Control of an under actuated unstable nonlinear object

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Abstract: This paper presents a comprehensive comparative study of several nonlinear controllers for stabilisation of the under actuated unstable nonlinear object known as the Acrobot in the literature. The object is a two DOF robot arm only actuated at the elbow. The study compares several control algorithms from the literature and a new algorithm developed during the study. The comparison is based on both simulation and real experiments for all controllers.

1. Introduction.

During the last decade the Acrobot has been used as example object in several studies of non linear control. There are several reasons for this. First of all having only two degrees of freedom the complexity of the equations of motion is low enough to allow analytical results of the proposed non linear methods and in spite of the low order of the object the non linearities are very decisive for the behavior which means that classical LQR-controllers designed using a linearization of the object have a very limited region of stability around the linearization point. Secondly stabilizing the Acrobot can be seen as a prestudy to bipedal walking machines as it simulates balancing of a person having no feet (standing on stilts) using only hip movements. As mentioned above the Acrobot has been the subject of several studies but most of these present only one algorithm and often only theoretical or simulation results. The main purpose of this study is to perform systematic comparison of several algorithms for stabilisation of the acrobot using both simulation and real experiments. The importance of testing different algorithms on the same physical acrobot is emphasized by the fact that the actual configuration of the acrobot (lengths and masses of the two arms) has great influence of the performance of different controllers thus making it difficult to compare results obtained on different physical systems directly. In some configurations the performance of a linear controller is equal to the performance of some non linear controllers.

The outline of the paper is as follows. In section 2 the basic equations for the acrobot are given and the difference between torque control and acceleration control is explained. In section 3 a short description of the five investigated control algorithms is presented along with simulation results for the ideal Acrobot i.e. simulation without physical constraints of the actuator and measuring systems and feedback from all states. Section 5 contains simulations of the full system including the dynamics and constraints of the actuator and quantization of the angle measurements and sampling of the digital controller. It also contains the measurements on the real system. Section 6 gives a description of the experimental setup and describes the implementation of the acceleration control.

2. The Acrobot.

The Acrobot is a two DOF planar robot. The first arm is connected to the fixed environment with a rotational joint, joint 1. The second arm is connected to the first arm through a second rotational joint, joint 2. Only joint 2 is actuated which means that the torque of joint 1 is zero. Figure 1 shows the acrobot with the definitions of the joint angles and some basic physical parameters.

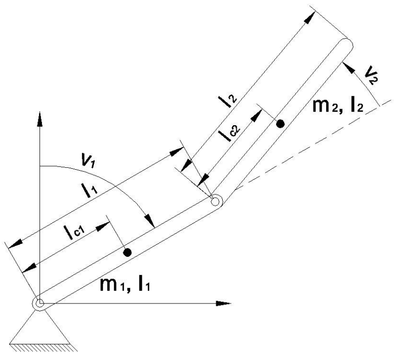

Figure 1. Acrobot model

As the torque of joint 1 is zero the equations of motion considering the torque of joint 2 as input are

$$
\begin{aligned}
-d_{11}\ddot{v}_1 + d_{12}\ddot{v}_2 + h_1 + \phi_1 &= 0 \\
-d_{21}\ddot{v}_1 + d_{22}\ddot{v}_2 + h_2 + \phi_2 &= \tau
\end{aligned}
\tag{1}
$$

Where

$$
\begin{aligned}
a_1 &= m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2) + I_1 + I_2 \\
a_2 &= m_2 l_1 l_{c2} \\
a_3 &= m_2 l_{c2}^2 + I_2 \\
a_4 &= g(m_1 l_{c2} + m_2 l_1) \\
a_5 &= g m_2 l_{c2}
\end{aligned}
\qquad
\begin{aligned}
d_{11} &= a_1 + 2a_2\cos(v_2) \\
d_{22} &= a_3 \\
d_{12} &= d_{21} = a_3 + a_2\cos(v_2) \\
h_1 &= -a_2\sin(v_2)(\dot{v}_2^2 - 2\dot{v}_1\dot{v}_2) \\
h_2 &= a_2\sin(v_2)\dot{v}_1^2 \\
\phi_1 &= a_4\sin(v_1) + a_5\sin(v_1 - v_2) \\
\phi_2 &= a_5\sin(v_1 - v_2)
\end{aligned}
\tag{2}
$$

