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Detecting Irregularities in Cyclic Motion

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Abstract

Real cyclic motions tend not to be perfectly even, i.e., the period varies slightly from one cycle to the next, because of physically important changes in the scene. A generalization of period *is defined for cyclic motions that makes periodic variation explicit. This representation, called the period trace, is compact and purely temporal, describing the evolution of an object or scene without reference to spatial quantities such as position or velocity. By delimiting cycles and identifying correspondences across cycles, the period trace provides a means of temporally registering a cyclic motion. In addition, several purely temporal motion features are derived, relating to the nature and location of irregularities. Results are presented using real image sequences and applications to athletic and medical motion analysis are discussed.*

1 Introduction

Periodic motions abound in nature as well as in the man-made world. Examples include the motions of a heart beating, an athlete running, or a wheel rotating. Although many real motions are intrinsically repeating, few are perfectly periodic. For instance, a walker's stride frequency may vary visibly from one cycle to the next and a heart may beat slower or faster according to changing activity levels. We use the term *cyclic* to describe motions that repeat but lack a constant period.

An analog of period can be defined for cyclic motions which we call *instantaneous period* that gives cycle length as a function of time. In applications such as cardiac image analysis the instantaneous period can be used to extract individual cycles and to obtain inter-cycle correspondences, providing a means of tracking changes in a heart across different cycles.

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Meaningful information about irregularities can be inferred by comparing different cycles of a cyclic motion. If two cycles are of different lengths, one is irregular with respect to the other. The exact point in time where an irregularity occurs can be determined by considering the derivative of cycle length (which we define in Section 2.3). Furthermore, the nature of the irregularity can be recovered, indicating how the motion varies locally relative to the norm.

Detection of irregularity information has applications in athletic and medical motion analysis. By analyzing the variance of the period in several cycles of a runner's gait or a swimmer's stroke, anomalies can be located providing feedback on specific areas which may need improvement [1]. Medical imagery can be analyzed using the same techniques, for instance to detect unevenness in a heart's beating motion. Finally, a motion's periodic variance provides a signature that can be used to detect certain qualities of a motion based on the pattern of irregularities that the motion exhibits.

The remainder of the paper is structured as follows. Section 2 formally defines cyclic motion and introduces the notion of *period trace*. Section 3 introduces several motion features that can be derived from the period trace. Section 4 presents a method for automatically recovering a motion's period trace using an optimization approach. Section 5 describes experiments on real image sequences.

2 Motion Classes

Most existing representations of motion describe how a set of intrinsic parameters of a specific object model change over time. Examples include point or line-based feature trajectories [2, 3], spatiotemporal surfaces [4], joint angle paths [5], and evolving physically-based representations [6]. In many applications, however, we are interested in the temporal evolution of an object or a scene and not its instantaneous shape. Consequently, in this section we introduce an alternative, purely temporal approach that is independent of an object's spatial properties. Our temporal motion representation, called the *period trace*, describes how the period changes throughout the course of a motion and contains a wealth of useful motion information. Because no assumptions are made about the spatial structure or representation of the scene, the period trace can be recovered from standard motion representations, including [2, 3, 4, 5, 6], and used to analyze both rigidly and non-rigidly moving objects and scenes. To motivate the period trace, we first define what it means for a motion to be *periodic* and, more generally, *cyclic*.

2.1 Periodic Motions

We define a *motion* $\mathbf{M}(t)$ to be a function whose value at time t is the instantaneous configuration of a continuously-moving object or scene[1]. We use the term *still* to refer to the instantaneous value of a motion at a particular point in time. We call a motion $\mathbf{P}$ *periodic* if it repeats with period p, i.e.,

$$\mathbf{P}(t+p) = \mathbf{P}(t) \tag{1}$$

holds for some constant $p > 0$ and all times t in a given time domain. The smallest such constant p is the *period* and the set $\mathbf{P}_{t_0} = \{\mathbf{P}(t) \mid t_0 \leq t < t_0 + p\}$ is called the *cycle* beginning at time t_0.

Examples of periodic motions include a rotating wheel, a spinning top, a beating heart, waving gestures, and an athlete running in place. However, many familiar motions that we intuitively characterize as being periodic do not satisfy Eq. (1). For instance, consider a runner who moves along an arbitrary path. The runner's motion can be decomposed into a periodic component (running in place) and a component consisting of whole-body movement along the path[2].

