robot Hands. *Proceedings of the 1984 American Control Conference*, pp. 32–38.

Goyal, S., Ruina, A., and Papadopoulos, J. 1991. Planar sliding with dry friction. Part 1. Limit surface and moment function. Part 2. Dynamics of motion. *Wear.* **143**:307–352.

Haug, E. J., Wu, S. C., and Yang, S. M. 1986. Dynamics of Mechanical Systems with Coulomb Friction, Stiction, Impact and Constraint Addition-Deletion. *Mechanism and Machine Theory.* **21**(5):401–425.

Hodgins, J. K., and Raibert, M. H. 1990. Biped Gymnastics. *International Journal of Robotics Research.* **9**(2):115–132.

Hong, J., Lafferriere, G., Mishra, B., and Tan, X. 1990. Fine Manipulation with Multifinger Hands. *Proceedings of the 1990 IEEE International Conference on Robotics and Automation*, Cincinnati, Ohio, pp. 1568–1573.

Howe, R. D., and Cutkosky, M. R. 1992. Touch Sensing for Robotic Manipulation and Recognition. In *The Robotics Review 2*, eds. Khatib, O., Craig, J., and Lozano-Pérez, T., Cambridge, MA: MIT Press.

Howe, R. D., and Cutkosky, M. R. 1993. Dynamic Tactile Sensing: Perception of Fine Surface Features with Stress Rate Sensing. *IEEE Transactions on Robotics and Automation.* **9**(2):140–151.

Inoue, H. 1974. Force Feedback in Precise Assembly Tasks. A.I. Memo 308. Cambridge, Mass.: Massachusetts Institute of Technology Artificial Intelligence Laboratory. Reprinted in Winston, P. H., and Brown, R. H., eds. 1979. *Artificial Intelligence: An MIT Perspective.* Cambridge, Mass.: MIT Press.

Kao, I., and Cutkosky, M. R. 1989. Dextrous Manipulation with Compliance and Sliding. *Fifth International Symposium on Robotics Research*, August 28–31, 1989, Tokyo, Japan.

Kao, I., and Cutkosky, M. R. 1992. Quasistatic Manipulation with Compliance and Sliding. *International Journal of Robotics Research.* **11**(1):20–40.

Kao, I., and Cutkosky, M. R. 1993. Comparison of Theoretical and Experimental Force/Motion Trajectories for Dextrous Manipulation With Sliding. *International Journal of Robotics Research.* **12**(6):529–534.

Latombe, J.-C. 1991. *Robot Motion Planning.* Boston: Kluwer Academic Publishers.

Li, Z., and Canny, J. 1990 Motion of Two Rigid Bodies with Rolling Constraint. *IEEE Transactions on Robotics and Automation.* **6**(1):62–72.

Lötstedt, P. 1982. Mechanical Systems of Rigid Bodies Subject to Unilateral Constraints. *SIAM Journal of Applied Mathematics.* **42**(2):281–296.

Lozano-Pérez, T. 1981. Automatic Planning of Manipulator Transfer Movements. *IEEE Transactions on Systems, Man, and Cybernetics.* **SMC–11**(10):681–698. Reprinted in Brady, M., et al., eds. 1982. *Robot Motion.* Cambridge, Mass.: MIT Press.

Lozano-Pérez, T. 1983. Spatial Planning: A Configuration Space Approach. *IEEE Transactions on Computers.* **C-32**(2):108–120.

Lynch, K. M. 1992. The Mechanics of Fine Manipulation by Pushing. *Proceedings of the 1992 IEEE International Conference on Robotics and Automation*, Nice, France, pp. 2269–2276.

Lynch, K. M., and Mason, M. T. 1995. Pulling by Pushing, Slip with Infinite Friction, and Perfectly Rough Surfaces. *International Journal of Robotics Research.* **14**(2):174–183.

Mani, M., and Wilson, W. 1985. A Programmable Orienting System for Flat Parts. *Proc., NAMRI XIII*.

Mason, M. T. 1982. Manipulator Grasping and Pushing Operations. AI-TR-690. Ph.D. thesis. Massachusetts Institute of Technology, Artificial Intelligence Laboratory.

Mason, M. T. 1986. Mechanics and Planning of Manipulator Pushing Operations. *International Journal of Robotics Research.* **5**(3):53–71.

Mason, M. T., and Lynch, K. M. 1993. Dynamic Manipulation. *Proceedings of the 1993 IEEE/RSJ International Workshop on Intelligent Robots and Systems*, Yokohama, Japan, pp. 152–159.

Mishra, B., Schwartz, J. T., and Sharir, M. 1987. On the Existence and Synthesis of Multifinger Positive Grips. *Algorithmica*. **2**(4):541–558.

