

# Contact Transition Control of a Single Flexible Link Using Positive Acceleration Feedback

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## Abstract

*The ability to control a robot in both constrained and unconstrained situations is important. One way to do this is using a switching controller which uses a force controller for constrained space and a position controller for unconstrained space. This paper develops a switching controller for a single flexible link manipulator and provides experimental results for a motion from unconstrained to constrained space. The effect of different levels of positive acceleration feedback on the force response, after a collision with an unexpected object occurs, is examined. It is concluded that small amounts of positive acceleration feedback reduces the force overshoot caused by the collision but that too much can cause instability.*

## 1 Introduction

Many useful robotics applications, such as grinding and assembly, require the robot to interact closely with the environment. During this interaction, it is imperative that the robot be able to sense and control the force of contact with the environment. This can be done using a force controller which is designed to provide a desired force at the contact point. However, in real world applications the robot must first operate in unconstrained space before it contacts the object. In general, the force controller designed to complete the constrained space task will be unsuitable while the robot is in unconstrained space. One solution to this problem is to use a position controller for unconstrained motions and a force controller for constrained motions. When the robot makes or breaks contact with the environment the control law is switched accordingly. This type of controller will be referred to as a switching controller.

Consider the case where the manipulator makes the transition from unconstrained motion to constrained motion. This requires that the control law be switched

from position to force control. In practice it is impossible to know the position of the environment or the robot exactly. Since the position of the object / manipulator contains some error the manipulator will contact the environment, and switch to force control, with a non-zero velocity. This results in an impact impulse force and could cause the manipulator to bounce off of the environment. In Tarn et. al. [15] this problem is looked at for rigid manipulators. They propose adding a positive acceleration feedback term to the force control law. This is shown to reduce the impact force and the chance of bouncing. They also show that it improves the force response for different environments.

This paper, examines experimentally, the effect of using positive acceleration feedback with a single flexible link robot. The manipulator executes a trajectory which moves it from unconstrained space to constrained space. The manipulator does not have any prior information about the object location, so it is not possible to slow down before the collision occurs. The force response of the manipulator after the collision is examined for different levels of positive acceleration feedback.

## 2 Position Control

The area of position control of flexible manipulators has received considerable attention from the research community. One of the first papers on the topic was that of Cannon and Schmitz [2] where a Linear Quadratic Gaussian (LQG) controller was designed. Direct endpoint sensing was used and the goal was to execute a robot motion as fast as possible without residual vibrations in the beam. Since this paper, many other control schemes have been developed which use different flexible mode information [6,7,14].

The single flexible link manipulator is illustrated in Figure 1. For position control it is common to define the output of the system as,

$$y(t) = q_1(t) + w(x,t) \quad (2.1)$$

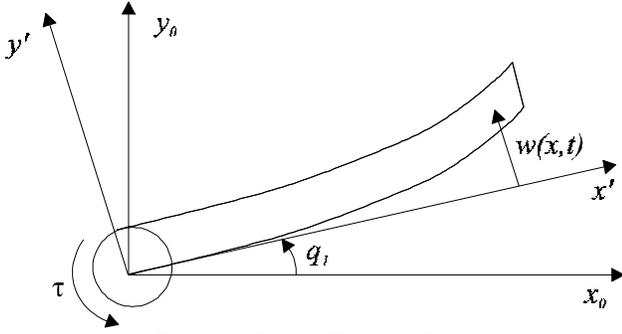


Figure 1 Single Flexible Link

where  $q_1(t)$  is the motor angle and  $w(x, t)$  is the deflection of the flexible beam from rest. Using assumed modes [9] the flexible deflection can be written as,

$$w(x, t) = \sum_{k=2}^{n+1} \phi_k(x) q_k(t) \quad (2.2)$$

where  $\phi_k(x)$  are the mode shape functions,  $n$  is the number of significant modes of oscillation and  $q_k(t)$  act as the generalized co-ordinates of the flexible beam. Substituting (2.2) into (2.1) gives,

$$y(t) = q_1(t) + \sum_{k=2}^{n+1} \phi_k(x) q_k(t). \quad (2.3)$$

It was shown in Wang and Vidyasagar [17] that the transfer function from the input torque,  $\tau$ , to  $y(t)$  does not have a well defined relative degree. They propose that the reflected output,

$$y_r(t) = q_1(t) - \sum_{k=2}^{n+1} \phi_k(x) q_k(t) \quad (2.4)$$

be used. The transfer function from the input torque to  $y_r(t)$  has a relative degree of two. Furthermore, the transfer function from  $\tau$  to  $\dot{y}_r(t)$  is strictly passive. This was used in Rossi et. al. [13], which showed that a PD controller operating on  $y_r(t)$  provides stability for the single flexible link. Based on [13] the position controller used in this paper has the control law,

$$\tau = K_p y_r(t) + K_d \dot{y}_r(t) + K_I \int_0^t y_r(a) da, \quad (2.5)$$

where  $K_p$ ,  $K_d$ , and  $K_I$  are constants determined in [16]. This controller guarantees stability, for small  $K_I$ , and has been shown experimentally to provide excellent position tracking and vibration suppression [16].

