A Divide-and-conquer Strategy in Recovering Surface Shape of Book from Shading

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Abstract A divide-and-conquer strategy in shape from shading is proposed for recovering book surface on fully perspective condition. It is shown that unique shape can be recovered despite of more unknowns than shade images by dividing original implicit SFS problem into explicit ones. Using invariance of shading, a transformed shading equation and a recurrence relation is derived for depth recovery. Whole process is divided into three sequential processes : preprocessing, apparent shape recovery, and ortho-image generation. Comprehensive algorithms are implemented for each process. The reliability of proposed strategy and accuracy of recovered results are shown through simulations and real experiment.

Key Word Shape from shading, Divide-and-conquer strategy, Transformed shading equation, Recurrence relation

1 Introduction

In this paper, a divide-and-conquer strategy is discussed in solving shape from shading (SFS) problem on fully perspective condition. The observing environments, being fixed by external requirements, are composed of two light sources and one camera which are the usual setting in industrial applications. The target object is surface of book which can be treated as double cylinder in global appearance and Lambertian reflectance model is assumed for this object.

The idea of this paper is based on a strategy which divides a implicitly connected problem into explicit ones, and solves it step by step. Each part of the idea is connected with several researches on perspective light condition[1][5][6], on recovery of book surface[9][10], and on perspective observation[3][4][6].

It has been generally considered hard to solve fully perspective SFS problem[6] because prior knowledge of depth, i.e. the solution itself, is needed to solve problem. Using only two light sources makes it difficult to solve problem owing to more unknowns than known conditions. Shape recovery of book surface is not simple because of obstacles like non-constant albedos, self-shadows and interreflections[7][9]. Whereas, assuming cylindrical shape and invariance of shading help it to reduce complexity of problem.

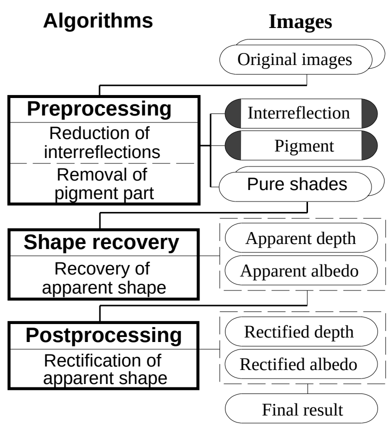

Figure 1: Flow diagram of whole process.

A new strategy is proposed to overcome difficulties of underlying problem. Fortunately, shading is invariant to observing position in Lambertian model. This results invariance of slope and depth. Therefore, it is possible to divide original implicit problem into explicit ones using invariances without violating generality of problem. So, whole recovery problem is divided into three : preprocessing, apparent shape recovery, and generation of ortho-image, as shown in fig. 1. In this strategy, for the reduction of unknowns which are five, a transformed shading equation and a re-

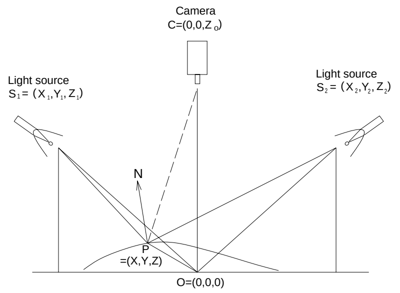

Figure 2: Lighting, observing and surface components defined in world coordinate.

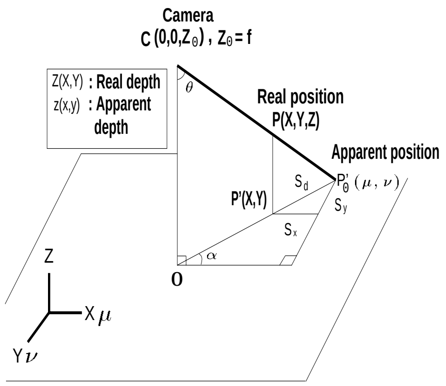

Figure 3: Apparent position of object defined by the relation of two coordinates.

currence relation which are essential in solving problem are derived. Using these equations, unknowns can be reduced into two, and consequently, unique shape recovery become possible.