Eq. (1) can be generalized to describe partially-periodic motions by replacing equality with equivalence under a certain class of transformations. Towards this end, we introduce the concept of a *match function*, d, which identifies any two images that belong to the same equivalence class, i.e., $d(I_1, I_2) = 0$ iff $I_1 \equiv I_2$. Various forms of invariance can be achieved using match functions. In particular, view-invariance can be achieved with an appropriate match function [7]. Eq. (1) is generalized to

$$d(\mathbf{P}(t+p), \mathbf{P}(t)) = 0 \tag{2}$$

for a given match function, d.

2.2 Cyclic Motions

The generalized notion of periodicity given in Eq. (2) has a very restrictive temporal constraint, namely, that the motion is perfectly regular from one cycle to the next. This constraint is relaxed with the introduction of a *period-warping* function, ϕ, as follows:

Definition 1 *A motion* $\boldsymbol{C}$ *is called* ***cyclic*** *if*

$$\mathbf{C}(\phi(t)) = \mathbf{C}(t)$$

for all times t *in a given time domain and some increasing continuous function* ϕ *satisfying* $\phi(t) > t$. *A function* ϕ *satisfying these properties is called* **C-warping**.

Intuitively, $\phi(t)$ corresponds to the start of the next cycle, where the current cycle begins at time t. The increasing condition on ϕ ensures that a cyclic motion is order preserving, i.e., $t_1 < t_2$ implies $\phi(t_1) < \phi(t_2)$. Notice that all periodic motions are cyclic and that a cyclic motion is periodic when Def. 1 is satisfied for $\phi(t) = t + p$, with p the period.

2.3 The Period Trace

A $\mathbf{C}$-warping function ϕ describes the temporal variation of a cyclic motion $\mathbf{C}$. Unfortunately, Def. 1 does not imply a unique $\mathbf{C}$-warping ϕ for a given cyclic motion $\mathbf{C}$. For instance, if $\mathbf{C}$ is periodic with period p, $\phi = t + kp$ for any positive integer k satisfies Def. 1. To get around this problem, we define the notion of *instantaneous period* as follows:

Definition 2 *Let* $\boldsymbol{C}$ *be a cyclic motion. Let*

$$\phi_1 = \text{pointwise-infimum } \{\phi \ \boldsymbol{C}\text{-}\textit{warping}\}$$

Define $\tau_1(t) = \phi_1(t) - t$. τ_1 *is called the* ***instantaneous period*** *of* $\boldsymbol{C}$.

The fact that ϕ_1 is $\mathbf{C}$-warping comes follows from the continuity of $\mathbf{C}$: Let $\mathbf{A} = \{\phi \ \boldsymbol{C}\text{-}\textit{warping}\}$ and let $\{\psi_i\}_i$ be a subset of $\mathbf{A}$ converging to ϕ_1 at t. Then $\mathbf{C}(\psi_i(t)) = \mathbf{C}(t)$ for each i, so $\lim_{i\to\infty} \mathbf{C}(\psi_i(t)) = \mathbf{C}(t)$. But $\lim_{i\to\infty} \mathbf{C}(\psi_i(t)) = \mathbf{C}(\phi_1(t))$ by continuity of $\mathbf{C}$. It follows that ϕ_1 is $\mathbf{C}$-warping.

[1] Although time is implicit, t could represent another quantity.

[2] Others have referred to this decomposition in terms of "relative" and "common" motion.

Intuitively, $\tau_1(t)$ is the length of the cycle beginning at time t. For instance, if $\mathbf{C}$ is periodic then τ_1 is the period. From τ_1, several useful quantities are computable, including τ_n, the instantaneous combined length of the next n cycles. Accordingly, let $\phi_n = (\phi)^n$ for integers $n > 0$, where product implies functional composition. Then the *nth instantaneous period*, τ_n, is defined as

$$\tau_n(t) = \phi_n(t) - t \tag{3}$$

In addition to being continuous, τ_n has the following property:

$$\frac{\tau_n(t+h) - \tau_n(t)}{h} > -1 \tag{4}$$

which follows from Eq. (3) and the increasing condition on ϕ_1. If τ_n is differentiable, Eq. (4) is equivalent to the condition $\tau_n' > -1$.

