Robot Localization using a Computer Vision Sextant

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Abstract

This paper explores the possibility of using Sun altitude for localization of a robot in totally unknown territory. A set of Sun altitudes is obtained by processing a sequence of time-indexed images of the sky. Each altitude constrains the viewer to a circle on the surface of a celestial body, called the circle of equal altitude. A set of circles of equal altitude can be intersected to yield viewer position. We use this principle to obtain position on Earth. Since altitude measurements are corrupted by noise, a least-square estimate is numerically calculated from the sequence of altitudes. The paper discusses the necessary theory for Sun-based localization, the technical issues of camera calibration and image processing, and presents preliminary results with real data.

1 Introduction

This work addresses the problem of robot localization: how can a robot make precise measurements of position in totally unknown territory? We focus on the use of celestial cues in order to obtain localization. Techniques for celestial navigation are fundamental for any sea navigator [3]. Celestial information is also used in spacecraft and satellite applications for attitude control and accurate positioning [5]. Yet celestial navigation has not been heavily mentioned in connection to autonomous rovers. This comes as a surprise since planetary rovers will need reliable means of checking dead reckoning data.

We pursue an automatic solution of the localization problem. Instead of using a sextant to obtain the altitude of a celestial body, we use a camera and a digital inclinometer. The measurements are processed by a non-linear least-squares optimization that replaces the tables used by mariners [3]. We use measurements of *Sun* altitude, but the basic ideas could be applied for any celestial bodies: satellites, planets, stars. Our primary target is navigation in cloudless atmospheres (not Earth) during long-duration missions.

In this paper, we describe a system in which these ideas were implemented and tested for a demonstration of feasibility. This system is based on a camera with telephoto lens and a digital inclinometer for measurement of attitude of camera. Results of real runs are shown in the paper.

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2 Celestial Navigation

This section summarizes basic concepts used in the celestial navigation literature. An extensive treatment is given by Hobbs [3].

We take position on Earth to be represented by a latitude/longitude pair. For calculations, we take south latitude as negative (zero at the equator) and west longitude as negative (zero at Greenwich).

Every celestial body can be projected onto the terrestrial surface by considering the line that goes from the center of Earth to the body. The point of intersection between the terrestrial surface and this line is the *geographic position*, denoted GP, of the body. The latitude of the GP of a body in the celestial sphere is denoted *declination*.

The Sun maintains an almost constant declination on a given day. Sun declination varies slowly between $+23$ and -23 degrees, during a year. Sun declination is tabulated in the Astronomical Almanac [1] for every day of a given year.

The longitude of the GP of the Sun, contrary to its declination, changes continuously during a day (since the Sun moves from East to West). We can recover longitude of the GP of the Sun if we know Greenwich Mean Time (GMT), since time is proportional to the motion of the Sun during the day. GMT is a mean value; there are drifts and variations during the year. For this reason, a small correction must be subtracted from GMT in order to obtain the longitude of the GP of the Sun. The correction is called *equation of time* and it is tabulated in the Astronomical Almanac for every day of a given year.

Once we obtain declination and equation of time, we can calculate the GP of the Sun for every given instant.

A measurement of Sun altitude constrains the observer to lie on a circle on the terrestrial surface, called the *circle of equal altitude*. As the name suggests, from any point on the circle, the celestial body will appear to have the same altitude. A circle of equal altitude is defined by a measurement of Sun altitude and the exact time of measurement. Two circles of equal altitude, arising from temporally distinct measurements, constrain the observer's position to two points (the points of intersection of the circles). Three circles of equal altitude constrain the observer's position to one point. Four or more measurements of altitude overconstrain the observer's position and can be used to check or correct previous measurements.

Figure 1 illustrates a situation where two stars are measured. Each measurement gives rise to one circle of equal altitude, so the two measurements give rise to two circles of equal altitude, which intersect at two points, thus yielding two possible solutions. In general the solutions are very far from each other and one can be discarded based on other information, for example, from dead reckoning.

Our system solves the localization problem in the following four steps:

1. Take a sequence of Sun images.
2. Obtain Sun altitude and the GP of the Sun for each image in the sequence.
3. Use Sun altitude and the GP of the Sun to generate a set of circles of equal altitude.
4. Intersect the circles of equal altitude numerically in a least-squares procedure.