### 3 Force Control

The area of flexible manipulator force control is not as established as that of flexible position control. In the papers by Chiou and Shahinpoor [3] and Fan and

Castelazo [5], control of a flexible robot with a force / torque sensor on the end effector is examined. In Choi and Krishnamurthy [4], a set of linear time varying equations are derived for a flexible robot in both constrained and unconstrained spaces. Separate LQR / LTR controllers were designed for specific times and control was switched between them. This required that the exact position of the object be known a priori and only simulation results were provided. In Pfeiffer [11], the feedforward torques to provide an optimal trajectory are computed off-line, while the vibrations in the flexible link are reduced on-line using a PD controller on the rigid body variables and a proportional controller on the beam curvature. Strain gauges are used to measure the forces of contact as well as the beam curvature. Finally, in Borowiec and Tzes [1], the effect of joint co-ordinate vs. end point sensing, environmental stiffness, input preshaping and sampling time on a two link flexible manipulator using an LQR controller with a rigid body feedforward term is investigated in simulation.

One difficulty which arises in force control of flexible manipulators is modeling the flexible beam during constrained motions. In position control it is common to use the clamped-free or pinned-free boundary conditions to solve the Bernoulli-Euler beam equation. In the constrained case the beam is in contact with an object and does not have a truly free end. Furthermore, the flexible beam may come into contact with the object at a location other than the tip, or the beam may bounce on and off the object, making the use of pinned-pinned boundary conditions difficult. In Matsuno et. al. [8], it is shown that the boundary conditions for the flexible beam in contact with the environment depend on the contact force and input torque, as well as the contact position. This makes the flexible beam equations very difficult to solve and simplifications must be made. To overcome these difficulties, most researchers simply use the clamped-free or pinned-free boundary conditions. This gives a poor approximation of the mode shapes, and thus a greater number of significant modes is required to predict the beam shape, to the same precision, than if the proper mode shapes were used. This increases the order of the system to be controlled. The approach used by Matsuno et. al. [8] makes several assumptions based on the static relationship between the elastic deformations and the contact force which simplify the boundary conditions enough to allow a solution to the equations of motion. The equations are still very complicated and not entirely accurate as static relationships are used to describe a dynamic system.

To help overcome some of these uncertainties system identification was performed on the single flexible link used in the experimental section to generate an

accurate model of the constrained system. The ARX method, which is based on the Least Squares Algorithm, was used to generate an eighth order model for the single flexible link in contact with a rigid environment. The model has the motor voltage as its input and force of contact as its output. As in Pfeiffer [11], strain gauges were used to measure the force of contact. This model was then used to design an LQR controller with a full order estimator which is used for the force control part of the switching controller [10].

#### 4 Positive Acceleration Feedback

To reduce the overshoot in the force response of the system after a collision with the environment has occurred, the use of positive acceleration feedback was proposed by Tarn et. al. in [15] for rigid robots. For the single flexible link using an LQR force controller the rigid body closed loop equation is, [10]

$$I_h \ddot{q}_1 + \rho \sum_{k=2}^{n+1} \ddot{q}_k \int_0^l x \phi_k(x) dx - \left( f^d - \sum_{j=1}^n k_j \hat{q}_j \right) + l f_e = 0 \quad (4.1)$$

where  $I_h$  is the motor hub inertia,  $k_j$  are the controller gains,  $\hat{q}_j$  are the estimator outputs,  $l$  is the flexible link length,  $\rho$  is the linear mass density of the beam and  $f_e$  is the force of contact. The presence of the second derivative terms means that the rigid body response may be highly oscillatory, which would make the force response oscillatory as well. According to [15], it is quite difficult to choose values for the controller gains which eliminate the overshoot and rigid body oscillations. Furthermore, these values will be highly dependent on the stiffness of the environment [15]. To overcome these two problems they propose adding a positive acceleration term to the feedback. If the control law

$$\tau = I_h \ddot{q}_1 + \rho \sum_{k=2}^{n+1} \ddot{q}_k \int_0^l x \phi_k(x) dx + f^d - \sum_{j=1}^n k_j \hat{q}_j, \quad (4.2)$$