The inverse functions exist and are defined as

$$\begin{aligned} \phi_{-n} &= (\phi_n)^{-1} \\ \tau_{-n}(t) &= \phi_{-n}(t) - t \end{aligned}$$

for $n > 0$. $\tau_{-n}(t)$ has the intuitive interpretation as the combined length of the previous n cycles ending at time t.

We refer to the set $\{\tau_n \mid n \neq 0\}$ as the *period trace* of a cyclic motion $\mathbf{C}$. For instance, the period trace of a periodic function is a set of constant functions: $\tau_n = np$, where p is the period (see Fig. 1). Since it consists of all of the instantaneous periods of a motion, the period trace provides a comprehensive description of cycles and their irregularities.

3 Detecting Irregularities

The period trace of a cyclic motion describes how it varies from one cycle to the next. Consequently, many important attributes of a motion can be computed directly from the period trace, without reference to the spatial structure of the underlying scene. In this section we present several features that quantify temporal irregularities in a cyclic motion.

3.1 Local Features

Because the period trace reflects the cumulative motion of n cycles, it is not obvious that local temporal information can be derived from it. In this section we show that local temporal irregularities can indeed be computed from the derivatives of τ_n. Any interval $[t, t+h]$ in which τ_n is not constant, for some n, is said to be an *irregular interval*. The quantity $\tau_n(t+h) - \tau_n(t)$ gives the cumulative change in the nth period and $\frac{\tau_n(t+h)-\tau_n(t)}{h}$ is the mean rate of change of τ_n in the interval $[t, t+h]$. Letting $h \to 0$, the mean rate of change converges to the instantaneous rate of change, $\tau_n'(t)$. Henceforth, we denote τ_n' as the one-sided derivative defined by $\tau_n'(t) = \lim_{h\to 0} \frac{\tau_n(t+h)-\tau_n(t)}{h}$.

Values of t for which $\tau_n'(t) \neq 0$ are *irregular points*, i.e., points where the motion is faster or slower at $\phi_n(t)$ than at t. Moreover, τ_n' provides both the sign and magnitude of the irregularity. For instance, $\tau_1'(t) = 1$ indicates that the period at time t is increasing at a rate of 1, i.e., motion is faster at t than at $\phi_1(t)$ by 1 unit. Because the motion is faster at the beginning of the cycle than at the end, it must slow down during the course of the cycle, thereby causing a net increase in the period.

Generally, not all points t where $\tau_n'(t) \neq 0$, for some $n \neq 0$, correspond to times where the motion is globally uneven (see Fig. 1). A nonzero value of $\tau_n'(t)$ indicates that motion is irregular at time t *relative* to time $\phi_n(t)$. A point t where the motion is globally irregular satisfies $\tau_n'(t) \neq 0$, for all $n \neq 0$.

The total first-order irregularity of a cyclic motion $\mathbf{C}$ at a point in time t can be defined as:

$$irreg_{\mathbf{C}}(t) = \underset{n\neq 0}{mean}\, |\tau_n'(t)| \tag{5}$$

Higher order irregularities may also be relevant in certain situations. For instance, consider an accelerating jogger whose period is steadily increasing. Due to the acceleration, τ_n' will be nonzero and second-order irregularities may be more interesting, i.e., where $\tau_n''(t) \neq 0$. Unless otherwise qualified, the term *irregularity* refers to a first-order irregularity.