This scheme is described in detail in the next sections.

3 Obtaining Circles of Equal Altitude

We need to measure Sun altitude and obtain the GP of the Sun. Solutions to these two problems are discussed in turn.

3.1 Obtaining Altitude

Sun altitude is measured using a camera and a digital inclinometer.

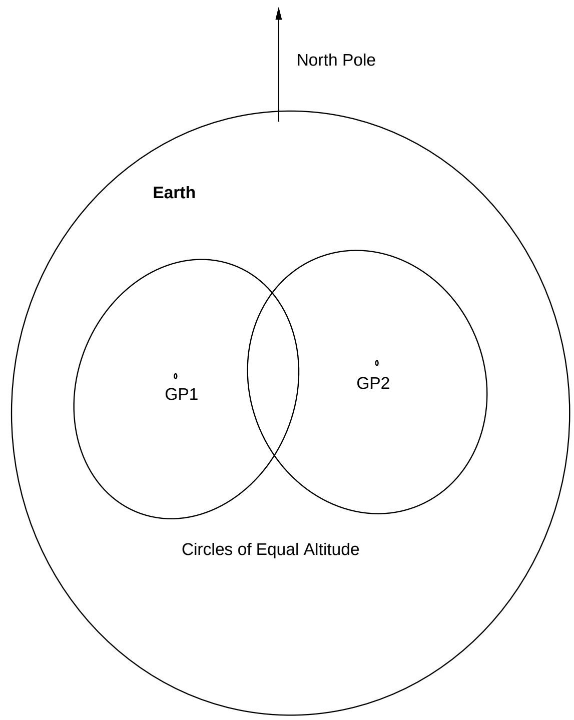


Figure 1: Two Measurements of Altitude: GPs and Circles of Equal Altitude

The camera, attached to a horizontal adjustable platform, is aimed at the Sun. A series of neutral density filters is used to eliminate excessive solar brightness. In order to reduce the influence of camera calibration, a small field of view is desired: with a large field of view, it would be necessary to know precisely the center of the image and the focal length. Telephoto lens (focal length 300mm) are used to produce a small field of view (Figure 2 depicts the hardware). Sun images are sent to a workstation, which performs the following operations:

1. A histogram is made with the pixel values, and the image is thresholded so that only the brightest 20% pixels remain valid. At this point the image is binary, but it contains several small regions (one or two pixels) around the main region (the Sun).
2. A grassfire transform is performed in order to destroy noisy features in the thresholded image. The grassfire transform is a fast algorithm for execution of morphological operations on images (for a discussion of this and other algorithms, see Xia's paper [6]). In a binary image, the grassfire transform calculates, for each pixel, the distance be-

Figure 2: Camera, Digital Inclinometer, Neutral Density Filters and Adjustable Platform

tween the pixel and the background. In an image, background can be taken either as the "0" pixels or the "1" pixels. Initially the grassfire transform is used to shrink regions. Regions with less than 4 pixels of radius are then deleted (by deleting all pixels with distances smaller than 4). The grassfire transform is then applied again to grow the regions. Large regions remain untouched, but smaller regions disappear in the process.

3. A region coloring algorithm is used to connect all remaining regions in the thresholded image. The algorithm scans the image and grows every region as much as possible.

4. Each connected region is analyzed. A region has its area and aspect ratio calculated; the largest region that obeys given constraints in aspect ratio is declared to correspond to the Sun.

5. The centroid of the chosen region is calculated and reported as the position of the Sun in the image.

Figure 3 shows a typical result: the region that corresponds to the Sun is white; the centroid of this region is marked with a cross.