Montana, D. J. 1988. The Kinematics of Contact and Grasp. *International Journal of Robotics Research.* **7**(3):17–32.

Murray, R. M., Li, Z., and Sastry, S. S. 1994. *A*

Mathematical Introduction to Robotic Manipulation. Boca Raton, Florida: CRC Press.

Nakamura, Y., Nagai, K., and Yoshikawa, T. 1989. Dynamics and Stability in Coordination of Multiple Robotic Mechanisms. *International Journal of Robotics Research.* **8**(2):44–61.

Natarajan, B. K. 1986. An Algorithmic Approach to the Automated Design of Parts Orienters. *Proceedings of the 27th Annual IEEE Symposium on Foundations of Computer Science*, pp. 132–142.

Nevins, J., Whitney, D., Drake, S., Killoran, D., Lynch, M., Seltzer, D., Simunovic, S., Spencer, R. M., Watson, P., and Woodin, A. 1975. Exploratory Research in Industrial Modular Assembly. Report R-921. C.S. Draper Laboratory, Cambridge, Mass.

Nguyen, V-D. 1988. Constructing Force-Closure Grasps. *International Journal of Robotics Research.* **7**(3):3–16.

Ohwovoriole, M. S., Hill, J. W., and Roth, B. 1980. On the Theory of Single and Multiple Insertions in Industrial Assemblies. *Proc. 10th International Symposium on Industrial Robots.* Bedford, U.K.: IFS Publications, pp. 545–558.

Ohwovoriole, M. S., and Roth, B. 1981. A Theory of Parts Mating for Assembly Automation. *Proceedings of the Robot and Man Symposium 81*, Warsaw, Poland.

Omata, T., and Nagata, K. 1994. Planning Reorientation of an Object with a Multifingered Hand. *Proceedings of the 1994 IEEE International Conference on Robotics and Automation*, San Diego, California, pp. 3104–3110.

Paljug, E., and Yun, X. 1993. Experimental Results of Two Robot Arms Manipulating Large Objects. *Proceedings of the 1993 IEEE International Conference on Robotics and Automation*, Atlanta, Georgia, Vol 1, pp. 517–522.

Paljug, E., Yun, X., and Kumar, V. 1994. Control of Rolling Contacts in Multi-Arm Manipulation. *IEEE Transactions on Robotics and Automation.* **10**(4):441–452.

Peshkin, M. A. 1986. Planning Robotic Manipulation Strategies for Sliding Objects. Ph.D. thesis. Carnegie Mellon University, Physics Department.

Peshkin, M. A., and Sanderson, A. C. 1988. Planning Robotic Manipulation Strategies for Workpieces that Slide. *IEEE Journal of Robotics and Automation.* **4**(5): 524–531.

Pingle, K., Paul, R., and Bolles, R. 1974. *Programmable Assembly, Three Short Examples*. Film. Stanford Artificial Intelligence Laboratory.

Ponce, J., Stam, D., and Faverjon, B. 1993. On Computing Two-Finger Force-Closure Grasps of Curved 2D Objects. *International Journal of Robotics Research.* **12**(3):263–273.

Ponce, J., Sullivan, S., Boissonnat, J.-D., and Merlet, J.-P. 1993. On Characterizing and Computing Three- and Four-Finger Force-Closure Grasps of Polyhedral Objects. *Proceedings of the 1993 IEEE International Conference on Robotics and Automation*, Atlanta, Georgia, Vol 2, pp. 821–827.

Raibert, M. H. 1986. *Legged Robots That Balance.* Cambridge, Mass.: MIT Press.

Rizzi, A. A., and Koditschek, D. E. 1993. Further Progress in Robot Juggling: The Spatial Two-Juggle. *Proceedings of the 1993 IEEE International Conference on Robotics and Automation*, Atlanta, Georgia, Vol 3, pp. 919–924.

Rus, D. 1992. Fine Motion Planning for Dexterous Manipulation. Ph.D. thesis. Department of Computer Science, Cornell University.

Salisbury, J. K. 1982. Kinematic and Force Analysis of Articulated Hands. Ph.D. thesis. Department of Mechanical Engineering, Stanford University.

Salisbury, J. K. 1987. Whole Arm Manipulation. *Proceedings 4th International Symposium of Robotics Research*, pp. 183–189.

Sawasaki, N., Inaba, M., and Inoue, H. 1989. Tumbling Objects Using Multi-Fingered Robot. *Proceedings of the 20th ISIR*, pp. 609-616.