3.2 Global Features

The period trace can also be used to compute various global features of a cyclic motion. These include

- period
- cyclic acceleration
- shortest and longest cycle
- regular and irregular points, cycles, and intervals

The simplest of these is the period. If $\tau_1' = 0$ then τ_1 is constant and corresponds to the period. Similarly, if $\tau_1' = c$, where $c > 0$, then the period is uniformly increasing at a rate of c. In the case of an accelerating jogger, τ_1' gives the rate of acceleration in terms of the rate of change of the stride period. Note that it is

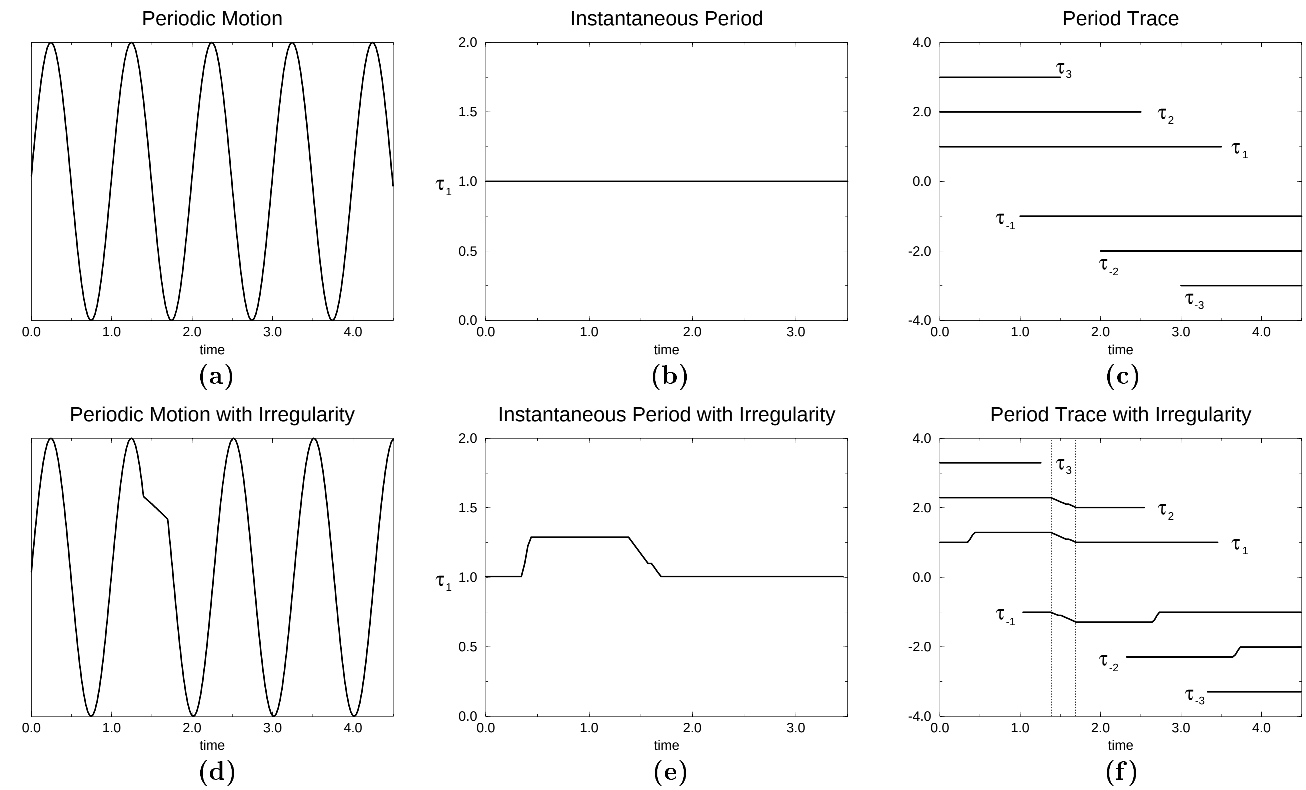

Figure 1: Effects of an irregularity on the instantaneous period. A periodic sinusoidal signal (a) has a constant period (b) and a period trace consisting of multiples of the period (c). A small irregularity is introduced (d) by temporarily slowing by a factor of 10. The irregularity shows up in two places in the instantaneous period (e), once where it enters the current cycle and once when it leaves. The ambiguity is resolved using the period trace (f) where the ramps "line-up" only at the true irregularity (i.e., in a small interval around time 1.5).

nontrivial to calculate the acceleration directly due to the nonrigid nature of the jogger's motion. The mean value of τ_1' gives the mean rate of increase of the period, which we call the *average cyclic acceleration*.

In addition to determining global motion properties such as the period and cyclic acceleration, the period trace can be used to compute globally significant points, cycles, or intervals in a motion. For instance, a longest (shortest) cycle is simply a value of t for which $\tau_1(t)$ is maximal (minimal).