Determination of the centroid of the Sun determines the angle between the optical axis and the ray that emanates from the center of the image to the Sun. This angle is:

$$\alpha = \arctan\left(\frac{v - v_o}{f_v}\right)$$

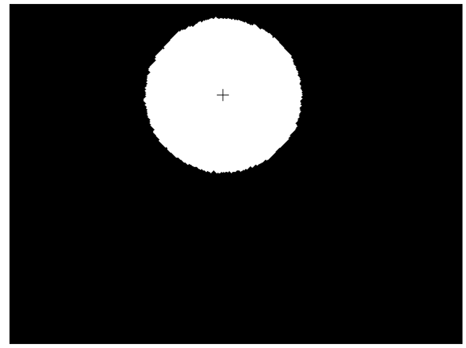


Figure 3: Typical Images in Altitude Measurement

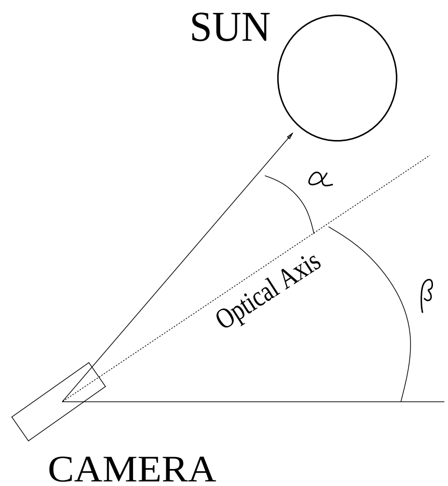


Figure 4: Geometry of Sun altitude measurements

where:

v Row of the centroid of the Sun (in pixels).

v_o Row of the center of the image (in pixels).

f_v Focal length (in pixels).

The parameters v_o and f_v are intrinsic parameters of the camera and their values were obtained through a calibration procedure (section 5).

Figure 4 illustrates the geometry of altitude measurement: it is necessary to obtain the angle between the Sun and the optical axis, *and* the angle between the optical axis and horizontal. The latter is measured with a digital inclinometer. Raw altitude is given by:

$$h_a = \alpha + \beta$$

where β is the inclinometer reading.

There are additional factors that must be taken into account in this type of measurement [3]. The gradual bending of an incoming light ray in the Earth's atmosphere is called *atmospheric refraction*; it must be corrected. Atmospheric refraction depends on the raw altitude, because rays bend in different ways when they reach the atmosphere at different angles. The refraction correction R in degrees is given by:

$$R = \frac{0.0167}{\tan\left(h_a + \frac{7.31}{h_a+4.4}\right)}.$$

The difference in the apparent altitude of a body within the solar system as viewed from the surface of the Earth and from its center is called *parallax*; it must be corrected since the images of the Sun are taken from its surface, not from its center. The parallax correction (referred to as PA [3]) for the Sun is:

$$PA = 0.0024 \cos h_a.$$

The final observed altitude is $h_o = h_a - R + PA$.

3.2 Obtaining GP of the Sun

The astronomical position of the Sun is fixed for a given Greenwich Mean Time value. A set of formulae is given in the Astronomical Almanac for automatic calculation of astronomical position of the Sun.

4 Computation of Localization Estimates

The result of the measurement process previously outlined produces:

- A sequence of altitudes, indexed by time.
- A sequence of GPs of the Sun, indexed by time.

From each pair altitude/GP, it is possible to define a circle of equal altitude on the surface of the Earth. If we parameterize the Earth surface in longitude/latitude coordinates, a circle of equal altitude corresponds to the loci of solutions of:

$$\sin l \sin l_{GP} + \cos l \cos l_{GP} \cos(\lambda_{GP} + \lambda) = \sin h_o, \quad (1)$$

where (l, λ) is a point in latitude/longitude space, (l_{GP}, λ_{GP}) is the GP of the Sun in this space and h_o is the measured altitude.

Since we have n pairs altitude/GP, we have a series of n equations in two unknowns. The intersection of all circles of equal altitude is performed numerically. We set the following non-linear least squares problem:

$$\begin{aligned}(\hat{l}, \hat{\lambda}) \quad = \quad & \arg\min_{l,\lambda} \Bigg[\sum_{i=1}^{n} (\sin l \sin l_{GPi} + \\ & \cos l \cos l_{GPi} \cos(\lambda_{GPi} + \lambda) - \sin h_{oi})^2 \Bigg] \quad (2)\end{aligned}$$

The minimization starts with a grid of starting points. The user defines a rectangular region (in terms of latitude/longitude) and spacings for a grid that covers a region on Earth. The size of the initial grid is arbitrary; the larger the grid, the longer the minimization will take. We have observed that the minimization, for a fixed starting grid, increases linearly with the number of images. This can be expensive after a great number of images has been acquired. We use the following procedure instead.