Underlying problem will be analyzed in section 2, and strategy for solution will be discussed in section 3. Practical algorithms will be explained in section 4, and experiments will be done in section 5. In section 6, estimation of proposed strategy will be discussed.

2 Problem formulation

Original Lambertian reflectance model[2] is defined by orthographic projection. Therefore, local slope, location of camera and light sources are defined in world coordinate. Whereas, observed images formed by perspective projection are defined in camera coordinate. These two coordniates have same origin as shown in fig. 2 and fig. 3. Position of world coordinate will be denoted by (x, y, z), and that of camera coordinate will be denoted (μ, ν) throughout this paper.

2.1 Equation of perspective shading

The shading of a Lambertian cylinder for i-th light source is described as

$$L_i = I_0\rho\frac{\cos\theta_{inc}}{D_i^2} = I_0\rho\frac{-\Delta_{Xi}p + \Delta_{Zi}}{D_i^3\sqrt{1+p^2}}, \qquad (1)$$

$$p \equiv \frac{\partial z}{\partial x}, \quad \frac{\partial z}{\partial y} \equiv 0, \quad D_i \equiv \sqrt{\Delta_{Xi}^2 + \Delta_{Yi}^2 + \Delta_{Zi}^2},$$

$$\Delta_{Xi} \equiv X_i - x, \quad \Delta_{Yi} \equiv Y_i - y, \quad \Delta_{Xi} \equiv Z_i - z.$$

Where I_0 and ρ denote the departed strength of light rays and the albedo of surface respectively. The (X_i, Y_i, Z_i) denotes location of light source and p denotes local slope.

2.2 Equation of perspective observation

Equation of perspective observation found in text books is described by two variables : focal length and height Z_o of camera. Because it doesn't violate generality to extend focal length as same with Z_o as shown in fig. 3, both coordinates can be described simultaneously on image plane.

In fig. 3, a point $P(x, y, z)$ on object surface is recorded at $P'_o(\mu, \nu)$ by apparent shift S_d. Since $S_d = z tan\theta$ and $\overline{OP'_o} = Z_o tan\theta$, the S_x becomes

$$S_x = S_d \cos\alpha = S_d\frac{\mu}{\overline{OP'_o}} = \mu\frac{z}{Z_o}, \qquad (2)$$

and S_y become $\nu z Z_o^{-1}$ in the same way. Consequently, following relation is derived

$$x = \mu(1 - \frac{z}{Z_o}), \quad y = \nu(1 - \frac{z}{Z_o}). \qquad (3)$$

2.3 Unknowns in recovery problem

Equation (1) contains three unknowns $I_0\rho$, p, $z(x, y)$ for known position (x, y). But by eq. (3), it is clear that (x, y) can't be determined before recovery of shape. So, two equations contain five unknowns (x, y, z, $I_0\rho$, p) and are connected implicitly.

3 Strategy and solution

Since shading is invariant to observing position in Lambertian model, it is possible to transform shading equation into new one. This reduces two unknowns and divides problem into two. By calculating photometric ratio, another unknown can be eliminated as well. As a result, unique recovery can be done using two shade images by determining two unknowns.

3.1 Invariant characteristics

Shape recovery is done at apparent position $P_o'(\mu,\nu)$ because depth is recovered from shade images. Where, angle between surface normal and light ray is invariant even in perspective observation because these are defined in world coordinate. The amount of shading is invariant as well by Lambertian assumption, that is, recorded brightness at P_o' is same with that at $P(x,y,z)$. Consequently, recovered slope at P_o' becomes same with slope at P. So, recovered depth at P_o' is same with original depth at $P'(x,y)$ even in perspective observation. This is very important advantage of SFS problem.