Highly irregular points can be found by maximizing Eq. (5). Similarly, the irregularity of a cycle is measured by integrating Eq. (5) over the entire cycle:

$$irreg(\mathbf{C}_t) = \frac{1}{\tau_1(t)} \int_t^{\phi_1(t)} irreg_{\mathbf{C}}(s)\ ds$$

Another useful feature is the median[3] cumulative irregularity of an interval:

$$irreg_{\mathbf{C}}(t_1, t_2) = median\{|\tau_n(t_2) - \tau_n(t_1)| \ \mid \ n \neq 0\}$$

[3]We have found *median* to be more robust than *mean* in this context.

3.3 Motion Signatures

Somes types of cyclic motion produce telltale temporal signatures. Examples include periodic motions and motions with uniform cyclic acceleration, both of which have period traces that are linear. The form of a motion's period trace may also indicate something about the distribution of irregularities. Cyclic motions that contain isolated irregularities have very distinct signatures (see Figs. 1-3). The instantaneous period of a motion of this sort is piecewise-constant except at isolated intervals whose location and extent correspond to the irregularities.

If a cyclic motion is known to have a certain form, this knowledge may be used to help recover the period trace. Specifically, if an *a priori* motion model is available, e.g., linear or piecewise constant, the model can ameliorate the task of deriving τ_n by reducing the search space and making the recovery procedure less sensitive to noise. Existing techniques for recovering periodicity information [7, 8, 9] implicitly use this strategy, exploiting the assumption of a constant period. The case where such a model is not available is treated in the next section.

4 Recovering the Period Trace

In order to detect cyclic motion features such as those introduced in Section 3, we must be able to recover τ_n for each integer $n \neq 0$ from a cyclic motion $\mathbf{C}$. Because the complete period trace can be computed from τ_1, it is sufficient to determine τ_1. The recovery of τ_1 can be posed as a constrained optimization problem. Specifically, given a cyclic motion $\mathbf{C}$, minimize τ_1 subject to the following constraints:

1. $\mathbf{C}(t) = \mathbf{C}(\phi^n_{\tau_1}(t))$ for $t \in \Re$, $n > 0$
2. τ_1 is continuous
3. $\tau_1 > 0$
4. $\tau_1' > -1$

where $\phi_{\tau_1}(t) = \tau_1(t) + t$.

For discrete data sampled on the interval $[t_1, t_2]$, the problem is simplified by introducing a positive match function and formulating an energy function to be minimized.

The energy function, $E(\tau_1)$, has two terms to enforce constraints 1 and 2 as follows:

$$\begin{aligned} E_1(\tau_1, t) &= \frac{1}{n}\sum_{i=1}^{n} d(\mathbf{C}(t), \mathbf{C}(\phi^i_{\tau_1}(t))) \\ E_2(\tau_1, t) &= |\tau_1'(t)| \\ E(\tau_1) &= \int_{t_1}^{t_2} E_1(\tau_1, t) + \alpha E_2(\tau_1, t)\, dt \end{aligned} \tag{6}$$

where α is a weighting factor. Care should be taken to include only those values of t and n such that $\phi^n_{\tau_1}(t) < t_2$. Constraints 3 and 4 are local and easily enforced, although the strict inequality in constraint 4 is relaxed (since τ_1' may be arbitrarily close to -1)[4].

4.1 Optimization Approach

We use a multiscale *snake* algorithm adapted from [10, 11] to minimize the energy function in Eq. (6). The snake is initialized to a rough estimate of τ_1 and is iteratively refined so as to reach a state of locally minimal energy. Constraints 3 and 4 are satisfied initially and maintained throughout the optimization process. We use a greedy method to update the snake in which $\tau_1(t)$ is adjusted (by ± 1 unit) to minimize E and maintain constraint satisfaction. Convergence is guaranteed since energy decreases monotonically. To avoid problems with local minima, the optimization approach is repeated at three scales of increasing resolution using the output of each stage as the initialization for the next.