1. Minimize expression (2) for the first two images, for all points in the starting grid.
2. Combine the solutions that are too close; use the solutions as new starting points.
3. Take a new image and augment the summation in expression (2).
4. Minimize expression (2), for the new starting points.
5. Go to the second step (while images are available).

The remaining solutions are the result of the whole procedure.

5 System Calibration

The system is sensitive to errors due to miscalibration. Three measurements are effected every time a pair altitude/GP is obtained:

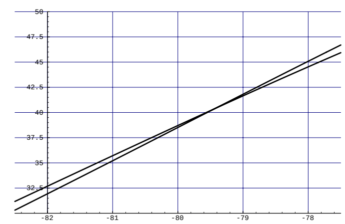
- An image is acquired. Camera parameters were obtained through a calibration method proposed by Robert [4]. The method generates the perspective projection matrix [2] which contains all the relevant information about the imaging system.
- An angle is obtained through a digital inclinometer. We use a digital inclinometer with recalibration, but limited to ± 0.2 degrees of accuracy (reduced to ± 0.4 for large inclinations).
- Greenwich Mean Time is stored. Measurement of time is provided by the computer network and can be easily calibrated through a network call to a NIST server.

6 Experiments

A preliminary version of the system was implemented and tested in real environments. A platform was built to which the camera/lens/filter apparatus was attached. The platform was leveled using the digital inclinometer.

Measurements were taken at different times of day, in periods of one hour. In each hour, six or eight measurements were taken. All experiments were performed at $(40°26', -79°59')$ latitude/longitude (ground truth obtained with GPS). Two runs of the system are presented below.

The first two measurements (Figure 5, top) were taken respectively at 22:50:24 and 23:0:41 (GMT), June 3 1994. The temporal distance between measurements was small, so the circles of equal altitude are almost superimposed. Minimizing equation (2) with both measurements yields $(40°22', -79°26')$ (maximum relative error of 0.68 %; each degree corresponds roughly to 100km). The second two measurements (Figure 5, bottom) were taken May 23 1994, respectively at 22:11:23 and 22:37:59 (GMT). Result of both measurements is $(40°37', 79°52')$ (maximum relative error of 0.44 %). These measurements are indicative of the accuracy we are experiencing with our system, still in the order of minutes of arc. In its current version, the system is not practical for real navigation, since such errors in latitude/longitude translate into large positional errors. The disturbances come from the fact that our platform is not completely rigid, suffering from perturbations from the wind. A heavier, more stable platform is necessary to increase accuracy. But the most critical component of our system is the digital inclinometer. For the measurements we describe, we had to use the inclinometer in its less sensitive range, achieving angular accuracy of ± 0.4 degrees. This alone is enough to corrupt our position calculations. A new, very precise inclinometer is the very next step, but in order to use it properly, a heavy and safe platform must be in place.



Measurements June 3 94

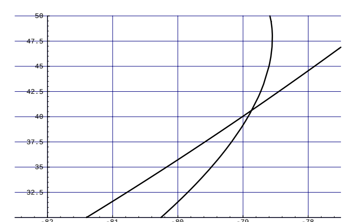


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Figure 5: Data from Sun Measurements: Latitude $\times$ Longitude (values in degrees).

7 Conclusion

The use of celestial cues for robot localization in unknown territory is a useful aid to traditional approaches such as dead reckoning. This work demonstrates the feasibility of this idea, by combining old ideas from the marine navigation community with image processing algorithms.

Further work will be required to deploy a system based on the computer vision sextant idea. Next steps for improvement of our system are: building a more stable platform and using a set of accurate inclinometers for measurement of altitude. The principal difficulty is to maintain the Sun within the field of view. We are considering two approaches: (1) physical tracking with telephoto lens; (2) providing a hemispherical field of view to the camera (with more sophisticated calibration), so that the Sun is always in sight.

The system does offer the perspective of a new source of information into an autonomous agent. The sky is almost always observable in outdoor areas, its properties are known with extraordinary accuracy, and it is independent from facts that occur in the vicinity of the agent. So celestial information can be a most valuable source of information in scenarios where the autonomous robot is lost, either because the territory is new or because traditional sensors are confused or broken.

References

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