3.2 Transformed shading equation

By substituting eq. (3) into (1), a transformed shading equation is derived

$$L_i = I_0\rho\frac{-\delta_{Xi}p+\Delta_{Zi}}{d_i^3\sqrt{1+p^2}}, \tag{4}$$

$$d_i \equiv \sqrt{\delta_{Xi}^2+\delta_{Yi}^2+\Delta_{Zi}^2},$$

$$\delta_{Xi} \equiv X_i-\mu(1-\frac{z}{Z_o}),\ \ \delta_{Yi} \equiv Y_i-\nu(1-\frac{z}{Z_o}).$$

This transform eliminates two unknowns (x,y), and replaces depth $z(x,y)$ into $z(\mu,\nu)$ maintaining generality of problem because depth is invariant. As a natural consequence, original problem become a problem of apparent shape recovery because eq. (4) is defined in camera coordinate.

3.3 Equation of slope

The equation of slope is derived by calculating photometric ratio[11] of two transformed shading equations in order to eliminate unknown $I_0\rho$

$$\frac{L_1}{L_2}=\frac{(-\delta_{X1}p-\Delta_{Z1})}{(-\delta_{X2}p-\Delta_{Z2})}\frac{d_2^3}{d_1^3}, \tag{5}$$

and by solving it with respect to slope p on the assumption of known $z(\mu,\nu)$. The equations of slope becomes

$$p^D=\frac{L_1\Delta_{Z2}d_1^3-L_2\Delta_{Z1}d_2^3}{L_1\delta_{X2}d_1^3-L_2\delta_{X1}d_2^3}. \tag{6}$$

When single image is available, this becomes

$$p_i^S=\frac{B\pm\sqrt{B^2-AC}}{A}, \qquad for \quad I_0\rho=1. \tag{7}$$

$$A\equiv\delta_{Xi}^2-L_i^2d_i^6,\ \ B\equiv\delta_{Xi}\Delta_{Zi},\ \ C\equiv\Delta_{Zi}^2-L_i^2d_i^6.$$

This equation is useful when image contains self-shadow. The $\pm$ sign in eq. (7) caused by mirror solution can be eliminated easily by comparing the slopes p^D and p_i^S on shadowless zone.

3.4 Recurrence relation

Since equation of slope is based on the assumption of known depth, depth should be determined prior to slope. Recurrence relation coming from definition of mean slope enables to determine depth priorily.

In discrete image, mean slope is defined as

$$p(\mu-1,\nu)=\frac{z(\mu,\nu)-z(\mu-2,\nu)}{2}. \tag{8}$$

So, following recurrence relation is derived with natural boundary condition

$$z(\mu,\nu)=z(\mu-2,\nu)+2p(\mu-1,\nu), \tag{9}$$
$$z(0,\nu)\equiv 0,\ \ z(1,\nu)\equiv p(0,\nu).$$

By this relation, z and p become connected. Therefore, shape recovery become possible using eqs. (6) and (9) from observed two shade images.

3.5 Equation of ortho-image

Recovered shape at apparent position should be corrected using equation of observation in order to generate ortho-image.

For interpolation, inverse of eq. (3) is used.

$$\mu=[x]\frac{Z_o}{Z_o-z}, \qquad \nu=[y]\frac{Z_o}{Z_o-z}, \tag{10}$$

where the $([x],[y])$ denotes the position of each pixel in ortho-image expressed by integer value.

3.6 Compatibility and uniqueness

The proposed shape recovery process is compatible with orthographic condition. When distance to light sources and camera become infinity, eq. (4) becomes equivalent with eq. (1) because (μ,ν) become same with (x,y).

The uniqueness of recovered shape is guaranteed naturally because recovery is done mechanically with no ad hoc constraint. Accurate result is expected for ideal Lambertian object because this is exact solution.

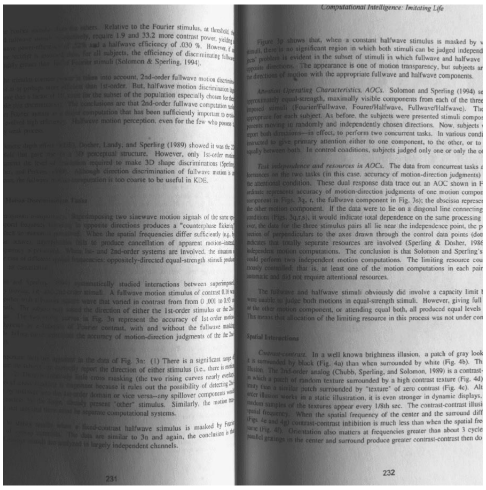

Figure 4: One of original shade image incident from right light source.

