Estimation of 3-D Parametric Models from Shading Image Using Genetic Algorithms

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Abstract

In this paper, a method for estimating parameters of a 3-D shape from a 2-D shading image using a genetic algorithms (GAs) is proposed. The shape of the object is represented by a superquadrics model, and then the model parameters are coded for being applied to GAs. The coded string is evaluated according to the similarity of the shading image calculated from the 3-D model shape represented by the parameters to the given 2-D shading image. By applying the GAs to the optimization of the evaluation value, the string having the minimum difference can be found. The parameters are estimated from some shading images of various 3-D shapes by using the proposed method, and the results are presented.

1 Introduction

The shape from shading [1], that is the method for reconstructing 3-D shape of an object from 2-D shading image, has long been studied in the field of computer vision. In this study, we reduce the shape from shading problem to the parameter estimation by modeling the object shape. The parameters of the model are estimated from a 2-D shading image using genetic algorithms (GAs) [2][3], which is an optimizing technique based on mechanisms of natural selection. By using parametric model, 3-D shapes can be simply represented by some parameters instead of a set of a lot of 3-D sampling points. The employed model is a superquadrics [4][5], which is proposed for representing 3-D smooth surfaces by using primitives expressed as a simple parametric equation. The equation of the superquadrics surface is expressed as

$$\left\{ \left(\frac{X'}{a_1}\right)^{\frac{2}{\varepsilon_2}} + \left(\frac{Y'}{a_2}\right)^{\frac{2}{\varepsilon_2}} \right\}^{\frac{\varepsilon_2}{\varepsilon_1}} + \left(\frac{Z'}{a_3}\right)^{\frac{2}{\varepsilon_1}} = 1, \qquad (1)$$

where a_1, a_2, and a_3 are scaling parameters in X', Y', and Z' directions, respectively, and ε_1 and ε_2 are squareness parameters. In addition to these parameters, Euler angle (θ, ϕ, ψ) and shift parameters (p_x, p_y, p_z) represent superquadrics for general orientation and position in the XYZ coordinate.

In this study, the orthographic projection imaging system is assumed, then the shading image is independent of p_z. Therefore p_z is excluded for estimation, and then ten parameters of $(a_1, a_2, a_3, \varepsilon_1, \varepsilon_2, \theta, \phi, \psi, p_x, p_y)$ are used for representing the superquadrics model of the 3-D object shape in this study. Those parameters are estimated by using the optimizing function of GAs.

In this paper, the principle of the proposed method for estimating the parameters of a 3-D shape from a 2-D shading image using GAs is described. Then, the results of estimating model parameters from synthesized shading images are shown.

2 Parameter Estimation Using GAs

2.1 Coding

The ten parameters are coded as a string of 64 bits.

The squareness parameters are quantized as the following equation.

$$\varepsilon_n = 2^{(128-k)\frac{6}{128}-5}, \qquad (n = \{1, 2\}), \quad (2)$$

where k is 7 bits integer ($0 \leq k \leq 128$), and then $2^{-5} \leq \varepsilon_n \leq 2$. It is defined so that the model shape can vary in proportional to the change of integer k.

The scale parameters a_1, a_2, a_2 are defined as the following equation so that they can be linearly quantized from 5 to 15.

$$a_n = 10\left(\frac{k}{64} + 0.5\right), \qquad (n = \{1, 2, 3\}), \quad (3)$$

where k is 6 bits integer ($0 \leq k \leq 64$), and then 5 $\leq a_n \leq 15$.

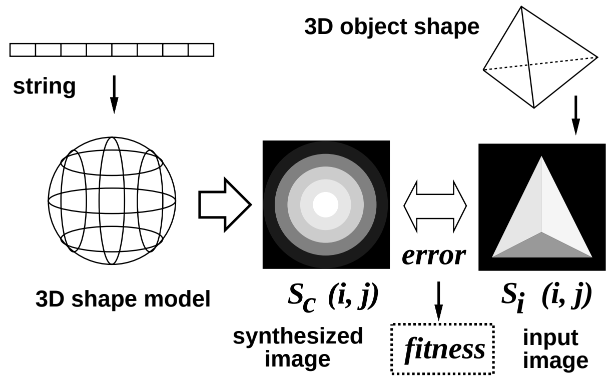

Figure 1: The fitness value is defined according to the similarity between the shading image determined by the 3-D shape and the given (input) shading image.

The orientation parameters are quantized as the following equation.

$$\{\theta, \phi, \psi\} = \frac{k}{128}\pi, \tag{4}$$

where k is 8 bits integer ($0 \leq k \leq 256$), and then $0 \leq \{\theta, \phi, \psi\} \leq 2\pi$.

The shift parameters are quantized as the following equation.

$$p_u = \frac{k-8}{4} + c_u, \qquad (u = \{x, y\}), \tag{5}$$

where k is 4 bits integer ($0 \leq k \leq 16$), and then $-2 + c_u \leq p_u \leq 2 + c_u$. Here, (c_x, c_y) represents the gravity point of object area in the input 2-D shading image.

2.2 Fitness value

Each string represents a modeled 3-D shape. The shading image of the modeled 3-D shape can be obtained by calculating the orientation of the surface normal of the 3-D model shape.

The fitness value of the string is defined according to the similarity between the shading image determined by the 3-D shape and the given (input) shading image as shown in figure 1. The fitness value f is expressed as the following equation:

$$f = e^{-1}, \quad e = \sum_x \sum_y \{S_i(x, y) - S_c(x, y)\}^2, \tag{6}$$

where S_i is the input shading image and S_c is the shading image calculated from the 3-D shape represented by each string. According to the definition of the fitness value, the string with larger fitness f represents the 3-D model closer to the object shape.