Due to the nature of cyclic motions, snake energy will be locally minimal at each of the n instantaneous periods (i.e., at τ_1, τ_2, τ_3, etc.). Therefore it is important that the snake be initialized in the vicinity of τ_1 so that it doesn't converge to τ_2 or τ_3. We use a simple strategy in which the snake is initialized to the smallest constant function with locally minimal snake energy. In other words, compute $E(c, t)$ for $c = 1, 2, \ldots$ until a local minimum is found, i.e., $E(c-1, t) < E(c, t) < E(c+1, t)$. We have found this method to be reliable and sufficient for the image sequences that we've considered.

[4] This modification poses minor invertibility problems for $\phi^n_{\tau_1}$.

5 Experiments

Two image sequences were used to evaluate the performance of the optimization method described in Section 4 and the detection of temporal motion features. The first sequence illustrates the use of match functions for view-invariant recovery of irregularity information. The second experiment uses X-ray data of a beating heart to show that the period trace can be calculated from medical imagery without pre-processing and can be used for temporal segmentation and image enhancement.

5.1 Turntable Motion Sequence

Three-dimensional motion analysis from image sequences requires separating 3D motion characteristics from artifacts arising from the projection process. For instance, consider the problem of measuring the period of a rotating phonograph turntable with a moving camera. Both camera motion and the rotation of the turntable contribute to the motion in the image sequence but only the contribution due to the turntable is relevant to the period. The problem is ameliorated with a match function that is invariant to changes in viewpoint. We introduced such a function in previous work [7], that equates two images if and only if they represent two views of the same object, under an affine projection model.

Accordingly, consider a motion representation consisting of the projected trajectories of n points on an object or in a scene. Represent a motion $\mathbf{C}(t)$ as a time-varying 2 by n matrix where each column is the instantaneous position of a point in image-coordinates at time t. Define the measurement matrix $\mathbf{M}_{s,t}$ of $\mathbf{C}$ as the concatenation of $\mathbf{C}(s)$ and $\mathbf{C}(t)$:

$$\mathbf{M}_{s,t} = \begin{bmatrix} \mathbf{C}(s) \\ \mathbf{C}(t) \end{bmatrix}$$

The measurement matrix can be registered to eliminate effects due to translation by subtracting each row's centroid from each element in the row [2]. Denote the registered measurement matrix as $\hat{\mathbf{M}}_{s,t}$. If σ_4 is the fourth singular value of $\hat{\mathbf{M}}(s,t)$, it can be shown [12] that

$$d_{\mathcal{A}}(\mathbf{C}(s), \mathbf{C}(t)) = \frac{\sigma_4}{2\sqrt{n}}$$

is a view-invariant match function. In other words, $d_{\mathcal{A}}$ equates two stills $\mathbf{C}(s)$ and $\mathbf{C}(t)$ precisely when they are views of the same object under an affine camera model such as orthographic or weak-perspective projection.

We filmed a rotating turntable using a hand-held video camera. Reflective markers were placed both on the turntable and elsewhere in the static scene. Note that although the motion of the turntable is rigid, the motion of the entire scene is non-rigid. Custom software was used to track the markers as the camera moved around the scene and the turntable simultaneously rotated. Twice the turntable was briefly touched to temporarily slow the rotation and artificially produce an irregularity.

Fig. 2 shows the recovered period trace. The figure illustrates that the optimization process successfully locates low-cost "valleys" in the cost space. The two regions with the highest irregularity values correspond to the two brief intervals in which the turntable was touched. The width of each interval indicates how long the turntable was touched and the irregularity value determines the extent to which the rotation was slowed. Notice that there are several ramps throughout the period trace but all coincide only in the intervals where the turntable was touched.

5.2 Heart X-ray Motion Sequence

We also tested our recovery method on a beating heart motion using a sequence of X-ray images obtained through coronary angiography. A few images from the sequence are shown in Fig. 3. Because of the difficulty of automatically segmenting these images, we chose to match the raw data directly, using the grayscale image intensities as the underlying spatial representation. The match function was just the mean squared pixel intensity difference between two images. The figure shows that the period changed from 26 frames to 27 to 24 and back to 27, but remained roughly constant for the duration of each cycle. A period trace of this form is indicative of a motion with isolated irregularities. Our results show that the irregularities occurred between contractions as the heart relaxed, pausing for slightly different lengths of time in different cycles.