4 Implementations

Algorithms have been implemented for covering real situation which is more complex than ideal owing to non-constant albedos, self-shadows and interreflections.

4.1 Preprocessing

In preprocessing, separation of pure albedo and shade images is done and effect of interreflection is reduced.

4.1.1 Separation of albedo and shading

Pure albedo image $\rho(\mu, \nu)$ is generated by dividing sum of two shade image $I(\mu, \nu)$ into local mean intensity $\overline{I}(\mu)$

$$\rho(\mu, \nu) = \frac{I_1(\mu, \nu) + I_2(\mu, \nu)}{\overline{I_1(\mu) + I_2(\mu)}}. \tag{11}$$

Pure shade images $L_i(\mu, \nu)$ are generated by dividing each shade image into pure albedo

$$L_i(\mu, \nu) = \frac{I_i(\mu, \nu)}{\rho(\mu, \nu)}. \tag{12}$$

This separation is based on a idea that intensity variation by shading is global but that by albedo is local.

4.1.2 Reduction of interreflections

Interreflections are occurred mainly on folded part of book and are hard to remove because of many unknown parameters[7][8][9]. Therefore, this effect is modeled following global reflectance pattern $L_i^G(\mu)$ (bold line in fig. 5) and is removed

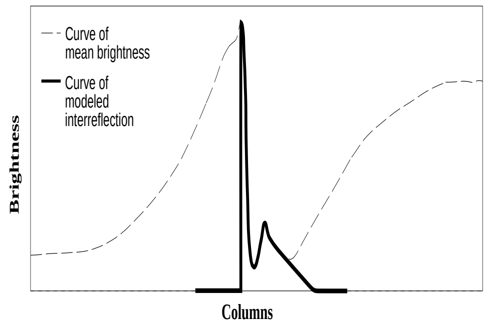

Figure 5: Modeled global interreflection pattern.

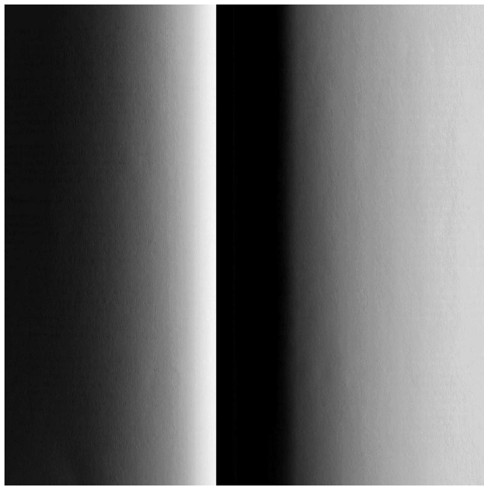

Figure 6: Generated pure shade image.

by calculating local effect. Local effect $L_i(\mu, \nu)$ is modeled as

$$L_i(\mu, \nu) \equiv a(\mu, \nu) L_i^G(\mu) + b(\mu, \nu), \tag{13}$$

and parameters (a, b) are calculated by minimizing error of fitting both sides of eq. 13.

Figure 6 shows pure shade image separated from fig. 4. Although this preprocessing is based on simple idea, acceptably reliable image of pure shade and albedo can be obtained.

4.2 Shape Recovery

Shape recovery can be done by proposed approach. However due to the self-shadow at margin and folded part, more comprehensive algorithm is required. This algorithm is based on a feed-back process which iteratively refines shape information.

Initial estimation of depth is done using eq. 6

Table 1: Results of simulation (512×512 image).