2.3 Evolutionary process

The string having the maximum fitness value is sought by the evolutionary process which is described as follows.

The initial population of n strings is first made by the random value.

Next, fitness value of each string in the population is calculated as described in **2.2**. The best n_e strings are first copied into the next generation. This process is called as the elitist strategy [3]. Using the elitist strategy, evolution speed can be made faster. The number of the elite strings n_e must be determined not to make the population trapped into a local minimum.

The rest strings of $(n - n_e)$ in the next population are reproduced by *crossover* operation. The pair of strings are selected according to the probability proportional to the fitness value. The two parents make two children by *two-point crossover* operation.

After n strings are reproduced by the crossover, n_m strings are selected randomly from the n strings for *mutation*. In each selected string, a bit is randomly selected and then the value of the bit is changed. The mutation is operated so that the population is not trapped into a local minimum.

From the n strings reproduced by the crossover and the mutation, the best $(n - n_e)$ strings are remained into the next generation with the n_e elite strings previously selected.

The flow of the process described above is shown in figure 2. After repeating this processes, the parameters having the highest fitness value is considered as the estimated parameters representing the superquadrics model of 3-D shape.

3 Parameter Estimation Experiments

To demonstrate the proposed method, parameter estimation experiment is performed. An input 2-D shading image is 32 × 32 pixels with 256 gray levels, which is synthesized by computer simulation.

The number of the strings in a population n is 500. The elite size n_e is 100. The number of the strings selected for the mutation in each generation n_m is 50. This means that the total mutation probability for each bit is 1/640 ($= n_m/n/64$). The generation is repeated 100 times.

Figure 3 shows the estimated results in the case for the cube. In accordance with the results, the estimated parameters are roughly close to the original. However, the estimation parameters of cube is not

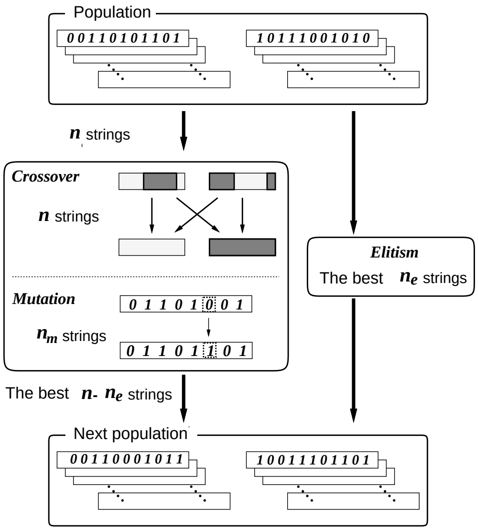

Figure 2: The flow of the evolutionary process in the proposed method.

close to the original. This is caused by the fact that cubes can be represented by several parameters sets because of the axial symmetry of superquadrics.

Figure 4 shows error transition in the cases for using various optimizing methods. Since there are many local minimum points because there are ten optimized parameters, the result by hill climbing tends to be trapped into local minimum. On the other hand, the result by GAs is not trapped into local minimum because of robustness of GAs.

4 Conclusion

In this paper, a new method for parametric reconstruction of 3-D shape from 2-D shading image using genetic algorithms is proposed. For demonstrating the performance of the proposed method, parameters of superquadrics model were estimated from shading images synthesized by computer simulations. The results shows that the proper estimation was obtained because GAs are robust for effectively searching the best parameters.

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References

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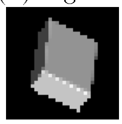
	a_1	a_2	a_3	ε_1	ε_2
ORG.	4.0	8.0	12.0	0.05	0.05
EST.	12.5	8.25	4.25	0.15	0.14

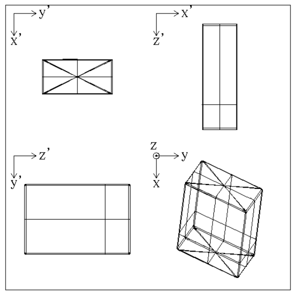
θ	ϕ	ψ	px	py
0.10π	0.20π	0.30π	0.0	0.0
0.85π	0.71π	0.46π	0.0	0.0

(a) Original and estimated parameters.

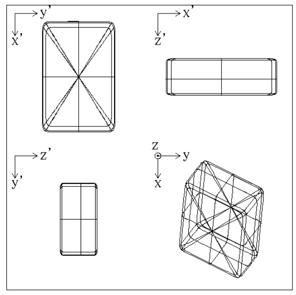
(b)original.



(c)estimated.



(d) Shape of the original parameters.



(e) Shape of the estimated parameters.

Figure 3: Parameter estimation results in the case for cube. Shading image (b) shows the input image used for estimating superquadrics model parameters, while (c) shows the synthesized shading image from the modeled shape by the estimated parameters.

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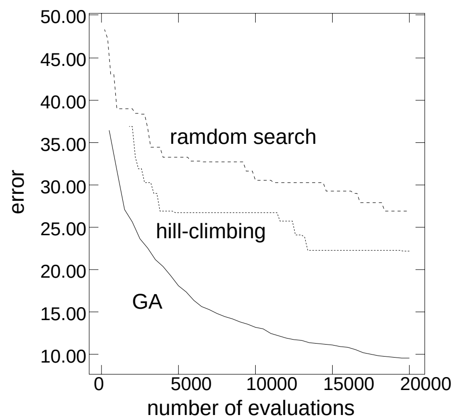

Figure 4: Comparison of GA with the other methods.