If the angular acceleration $\ddot{v}_2$ is considered as input the equations reduce to

$$
\begin{aligned}
-d_{11}\ddot{v}_1 + d_{12}u + h_1 + \phi_1 &= 0 \\
\ddot{v}_2 &= u
\end{aligned}
\tag{3}
$$

which is the form used in this study. This can be thought of as a mere mathematical reformulation of the problem as the torque can be calculated knowing u and the present state of the Acrobot but it can also be seen as a change of control paradigm from torque control to acceleration control with great impact on the achievable control performance. The acceleration control of joint two can be implemented with a high precision position servo calculating position and speed references digitally from the acceleration given by the controller. Using an analog high bandwidth speed controller in the position servo makes it possible to suppress ill described non linearities such as coulomb friction along with some other non linearities thus obtaining a control object which is near to the mathematical description assuring a greater likelihood for success with the non linear controllers found based on the ideal equations.

Several authors have found that the actual physical configuration of the the acrobot has great impact on the influence of the non linearities on the control. Timcenko [1] found that if $a_1 = 2\ a_3$ her non linear controller was equal to a linear controller. In this study the physical dimensions have been chosen so that this equality is far from fulfilled thus emphasizing the non linearities of the Acrobot.

3. The investigated stabilisation algorithms.

In this section a short description of the investigated algorithms are given. For a in depth presentation the reader is referred to the references. Five different algorithms have been investigated. For each controller a region of attraction (ROA) and initial state time response are shown. These are based on simulation results with the ideal acrobot. The ROA is the 4 dimensional space of initial states from which the controller is able to stabilize the system. As the ROA is found by simulation only the two dimensional slice with initial angular speeds equal to zero is found.

3.1. LQR-controller.

The system has been linearized at the vertical equilibrium point and a classical LQR-controller has been designed. The performance of this controller is used to emphasize the poor results of a linear controller for the non linear system and as a reference for the improvements obtained by the non linear controllers. Figure 2 shows the time response for the largest value of v_1 in the ROA and it is seen that the linear controller only is able to stabilize the system from 1.2 deg. This is also seen in figure 3 which shows a very small ROA.

3.2. Pseudolinearization.

Pseudolinearization is described in [2]. The method is used for systems with more equilibrium points. This makes it suited for the Acrobot. Bortoff [3,4] states that it is not possible to pseudolinearize the Acrobot, so he presents a further development of the method where numerical approximation with splines are used. The method is demonstrated on the Acrobot. In this study an analytical solution to the pseudolinearization of the Acrobot has been found. This has been possible due to the simpler equations of motion obtained considering the angular acceleration of v_2 as input instead of the torque of joint two. The algorithm has been tested by simulation and