Due to certain spatial irregularities of the heart's motion, the motion was not precisely cyclic. These anomalies manifested themselves as unusual patterns in certain intervals of the period trace. Notice that at around frame 20 the period drops sharply and then abruptly shoots up. This corresponds to a *spatial* irregularity where the heart relaxed at a slightly different location in the first and second cycles.

The τ_1 curve is qualitatively similar to an electrocardiogram (ECG), which can also be used to detect heart periods, although different information is conveyed. An ECG measures electric activity of the heart so period information is not explicit, in contrast to the period trace which gives period as a function of time.

5.2.1 X-ray Enhancement

To evaluate the accuracy of the period trace for the heart data we generated a composite cycle of the heart's motion. Recall that each still of a cyclic motion appears once in each cycle. The functions ϕ_n provide this correspondence; $\phi_2(3) = 56$ indicates that frame 3 recurs two cycles hence at frame 56. Therefore, for each time t, we can obtain a composite image that is the median of frames $\phi_n(t)$, $n \neq 0$.

Angiographic imaging involves a radioactive fluid that flows through the heart and shows up under X-ray. The fluid takes some time to initially propagate through the heart and washes out after a few seconds, so visible structure varies throughout the sequence. Hence, the composite images reveal structure that is not readily apparent in a single frame (see Fig. 4).

6 Conclusion

We have presented a new motion representation called the *period trace* that is purely temporal and can be used to detect irregular points and regions in a cyclic motion. An optimization method for recovering the period trace of a motion was described which incorporates invariance to handle motions that are only partially cyclic. The algorithm was evaluated on real image sequences. Future work will consider other temporal motion features and will consider other applications such as motion recognition.

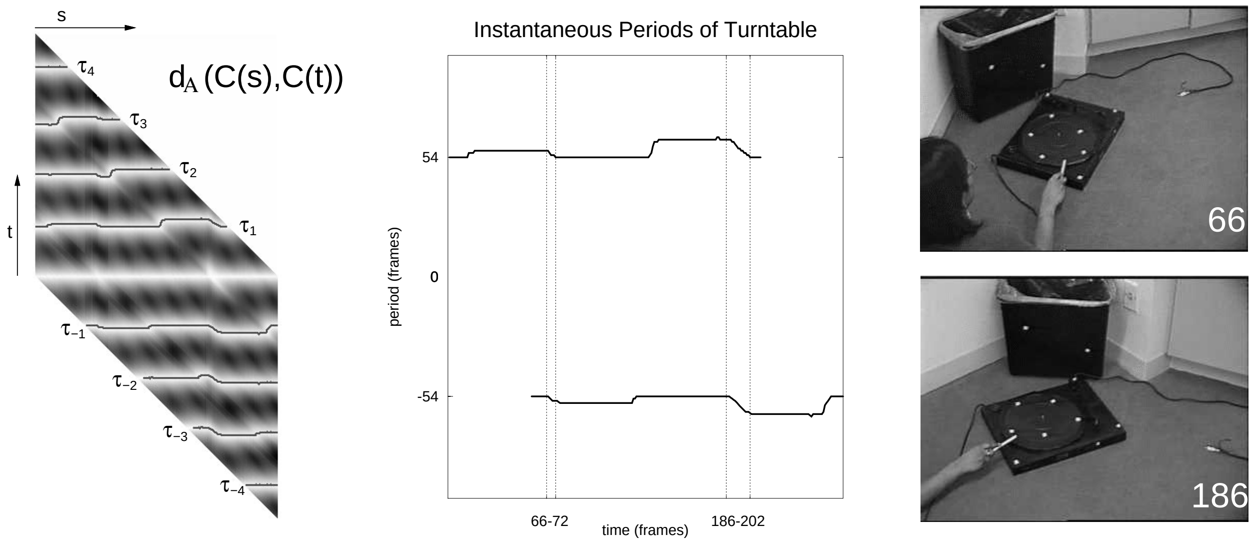

Figure 2: Period trace of turntable motion sequence. Left: Cost plot of $d_{\mathcal{A}}(C(s), C(t))$ with values shown as image intensity (white – low cost, black – high cost). The recovered period trace is superimposed on the cost plot. Middle: A graph of τ_1 and τ_{-1} with two most irregular intervals marked. Notice that discontinuities appear in various places but only "line up" at actual motion irregularities. Right: Selected images in the two irregular intervals. The ground truth turntable frequency is $33\frac{1}{3}$ revolutions per minute, or 54 frames per revolution at NTSC video rate.