were used, then the closed loop equation would become,

$$-f^d + \sum_{j=1}^n k_j \hat{q}_j + l f_e = 0. \quad (4.3)$$

This is a first order system so overshoot is eliminated. In practice however the flexible mode variables,  $q_k$   $k = 2 \dots n+1$ , are measured using strain gauges. These readings are fairly noisy and thus it is not possible to calculate the acceleration of the flexible modes with any accuracy. Furthermore the rigid body acceleration measurement may have noise present or  $I_h$  may not be known precisely. This means that it is not possible to exactly cancel the accelerations. Therefore if (4.2) is

used,  $\ddot{q}_1$  may appear in (4.3) with a negative sign. This will cause instability by the Routh-Hurwitz Criterion. To overcome this problem the rigid body acceleration is multiplied by a gain term,  $\psi$ , to ensure that

$$\psi \hat{\ddot{q}}_1 < I_h \ddot{q}_1, \quad (4.4)$$

where  $\hat{\ddot{q}}_1$  is the measured rigid body acceleration. Therefore the control law used in the experimental results is,

$$\tau = \psi \hat{\ddot{q}}_1 + f^d - \sum_{j=1}^n k_j \hat{q}_j. \quad (4.5)$$

This will eliminate some of the effects of the rigid body acceleration and help to reduce the overshoot in the force response.

#### 5 Experimental Results

The position controller from section 2 and the force controller from section 3 were implemented on a single flexible link manipulator. The flexible link used two sets of strain gauges to measure the force of contact during constrained motion. This force measurement was used to determine when to switch between controllers. The flexible oscillations, which occur in the beam during unconstrained motion, also generate strain in the beam. This produces an apparent force reading which is calculated from the strain gauge readings. Thus it is important that the switching threshold be set high enough so that these deflections do not trigger a switch to force control. For the following experimental results the threshold was chosen as 0.05N. Therefore, when the force measurement exceeds this value the force controller is used. Otherwise the position controller is used. To collect the experimental data found in this chapter, the robot was commanded to follow a trajectory in free space. The trajectory, shown in Figure 2, is a cubic spline which commands joint  $q_1$  to go from 0 to -1 radians in 5 seconds. An object is placed on this path so that the tip of the flexible link will contact the object. When the manipulator detected contact with the environment the desired trajectory changed from the cubic spline to maintaining a constant 1N force of contact. With the experimental apparatus it was not possible to hit the object at exactly the same point in the cubic spline trajectory every time. However, the collision occurs at approximately 2.5 seconds into the trajectory. At this time the velocity of the manipulator is 0.3 rad/s. The results presented in this section do not use any prior knowledge about the object's position. This means that the manipulator does not make any attempt to slow down before hitting the object. If it was possible to slow down

before hitting the object, the results would be much better since the impulse force of contact would be reduced. Therefore, the results here are for the toughest test of the switching control strategy.

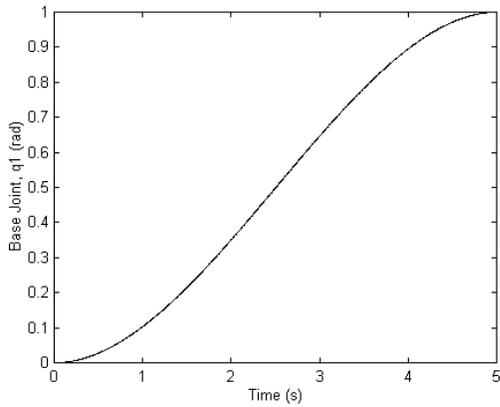


Figure 2 Position Cubic Spline Trajectory

To see the effect of adding positive acceleration feedback to the LQR force controller several tests were performed varying  $\psi$ , the amount of acceleration feedback. The results can be found in Figures 3 through 7. As these figures show, adding positive acceleration feedback helped to reduce the overshoot in the force responses up to a point. Increasing the feedback past this point degraded the response and even made the system go unstable as shown in Figure 7.