Case	1	2	3	4
Shape	$\frown$	$\frown$	$\frown$	$\frown\frown$
Shading	$\Vert\wedge\times$	$\Vert\wedge\times$	$\Vert\wedge\circ$	$\wedge\wedge\circ$
Recovery	$\Vert\,\Vert\times$	$\Vert\wedge\times$	$\Vert\wedge\circ$	$\wedge\wedge\circ$
Time (sec)	13	22	179	345
Error (%)	48.4	$2 \cdot 10^{-6}$	$7 \cdot 10^{-5}$	3.53

$$\text{Error } (\%) = \frac{|Z_{real} - Z_{recovered}|}{Z_{real}}$$

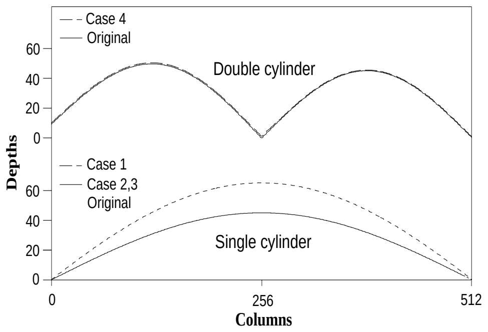

Figure 7: Simulation results of shape recovery.

by assuming no self-shadow. Using recovered information, zone over the line of incident light ray

$$p_i^L = \frac{\sqrt{\delta_{Xi}^2 + \delta_{Yi}^2}}{\Delta_{Zi}} \qquad (14)$$

is determined as shadowless and other part is done as self-shadowed. Within shadowless zone, adaptation of observed albedo to theoretical reflectance calculated by recovered shape is done. In next process, same way as initial calculation is done on shadowless zone, but p_i^S of eq. 7 is calculated by a shadowless image when another image is self-shadowed. Combining depth recovered from both zone, shadowed and non-shadowed, whole shape of book is produced, then this is used for refining albedo by adaptation, again.

This feed-back process iteratively refines shape of book. In folder part of book, where no direct light reaches, extrapolation of depth from known part is done. This feed-back process shall be converged within acceptable error when uniqueness and existence of SFS solution are guaranteed.

5 Experiments

Table 1 and fig. 7 show simulated results of shape recovery done by Pentium 133 MHz Unix system.

Table 2: Experimental environments.

Size of book	20.5 cm (998 pixel)
Pixel size	0.24 mm (59.7")
Light sources	$(\pm 109, 0, 48) cm$ $(\pm 4504, 0, 1993)$ pixel
Camera	$83 cm$ (3428 pixel)

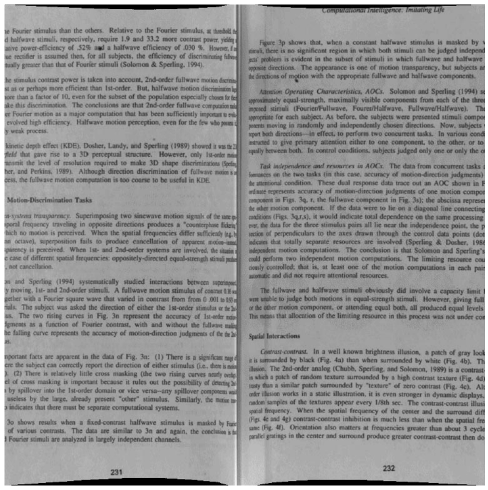

Figure 8: Finally obtained ortho-image.

Light sources are located at $(\pm 2000, 0, 4000)$ for *Case 1* and *2*, and at $(\pm 9000, 0, 4000)$ for *Case 3* and *4* in pixel unit. Camera is located at 4000.

The $\frown$and $\frown\frown$ symbols mean single and double cylinder respectively as shown in fig. 7. Three symbols in *Shading* and *Recovery* fields indicate status of camera, light sources and self-shadow respectively. Symbol $\Vert$ and $\wedge$ mean orthographic and perspective condition respectively, and symbol $\circ$ and $\times$ mean existence and non-existence of self-shadow respectively.

Case 1 shows high error because perspective light condition is ignored, whereas *Case 2* and *3* show near exact results. This means that ignoring perspective light condition gives high error even when distance is acceptably long (8.7 times than object size). *Case 4* shows acceptable error for double cylinder even in fully perspective condition. In comparing *Case 2* and *3*, it is clear that self-shadow is overcome successfully.