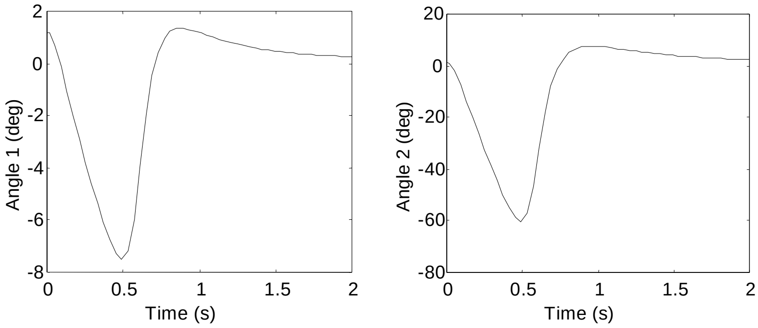

Figure 2. Initial state time response of the LQR controller

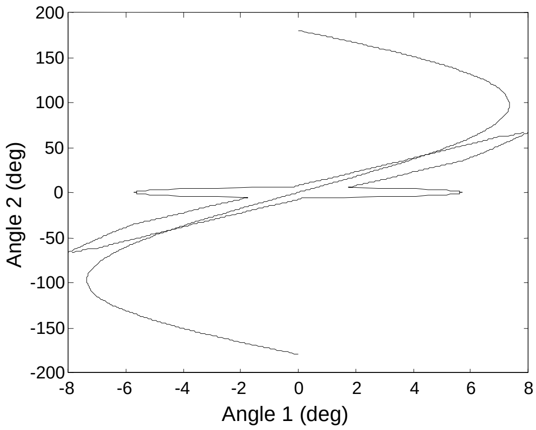

Figure 3. ROA for the LQR controller

experiment. Figure 4 shows an initial state time response for the largest possible value of v_1 = 3.4 deg. It is seen that this angle is three times greater than for the linear controller.

3.3. Partial feedback linearization and linearization of the remaining non linear system.

Timcenko [1] and Olfati-Saber and Megretski [5] describe a solution where a non linear transformation is found that transforms the system to another ‘less non linear system’. This system is then linearized and a LQR controller is designed for the linearized system. The method has been tested by simulation in these investigations. In this study the method has been used on the acceleration controlled Acrobot and tested both by simulation an experiment. Figure 5 shows an initial state time response for the largest possible value of v_1 = 4.6 deg. It is seen that this angle is four times

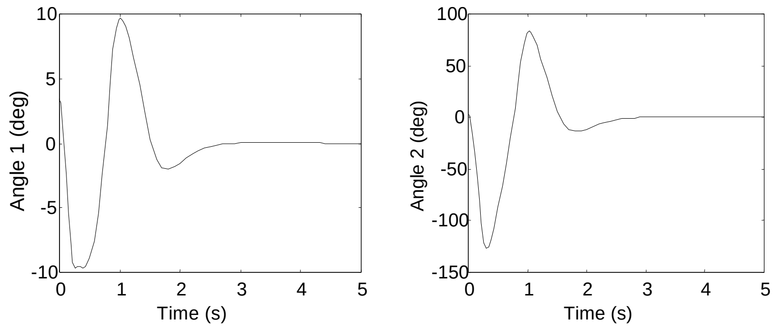

Figure 4. Initial state time response of the pseudolinearized controller

greater than for the linear controller. The ROA in figure 6 shows this improvement is general for all initial values.