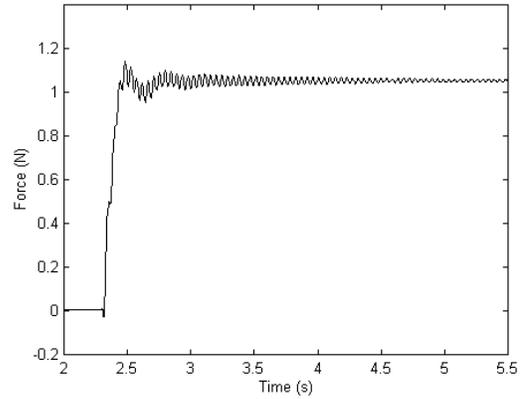


Figure 4  $\psi = 0.1$

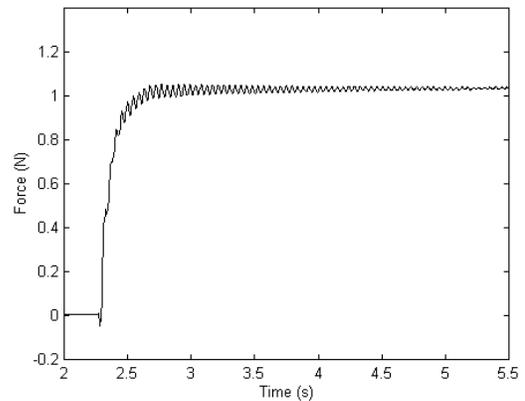


Figure 5  $\psi = 0.2$

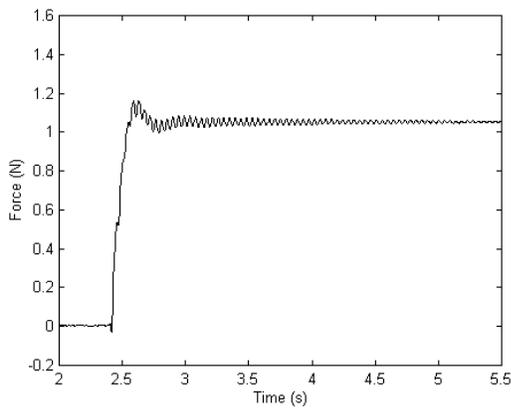


Figure 3  $\psi = 0.0$

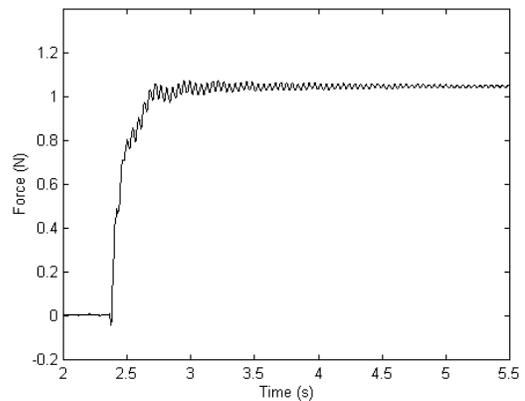


Figure 6  $\psi = 0.3$

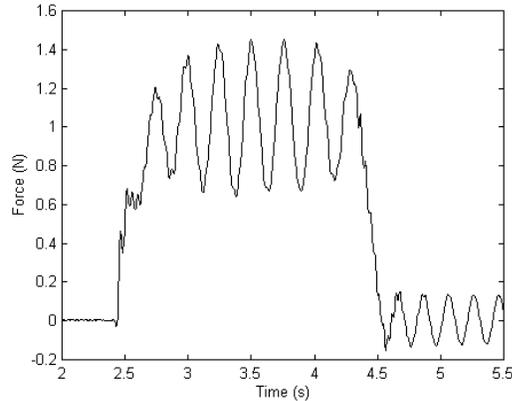


Figure 7  $\psi = 0.5$

The high frequency oscillations in the force response are due to vibrations in the flexible beam which the LQR force controller successfully eliminates.

## 6 Conclusions

This paper has demonstrated a switching control system for a single flexible link manipulator. While the manipulator is in unconstrained space a position controller is used. Control is switched to an LQR force controller when the strain gauges mounted on the flexible link indicate that a force greater than 0.05N is applied to the link. This controller was used on the single flexible link to successfully make the transition from position control to force control after striking an object of unknown position. The force response of this motion was very good and the impulse force of collision was fairly small. One reason for this is that the flexible link has a natural compliance which reduces the force of impact and lowers the chance of bouncing. This is a major advantage of flexible manipulators over rigid manipulators for contact applications. The use of positive acceleration feedback was investigated to reduce the overshoot present in the force response, caused by the collision with an object. It was found that small amounts of positive acceleration feedback did reduce the overshoot, however too much feedback could cause instability.

The stability of the switching control system is not easy to prove because of the discontinuous control signal when a transition occurs. Individually, the position and force controllers are each asymptotically stable. Therefore, if the manipulator trajectory eventually remains in either the constrained or unconstrained regions, the system will be stable. However, there is the possibility for instability if the manipulator bounces between the two regions an infinite number of times. In Tarn et. al. [15], a proof is given which establishes some

constraints on the position and force controller gains for a rigid robot, which eliminates the possibility of an infinite number of switches. Thus stability is guaranteed. This proof has been extended to a single flexible link in [10] with some restrictive assumptions. The proof in [10] assumes that a rigid body PID position controller is used with an implicit force controller to form the switching controller. A rigid body position controller only operates on the motor angle and ignores the flexible oscillations. An implicit force controller, models the environment as a spring and can thus use position control to control the force of contact. These controllers are much simpler than the controllers used in this paper. Furthermore the proof requires that the single flexible link has only one significant mode of oscillation and that these oscillations are small.

## 7 References

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