Environment of real experiment are shown in tab. 2. Pure albedo image is separated from observed shade images and apparent warp is corrected using recovered depth. This final ortho-

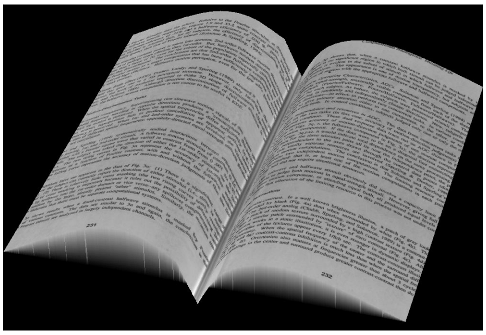

Figure 9: Three dimensional perspective view of book produced by recovered albedo and depth.

image of albedo is shown in fig. 8. When it is compared to fig. 4, reliability of separating albedo and shading, and that of depth recovery can be easily acknowledged. In order to show recovered informations simultaneously, image of three dimensional perspective view is generated and is shown in fig. 9.

Results of simulation and real experiment show accuracy and reliability of proposed approach.

6 Conclusions

A strategy and solution in shape recovery of book surface from shading on fully perspective condition is proposed in this paper.

Following self-estimation is done.

First, proposed strategy is a proper way in solving fully perspective SFS problem. It is more efficient in time and is more probable to obtain unique and exact solution than frequently used optimizing strategy (see Zhang[12] for review). Second, since derived equations are compatible with orthographic condition, these may be used as a subset of other SFS approaches. Third, due to the recurrence relation which propagates recovered shape information, proposed approach can cause hard error when there is severe quantization error during observation. Finally, for applying to shape recovery of general object, another constraint or invariance should be added. This one must be orthogonal to solution space of proposed approach which is spanned by slope p.

The proposed approach is expected useful in constructing virtual library of *rare books* where the restriction of non-contact observation is highly required.

References

[1] C. Cho and H. Minamitani, "Obtaining 3-D Shape from Silhouette Informations Interpolated by Photometric Stereo", MVA, pp.147-150, 1994.

[2] B.K.P. Horn and R.W. Sjoberg, "Calculating the Reflectance Map", Applied Optics, Vol.18, No.11, pp.1770-1779, 1979.

[3] K. Kanatani and T.C. Chou, "Shape from Texture : General Principle", Artificial Intelligence, Vol.38, pp.1-48, 1989.

[4] K. Kanatani, "Computational Projective Geometry", CVGIP (IU), Vol.54, No.3, pp.333-348, 1991.

[5] B. Kim and P. Burger, "Depth and Shape from Shading Using the Photometric Stereo Method", CVGIP (IU), Vol.54, No.3, pp.416-427, 1991.

[6] K. M. Lee and C.-C. J. Kuo, "Shape from Shading with Perspective Projection", CVGIP (IU), Vol.59, No.2, pp.202-212, 1994.

[7] W. S. Macchi, N. V. Lobo, "Interreflections With Rough Surfaces", ICPR, pp.602-605, 1994.

[8] S. K. Nayar, K. Ikeuchi and T. Kanade, "Shape from Interreflection", IJCV, Vol.6, No.3, pp.75-104, 1991.

[9] H. Ukida, T. Wada and T. Matsuyama, "3D Shape Reconstruction of Unfolded Book Surface from A Scanner Image", MVA, pp.409-412, 1994.

[10] T. Wada, H. Ukida, and T. Matsuyama, "Recovering 3D Shape of Unfolded Book Surface from a Scanner Image (II)", Trans. IEICE, Vol.J78-D-II, No.2, pp.311-320. 1995. (Japanese)

[11] L.B. Wolff and E. Angelopoulou, "Three-Dimensional Stereo by Photometric Ratio", JOSA (A), Vol.11, No.11, pp.3069-3078, 1994.

[12] R. Zhang, P-S. Tsai, J. E. Cryer and M. Shah, "Analysis of Shape from Shading Techniques", Technical Report, Univ. of Central Florida, 78p, 1994.