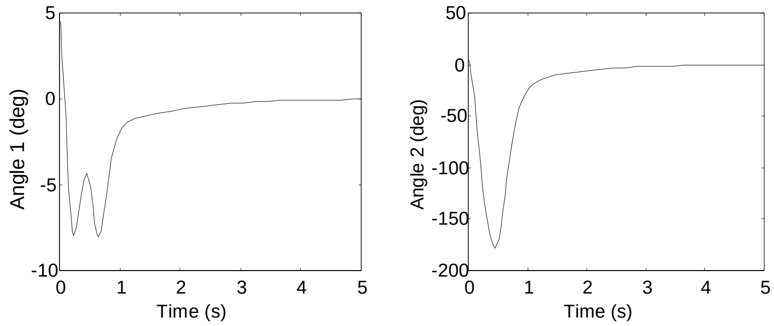

Figure 5. Initial state time response of the Timcenko controller

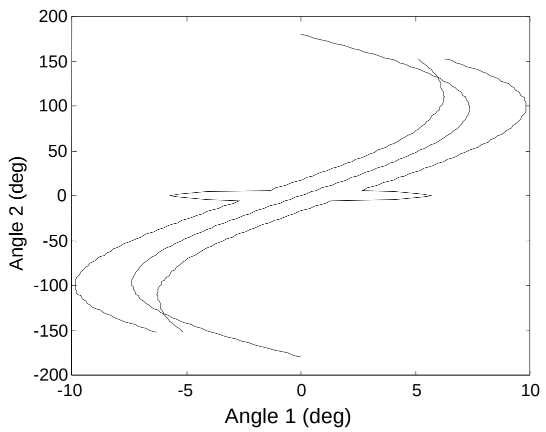

Figure 6. ROA for the Timcenko controller

3.4. Partial feedback lineariztion with relative degree 2, unstable zero dynamics.

De Luca and Oriolo [6] proposes a three step controller for the Acrobot. First step controls the swing up until the lower arm is pointing upwards, the second step brings the system near to the equilibrium manifold and third step stabilizes the system in vertical position. In the second step they use a partial feedback linearization with relative degree 2. They do not analyze the zero dynamics of the system. The simulation in the present study shows that it is unstable so this controller has not been tested experimentally.

3.5. Partial feedback linearization with relative degree 3, stable zero dynamics.

As it has been proved that the Acrobot is not feedback linearizable the maximum relative degree for an output function is three. Following the methods of [7] an output function with relative degree 3 and stable zero dynamics has been found. The reason for success probably is the reduced complexity of the acceleration controlled system. Based on this transformation a controller has been designed and tested both with simulation and experiment. Figure 7 shows a time response for v_1 = 10 degrees. This is not the maximum value but is chosen because it is the maximum value for the experimental system limited by the actuator constraints. Figure 8 shows the theoretical ROA of the new controller. This shows a dramatic improvement of approximately a factor 40 compared to the linear controller.

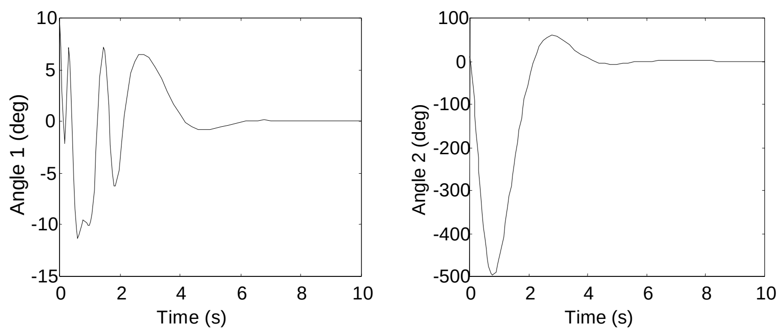

Figure 7. Initial state time response of the new controller

4. Comparison based on simulation and experiments.

All the controllers have been tested using both simulation and experiment except 4 which has only been tested by simulation. The reason for not implementing 4 experimentally was that the simulation showed that the zero dynamics of the system was unstable thus ending with a very high angular velocity $\dot{v}_2$.

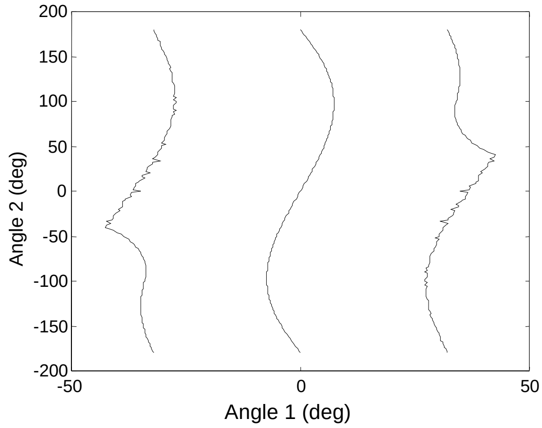

Figure 8. ROA of the new controller

In the experiments the special start configuration with $v_1=v_2$ has been used and the largest value from which the controller can stabilize the system has been found. The results are shown in table 1.