Schaal, S., and Atkeson, C. G. 1993. Open Loop Stable Control Strategies for Robot Juggling. *Proceedings of the 1993 IEEE International Conference on Robotics and Automation*, Atlanta, Georgia, Vol 3, pp. 913–918.

Schwartz, J. T., and Sharir, M. 1983. On the Piano Movers' Problem: II. General Techniques for Computing Topological Properties of Real Algebraic Manifolds. *Advances in Applied Mathematics.* **4**:298–351.

Simunovic, S. N. 1979. An Information Approach to Parts Mating. Sc.D. thesis. Massachusetts Institute of Technology, Department of Mechanical Engineering.

Sturges, R. H., Jr. 1988. A Three-Dimensional Assembly Task Quantification with Application to Machine Dexterity. *International Journal of Robotics Research.* **7**(4):34–78.

Teichmann, M., and Mishra, B. 1994. Reactive Algorithms for 2 and 3 Finger Grasping. *Proceedings of the 1994 IEEE/RSJ International Workshop on Intelligent Robots and Systems*, Grenoble, France, July 11–14.

Trinkle, J. C. 1992. On the Stability and Instantaneous Velocity of Grasped Frictionless Objects. *IEEE*

Transactions on Robotics and Automation. **8**(5):560–572.

Trinkle, J. C., Abel, J. M., and Paul, R. P. 1988. An Investigation of Frictionless Enveloping Grasping in the Plane. *International Journal of Robotics Research.* **7**(3):33–51.

Trinkle, J. C., and Paul, R. P. 1990. Planning for Dexterous Manipulation with Sliding Contacts. *International Journal of Robotics Research.* **9**(3):24–48.

Trinkle, J. C., and Hunter, J. J. 1991. A Framework for Planning Dexterous Manipulation. *Proceedings of the 1991 IEEE International Conference on Robotics and Automation*, Sacramento, California, pp. 1245–1251.

Trinkle, J. C., Ram, R. C., Farahat, A. O., and Stiller, P. F. 1993. Dexterous Manipulation Planning and Execution of an Enveloped Slippery Workspace. *Proceedings of the 1993 IEEE International Conference on Robotics and Automation*, Atlanta, Georgia, Vol 2, pp. 442–448.

Trinkle, J., Pang, J.-S., Sudarsky, S., and Lo, G. 1995. On Dynamic Multi-Rigid-Body Contact Problems with Coulomb Friction. Technical Report TAMU-CS TR95-003. Department of Computer Science, Texas A&M University.

Wang, Yu, and Mason, M. 1987. Modeling Impact Dynamics for Robotic Operations. *Proceedings of the 1987 IEEE International Conference on Robotics and Automation*, pp. 678–685.

Whitney, D. E. 1982. Quasi-Static Assembly of Compliantly Supported Rigid Parts. *Journal of Dynamic Systems, Measurement, and Control.* **104**:65–77. Reprinted in Brady, M., et al., eds. 1982. *Robot Motion.* Cambridge, Mass.: MIT Press.

Xue, Y., and Kao, I. 1994. Dextrous Sliding Manipulating Using Soft Fingertips. *Proceedings of the 1994 IEEE International Conference on Robotics and Automation*, San Diego, California, pp. 3397–3402.

Yokokohji, Y., Yu, Y., Nakasu, N., and Yoshikawa, T. 1993. Quasi-Dynamic Manipulation of Constrained Object by Robot Fingers in Assembly Tasks. *Proceedings of the 1993 IEEE/RSJ International Workshop on Intelligent Robots and Systems*, Yokohama, Japan, pp. 144–151.

Yun, X. 1993. Object Handling Using Two Arms Without Grasping. *International Journal of Robotics Research.* **12**(1):99–106.

Yoshikawa, T., Yokokohji, Y., and Nagayama, A. 1993. Object Handling by Three-Fingered Hands Using Slip Motion. *Proceedings of the 1993 IEEE/RSJ International Workshop on Intelligent Robots and Systems*, Yokohama, Japan, pp. 99–105.

Yoshikawa, T., and Zheng, X.-Z. 1993. Coordinated Dynamic Hybrid Position/Force Control for Multiple Robot Manipulators Handling One Constrained Object. *International Journal of Robotics Research.* **12**(3):219–230.

Zumel, N. B. 1995. Analysis and Planning for Nonprehensile Two Palm Manipulation. Ph.D. thesis proposal. The Robotics Institute, Carnegie Mellon University.

Zumel, N. B., and Erdmann, M. A. 1996. Nonprehensile Two Palm Manipulation with Non-Equilibrium Transitions between Stable States. *Proceedings of the 1996 IEEE International Conference on Robotics and Automation*, Minneapolis, Minnesota.