Figure 3: Period trace of heart X-ray motion sequence. Left: Period trace superimposed on cost plot. Middle: Instantaneous period. Right: X-ray images of a heart relaxed (frame 30) and contracted (frame 40). Notice that the instantaneous period is roughly piecewise-constant corresponding to the predicted model of a motion with isolated irregularities. Qualitatively the motion is seen to be relatively even except in certain small intervals (frames 20, 40, 70), suggesting that the period variation is due to uneven motion solely in these small intervals.

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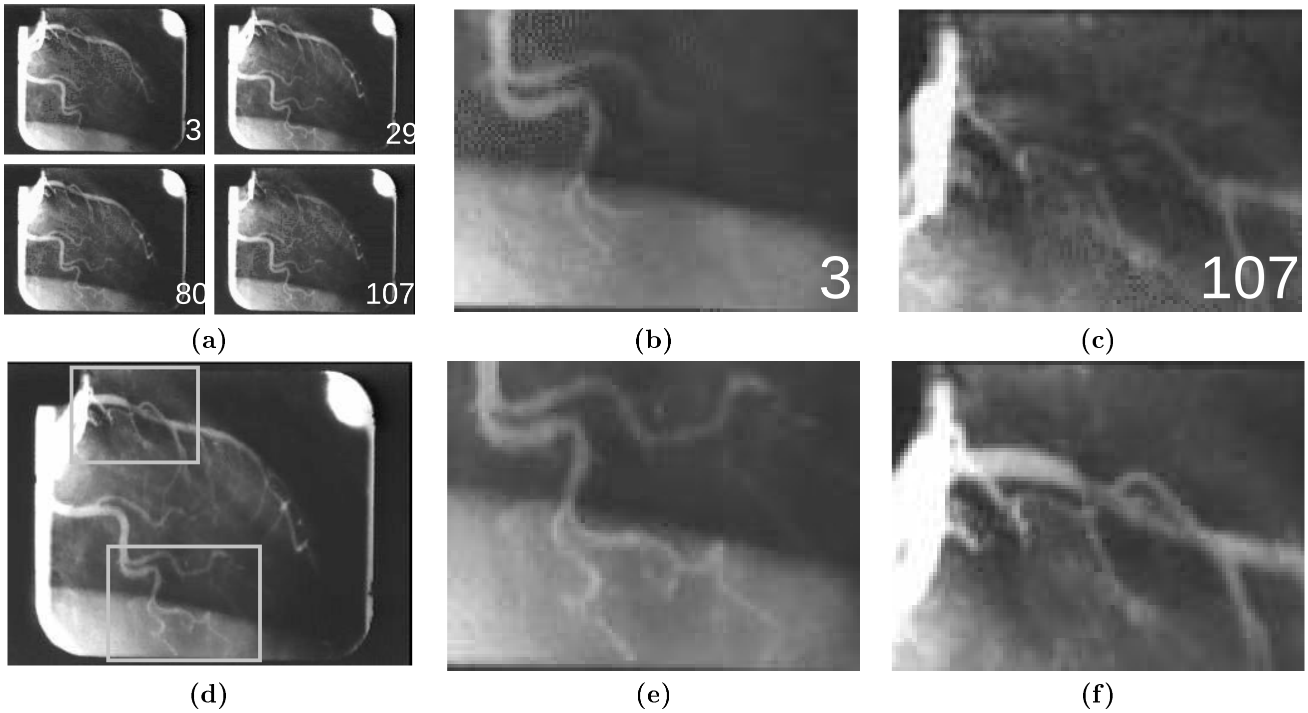

Figure 4: Image enhancement using the period trace. Four corresponding images (a) detected from the period trace and a composite (d) formed from five such images with two regions of interest marked. Compare enlarged regions of individual frames (b) and (c) with corresponding regions (e) and (f) of composite. Note that the composite reveals structure not readily apparent in individual images.

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