TABLE 1. Catching zone

Controller	Theoretical	Simulated	Measured
Linear	1.2^o	-1.0^o-1.5^o	-0.8^o-1.3^o
New	41^o	10^o	12^o
New (-180)	4.5^o	5.2^o	7.0^o
Timcenco	4.6^o	5.6^o	5.6^o
Bortoff	3.4^o	3.4^o	4.0^o

The third row, new (-180) is the limitation of the new controller if v_2 must stay within +- 180 degrees. The values demonstrate a good correspondence between theory (simulation of ideal system), simulation (with physical constraints) and measurements. The big difference between theoretical and simulated values for the new controller is due to motor constraints in the physical system. It is clear that all the non linear controllers have much better performance than the linear controller and that the new controller is clearly the best from a ROA view. Figure 9 to 12 show simulated (thin line) and measured (bold line) initial state time responses for the four stable controllers. Simulation and experiments have shown that the exact form of the time response is very sensitive to small moments on joint 1, so the differences between measured and simulated response is probably mostly due to the moment caused by the wires on the acrobot.

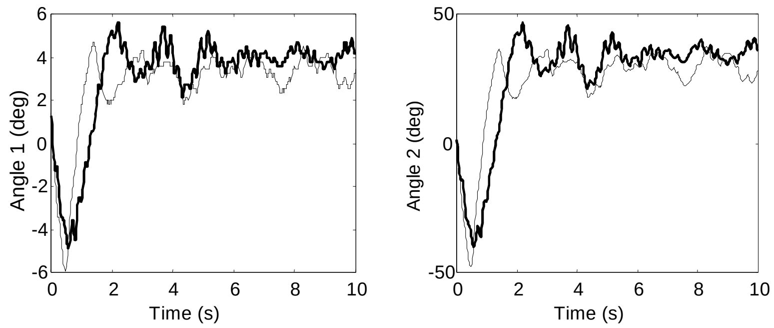

Figure 9. Measured and simulated output of the linear controller

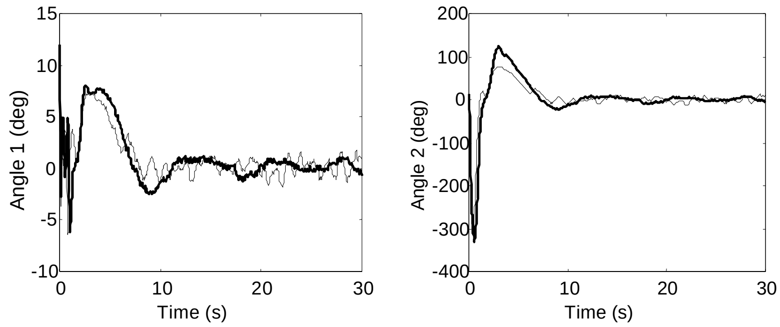

Figure 10. Measured and simulated output of the new controller

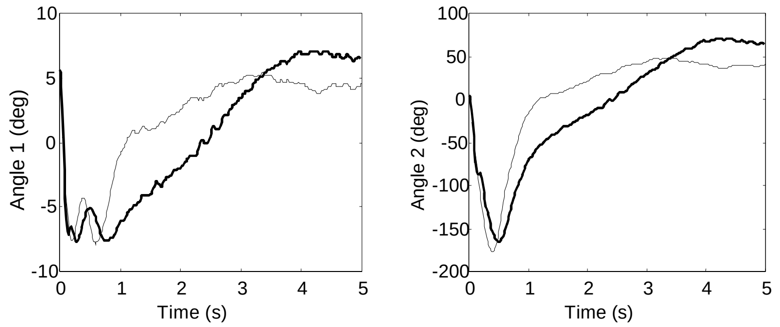

Figure 11. Measured and simulated output of the Timcenco controller

5. Experimental setup.

The implemented acrobot is shown on Figure 13. The lower arm consists of two parts between which the upper arm swings. The advantage of this is that the upper arm will not cause any torsion moment on the lower arm during operation thus allowing a

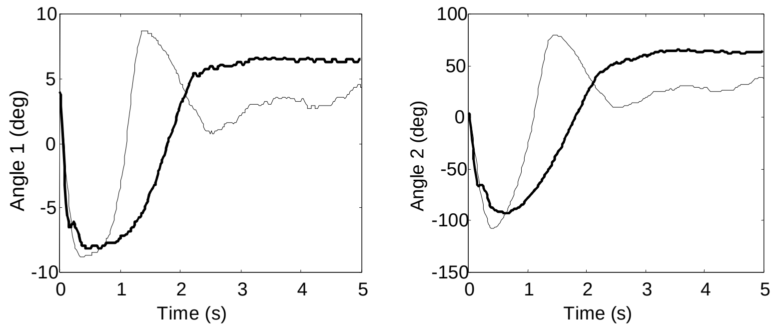

Figure 12. Measured and simulated output of the Bortoff controller

lighter construction. The disadvantage is that the upper arm must be shorter than the lower thus limiting the possible configurations in the investigation. The physical parameters of the Acrobot is given in table .

TABLE 2. Parameters for the Acrobot

	Mass	Inertia	Length
Arm 1	3.49 kg	$91.3\ 10^{-3}$ kgm^2	438 mm, 102 mm
Arm 2	1.03 kg	$17.5\ 10^{-3}$ kgm^2	410 mm, 100 mm

The joint between the upper and the lower arm is driven by a 12 V DC-motor (Minimotor type 3757 CR) with tachogenerator through a two stage tooth belt drive giving a total gearing ratio of 20. The tooth belt drive has been chosen because of its low backlash. The motor and the first drive stage is mounted on the lower arm near joint 1 to reduce the forces on this joint. The angles of joint 1 and joint 2 are measured by optical encoders with 500 lines thus giving a resolution of 0.18 degrees. The reference for the power amplifier is given using a 12-bit DA-converter. The controllers are implemented in C on a 100 MHz 80486DX4 PC using a real time operating system developed at Institute of Automation based on OSkit. A sample frequency of 64 Hz is used acceleration control is implemented by finding speed and position references digitally from the acceleration given by the control algorithm. These references are the given to a high precision position servo based on the encoder feedback and a high bandwidth analog speed controller using the motor tacho feedback.

6. Conclusion.

Five algorithms for stabilization of the Acrobot have been investigated. Three of these have been presented previously by other authors, but two of them have been verified experimentally for the first time in this study. A new algorithm based on partial feedback with relative degree of 3 and stable zero dynamics has been developed an tested both by simulation and experiment and shows far superior performance with respect to stability region (ROA).

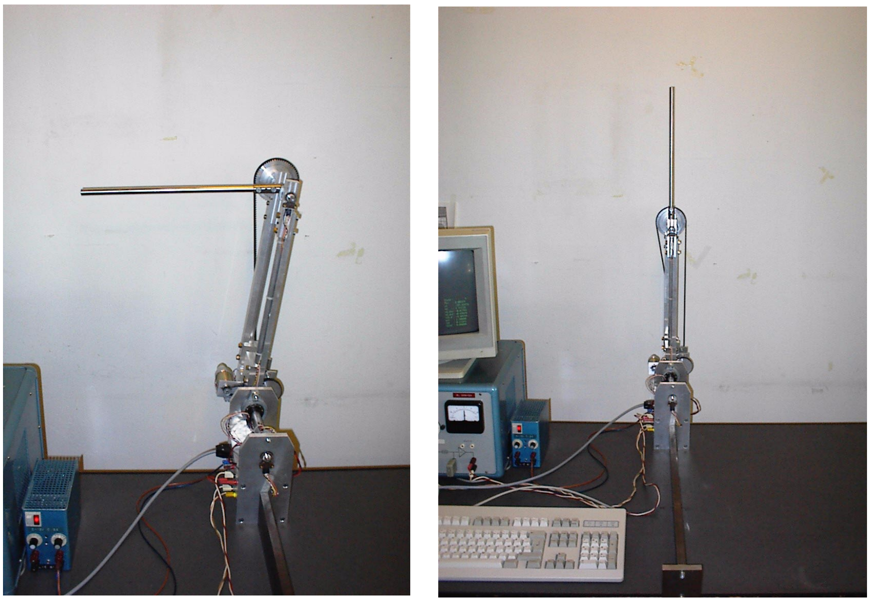

Figure 13. The Acrobot

7. References.

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